Errors in Petrosian Quantities

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1 Introduction

The SDSS has decided to use Petrosian quantities to characterise galaxies. Section 2 defines them, section 3 analyses their statistical properties, and a discussion follows in section 4

2 Definitions

Let $I(r)$ be the measured azimuthally averaged surface brightness profile of an object in r'. Define the Petrosian ratio $\mathcal{R}_P(r)$ as the ratio of the local surface brightness at radius r to the mean within r:

$$\mathcal{R}_P \equiv \frac{\int_{\alpha r}^{\beta r} I(r') 2\pi r'\, dr' / [\pi(\beta^2 - \alpha^2) r^2]}{\int_0^r I(r') 2\pi r'\, dr' / [\pi r^2]}.$$

We have provisionally adopted $\alpha = 0.8$ and $\beta = 1.25$.

The Petrosian radii are the values r_i where $\mathcal{R}_P$ falls to a specified value f_1 (strawman value 0.125).

If there's more than one such radius, the largest of the r_i will be taken as the Petrosian radius r_P.

The Petrosian flux F_P is defined as the total flux as measured within a certain number of Petrosian radii:

$$F_P = \int_0^{\gamma r_P} I(r') 2\pi r'\, dr'.$$

The Petrosian half-light r_{50} is defined by the implicit equation:

$$\int_0^{r_{50}} I(r') 2\pi r' \, dr' = 0.5 F_P.$$

The Petrosian 90% radius is defined by the implicit equation:

$$\int_0^{r_{90}} I(r') 2\pi r' \, dr' = 0.9 F_P.$$

3 Statistical Properties of Petrosian Quantities

Let us define

$$C(r) \equiv \int_0^r I(r') 2\pi r' \, dr'$$

so we may write the Petrosian ratio as

$$\mathcal{R}_P \equiv \frac{1}{\beta^2 - \alpha^2} \frac{C(\beta r) - C(\alpha r)}{C(r)}$$

We can write $I(r)$ as the sum of the true intensity $I_0(r)$ and a noise term:

$$I(r) = I_0(r) + d(r)$$

with the noise d having $\langle d(r) \rangle = 0$ and $\langle d(r) d(s) \rangle = \epsilon(r)^2 \delta(r - s)$. We then have:

$$\begin{aligned}
\langle C(r) \rangle &= \int_0^r I_0(r') 2\pi r' \, dr' \\
&\equiv C_0(r) \\
\mathrm{Var}(C(r)) &= \int_0^r \epsilon^2(r') 2\pi r' \, dr' \\
&= C_0(r) + \pi r^2 S
\end{aligned}$$

where the last line follows from assuming Poisson statistics with a sky level of S. Two Cs are correlated as they share some pixels, so

$$\mathrm{Covar}(C(ar)C(br)) = \begin{cases} C_0(br) + \pi r^2 S b^2 & \text{if } b \le a \\ C_0(ar) + \pi r^2 S a^2 & \text{otherwise} \end{cases}.$$

If we consider the quantity $R \equiv x/y$, to linear order its mean and variance are given by

$$\langle R \rangle = \frac{\mu_x}{\mu_y}$$

and

$$\mathrm{Var}(R) = \frac{\mu_x^2}{\mu_y^2}\left(\frac{\sigma_x^2}{\mu_x^2} - 2\frac{\sigma_{xy}}{\mu_x\mu_y} + \frac{\sigma_y^2}{\mu_y^2}\right)$$

where μ_x, σ_x^2, μ_y, ans σ_y^2 are the means and variances of x and y, and σ_{xy} is the covariance of x and y. (a more complete discussion is given in my note *The Estimation of Object's Ellipticities*).

In the case of the Petrosian ratio $\mathcal{R}_P$,

$$x \equiv \frac{1}{\beta^2 - \alpha^2}\left(C(\beta r) - C(\alpha r)\right)$$

and

$$y \equiv C(r)$$

so

$$\langle R_P \rangle = \frac{1}{\beta^2 - \alpha^2}\frac{C_0(\beta r) - C_0(\alpha r)}{C_0(r)}$$

and

$$\begin{aligned}\mathrm{Var}(R_P) = {} & \frac{1}{(\beta^2 - \alpha^2)^2}\frac{\left(C_0(\beta r) - C_0(\alpha r)\right)^2}{C_0(r)^2} \times \\ & \left(\frac{C_0(\beta r) - C_0(\alpha r) + \pi r^2 S(\beta^2 - \alpha^2)}{\left(C_0(\beta r) - C_0(\alpha r)\right)^2} - \right. \\ & \left. 2\frac{C_0(r) - C_0(\alpha r) + \pi r^2 S(1 - \alpha^2)}{C_0(r)\left(C_0(\beta r) - C_0(\alpha r)\right)} + \frac{C_0(r) + \pi r^2 S}{C_0(r)^2}\right)\end{aligned}$$

These results, although simple enough, are rather unedifying. The accompanying figures show the Petrosian ratio, along with the curves corresponding to $\pm 1\sigma$ for a variety of radial profiles (all models are assumed to have circular symmetry). All intensities are in DN (assuming unit gain), and all radii in pixels. No seeing corrections have been applied.

