SDSS Profile Extraction

Robert Lupton & Jim Gunn

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1 What is that Photo Measures?

The question of what it is that photo measures is a little subtle.

Let us consider an image above the atmosphere; if it's a star it'll simply be a delta function. On passing through the atmosphere and the telescope the input image is convolved with the PSF, and upon falling upon the CCD is it further convolved with the pixel response function, and sampled at a discrete set of points.

The PSF dramatically curbs the images high spatial frequencies, and results in a (nearly) band-limited signal.

The pixel response function has its first zero at twice the Nyquist frequency for marginally sampled data, so it leads to some suppression of high spatial frequencies but nothing too catastrophic. It does, however lead to images that have a four-fold symmetry rather than the presumed circular symmetry of the PSF (if we take the TDI mode of taking SDSS data into account, the symmetry is only two-fold).

Because the post-PSF signal is band-limited, and because the pixels are small enough to properly sample it, the discrete sampling leads to no loss of information; we are able to recover the exact value of the PSF-and-Pixel convolved image at any point in the chip — but note that it is *this* image, not the image incident on the camera, that we are able to recover and measure. Any attempt to undo the effects of pixellation is a deconvolution with all the attendent problems and uncertainties.

2 Sector-Array Profile Extraction

(In the following discussion, we assume that the object is centred on the centre of a pixel; the real world is discussed in section 3).

One of the fundamental properties of an object is its radial profile. Unfortunately, individual pixels in the outer parts have too low a signal to noise to be useful, so some sort of binning of the data is needed. The size of the bins should clearly increase with radius so that the signal to noise ratio is approximately constant as a function of radius. For flattened objects, we are also interested in the distribution of flux with position angle.

The way that photo extracts radial profiles is to divide an object into 169 *cells* — 15 annuli, each divided into 12 sectors. [1] The complete collection of cells is referred to as a *sector array.*

The boundaries between annuli are spaced approximately exponentially in radius (the ratios of inner to outer radii is approximately $1.25/0.8 \approx 1.56$). In the inner parts the pixellation of the image complicates matters somewhat; tables 1 and 2 show the assignment of pixels to cells. The ith (0—14) annulus consists of cells $12(i-1)...12i$. In the tables, each number (or comma-separated pair) corresponds to a single pixel, and the number is the cell to which it belongs. Cell 0 is the entire innermost annulus, which contains a single pixel. In some cases pixels are divided between cells; this is indicated in table 2 by comma-separated pairs of cell numbers (for example, the pixel to the north east of the central pixel is assigned to both cells 8 and 9).

The number of pixels in each annulus, and the radius of the circular aperture containing the same number of pixels, are given in table 3. [2]

3 How Photo Fills the Sector Arrays

Section 2 discussed how photo assigns pixels to cells if the object under consideration happens to be positioned at the centre of a pixel; here we deal

[1] All except the innermost annulus, that is, which is a small undivided disk at the centre of the object: $1 + 14 \times 12 = 169$.

[2] The outer radius of about 653 pixels comes from the requirement that there be no more than 2^{16} pixels in a single cell; this is set by the choice of short accumulator arrays in the profile extraction code and could be relaxed if there were sufficient demand, but it's probably simpler (and faster) to bin the image and then use the current code.

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						59	59	59	58	58	58	58	58	57	57	57						
				59	59	59	59	59	58	58	58	58	58	57	57	57	57	57				
			59	59	59	59	59	59	58	58	58	58	58	57	57	57	57	57	56			
		60	60	59	59	59	59	59	47	46	46	46	45	57	57	57	57	56	56	56		
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	60	60	60	60	48	48	47	47	47	34	34	34	45	45	44	44	44	56	56	56	56	
60	60	60	60	60	48	48	48	35	35	35	34	33	33	32	44	44	44	56	56	56	56	56
49	49	49	49	48	48	48	48	36	35	23	22	21	32	32	44	44	44	44	55	55	55	55
49	49	49	49	37	37	37	25	36	24	11	10	8	20	32	31	43	43	43	55	55	55	55
49	49	49	49	37	37	37	25	25	13	1	0	7	19	31	31	43	43	43	55	55	55	55
49	49	49	49	37	37	37	25	26	14	2	4	5	18	30	31	43	43	43	55	55	55	55
49	49	49	49	38	38	38	38	26	26	15	16	17	29	30	42	42	42	42	55	55	55	55
50	50	50	50	50	38	38	38	26	27	27	28	29	29	29	42	42	42	54	54	54	54	54
	50	50	50	50	38	38	38	39	39	28	28	28	41	41	41	42	42	54	54	54	54	
	50	50	50	50	50	38	39	39	39	40	40	40	41	41	41	41	54	54	54	54	54	
		50	50	50	50	51	39	39	39	40	40	40	41	41	41	53	53	54	54	54		
		50	50	50	51	51	51	51	39	40	40	40	41	53	53	53	53	53	54	54		
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				51	51	51	51	51	52	52	52	52	52	53	53	53	53	53				
						51	51	51	52	52	52	52	52	53	53	53						
								51	52	52	52	52	52	53								

Table 1: The assignment of pixels to cells for the innermost six annuli.

58	58	58	58	58	57								
58	58	58	58	58	57	57	57						
58	58	58	58	58	57	57	57	57	57				
58	58	58	58	58	57	57	57	57	57	57,56			
	46	46	46	45	57	57	57	57	57,56	56	56		
	46	46	46	45	45	45	57	57,56	56	56	56		
	46	46	46	45	45	45	45,44	56	56	56	56	56	
	34	34	34	45	45	45,44	44	44	56	56	56	56	
		34	33	33	32,33	44	44	44	56	56	56	56	56
		22	21	32,33	32	44	44	44	44	55	55	55	55
		10	8,9	20	32	31	43	43	43	55	55	55	55
		0	7	19	31	31	43	43	43	55	55	55	55
						31	43	43	43	55	55	55	55
										55	55	55	55

Table 2: The assignment of pixels to cells for the first quadrant of the innermost six annuli. The boundaries between the cells are indicated.

with the real case where the object's centre is at some arbitrary place within a pixel.

The inner and outer parts of the object are treated differently. The inner parts are sinc-shifted to place the centre of the object at the centre of a pixel, after which the cell assignments of table 1 are used; in the outer parts pixels are assigned to the cell in which their centres lie (it would take far too long to sinc shift the entire object). Table 3 indicates which method is used for each annulus.

The pixel assignment in the inner parts is independent of the object's position within a pixel (if we neglect the non-band-limited nature of Poisson noise), whereas the assignment in the outer parts is somewhat arbitrary. As an object moves around, the effective boundaries of the outer annuli vary a little (by maybe a twentieth of a pixel), depending on exactly which pixel falls into which cell.

This leads to problems at the interface between the two techniques, leading to the double- or non- counting of flux in some pixels (you can think of the central region as being a circular disk that is smoothly moved around relative to the pixellation, while the outer parts are tied to the pixels).

It appears that the effect is at worst a couple of percent in the total flux, but there is the possibility of systematic differences between large and small

Annulus	Cell	Sinc?	r_{inner}	r_{outer}	A_{ann}	A_{disk}
0	0	y	0.00	0.56	1	1
1	1	y	0.56	1.69	8	9
2	13	y	1.69	2.59	12	21
3	25	y	2.59	4.41	40	61
4	37	y	4.41	7.51	116	176
5	49	y	7.51	11.58	244	421
6	61	n	11.58	18.58	664	1085
7	73	n	18.58	28.55	1476	2561
8	85	n	28.55	45.50	3944	6505
9	97	n	45.50	70.51	9114	15619
10	109	n	70.51	110.53	22762	38381
11	121	n	110.53	172.49	55094	93475
12	133	n	172.49	269.52	134732	228207
13	145	n	269.52	420.51	327318	555525
14	157	n	420.51	652.50	782028	1337553

Table 3: Radii of boundaries of annuli, and number of pixels involved. A_{ann} is the area of the annulus, and A_{disk} is the area of the disk out to r_{outer}. The second column gives the first cell in the annulus, and the third indicates if the values in that annulus are derived from sinc shifting the image to centre it on a pixel.

objects.

Photo corrects for this effect by looking at the total flux from the inner parts, *and* separately finding the total flux that isn't assigned to any cell in the outer parts. It then adjusts the mean flux levels in the first 'outer' annulus (annulus 6), so that the "missing flux" is accounted for. This can be thought of, I believe correctly, as effectively adjusting the annulus's inner boundary to be at its nominal position.

4 Extraction of Exact Sector Means using Sinc Interpolation

As discussed in the document *Aperture Photometry in Band Limited Data*, for band limited data it is possible to define weights that allow us to determine the integral of the measured intensity distribution over a any area, including a sector defined by two radii and two angles. For the inner part of the sector arrays (the sinc shifted annuli), we have used these weights to find mean fluxes rather than the definitions of sectors given above.

Note well that these "exact" sector means are exact in the sense that they are the true mean of a sector laid down on the PSF-and-pixel convolved image that the imaging camera measures.

5 The Determination of Radial Profiles and Errors from Cell Arrays

Given measurements in a complete set of cells, how should we estimate the mean, median, and variance of the image in annuli?

The case of the mean is straightforward; add up all of the counts within the annulus and divide by the total number of pixels. If a clipped mean is desired, first reject the cells that are to be clipped, and then add up the counts in the remaining cells.

It is not obvious what we mean by the median of an annulus. The simple median of all the pixels is not what is desired (e.g. for a flattened system, the median of the annulus is close to sky and very insensitive to the object under study). We have chosen to use the (possibly clipped) mean of the medians of the cells within the desired annulus to define a 'median' value; in this way we preserve most of desired insensitivity of the median to neighbouring objects, without badly underestimating the profile of flattened objects.

If the object under study is not circularly symmetrical, a straightforward estimate of the variance as

$$\frac{1}{n} \cdot \frac{n}{n-1} \cdot \frac{1}{n} \sum_i (v_i - \bar{v})^2$$

or

$$\frac{1}{n} \cdot \frac{n}{n-1} \cdot \frac{1}{\sum_i \sigma_i^2} \sum_i \frac{(v_i - \bar{v})^2}{\sigma_i^2}$$

includes a contribution from the object's radial profile. Under some circumstances this may be what is desired, but for fitting radial profiles (as used in determining Petrosian quantities) it leads to drastic overestimates, which in turn make the profile seem less well determined towards the centre, rather than out in the wings where the sky dominates.

An alternative definition of the variance is given by defining

$$\begin{aligned} T &\equiv \sum_{i=0}^{n} \left(v_i - \frac{1}{2}(v_{i+1} + v_{i-1}) \right)^2 \\ &= \frac{3}{2} \sum_i v_i^2 - 2 \sum v_i v_{i+1} + \frac{1}{2} \sum v_{i-1} v_{i+1} \end{aligned}$$

where we interpret $v_0 \equiv v_n$ and $v_{n+1} \equiv v_1$. If we assume that $v_i = V + \epsilon_i$ with $\langle \epsilon_i \rangle = 0$ and $\langle \epsilon_i \epsilon_j \rangle = \sigma^2 \delta_{ij}$ we find that

$$\begin{aligned} \langle T \rangle &= \frac{3n}{2} \left(V^2 + \epsilon^2 \right) - 2nV^2 + \frac{1}{2} nV^2 \frac{1}{2} \sum v_{i-1} v_{i+1} \\ &= \frac{3n}{2} \sigma^2 \end{aligned}$$

so we may define $\tilde{\sigma}^2 = 2/3n\,T$ with

$$\langle \tilde{\sigma}^2 \rangle = \sigma^2$$

and

$$\mathrm{Var}(\tilde{\sigma}^2) = \frac{1}{n} \left(\langle \epsilon^4 \rangle + \frac{8}{9} \sigma^4 \right)$$

for a Gaussian $\langle \epsilon^4 \rangle = 3\sigma^4$, and this becomes

$$= \frac{35}{9n} \sigma^4,$$

almost exactly twice that for the usual estimate (which has variance $2\sigma^4$).

One could obviously define higher-order analogues of T which would remove more of a smoothly varying background (and have higher variance), but experiment shows that $\tilde{\sigma}^2$ defined above works well in practice.

For example, if we write $v_i = V + A\sin(2\pi i/n) + \epsilon_i$,

$$\begin{aligned}\langle\tilde{\sigma}^2\rangle &= \sigma^2 + \frac{2}{3}A^2\left(\frac{3}{4} - \cos\frac{2\pi}{n} + \frac{1}{4}\cos\frac{4\pi}{n}\right)\\ &\approx \sigma^2 + \frac{4\pi^2A^2}{3n^4}\end{aligned}$$

or $\sigma^2 + (A/9.1)^2/2$ for $n = 12$. The estimated variance using a single mean for the entire annulus is of course $\sigma^2 + A^2/2$, so the use of a local mean has reduced the effect of elliptical isophotes by a factor of nearly 10.