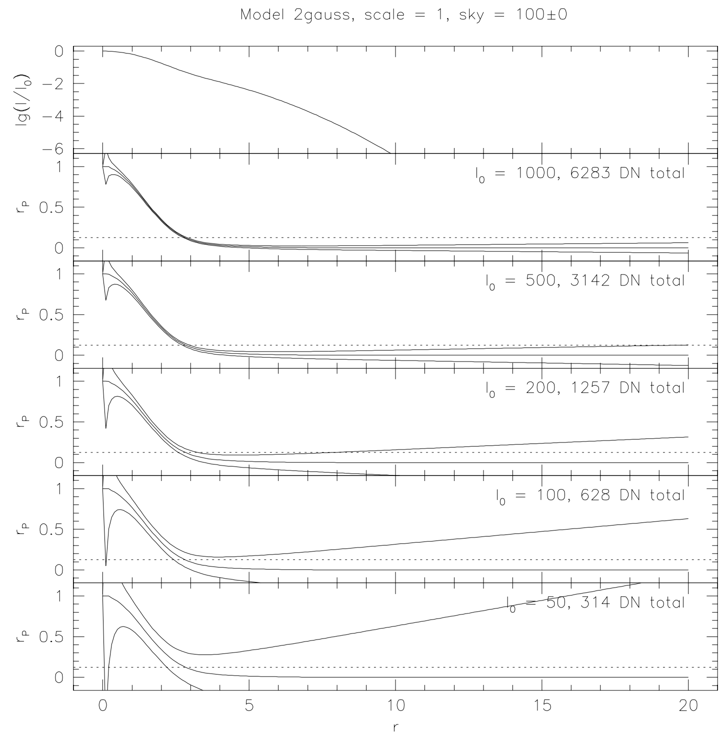

Figure 1: A double-Gaussian 'stellar' profile with the larger component having twice the width of the smaller, and 40% of its power. This is a reasonable approximation to the PSF produced by Kolmogorov turbulence in the atmosphere.

4 Discussion

All the following magnitudes assume that a 14^{th} magnitude object has 10^6 counts.

Looking at the figures, it is clear that with the adopted Petrosian ratio of 0.125, the resulting Petrosian radii occur at points in the object's profiles where there is not much light, rendering the resulting magnitudes sensitive to bad sky subtraction and the presence of nearby objects.

Figure 1 implies that we will not be able to measure Petrosian magnitudes for stars much fainter than magnitude 21.5. This is perhaps not too disturbing as the SDSS photometric pipeline measures two primary magnitudes for each object, a PSF magnitude appropriate for stellar objects, and the Pet-

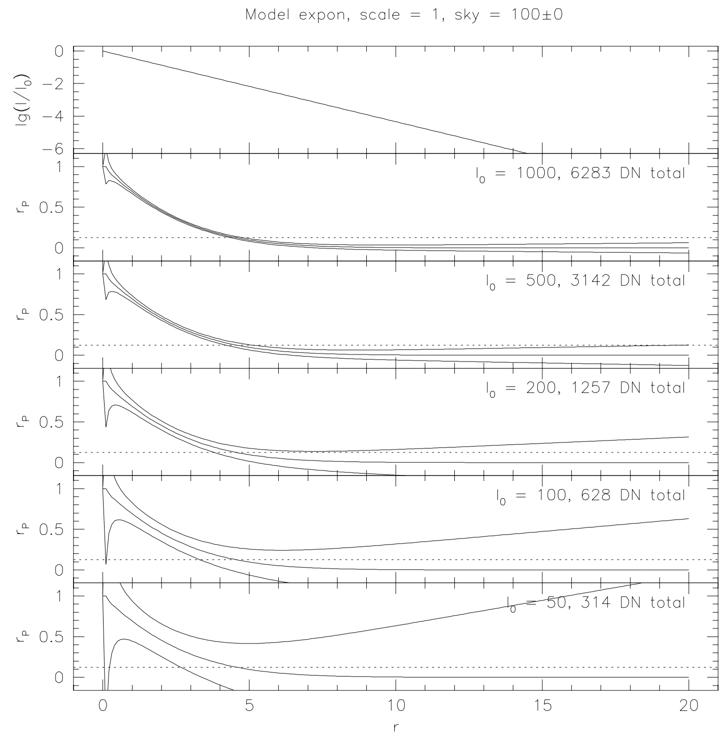

Figure 2: An exponential disk with scale length $\lambda = 1$.

rosian magnitude which we hope will be useful for all non-stellar sources, or failing that at least for those bright enough that they are more than just a fuzz on the images.

Turning to the exponential disks, we are unable to measure Petrosian radii for $\lambda = 1$ or 2 disks fainter than about 20.5, and including a small uncertainty in the sky level makes things significantly worse.

De Vaucouleurs models are very centrally concentrated, and the one in figure 5 is perhaps too small to be interesting, but figure 6 has an r_e of 4 and it's clear that the Petrosian ratio is poorly determined; figure 7 is still worse. Note that these are bright galaxies; the faintest shown has a magnitude of about 18.5.

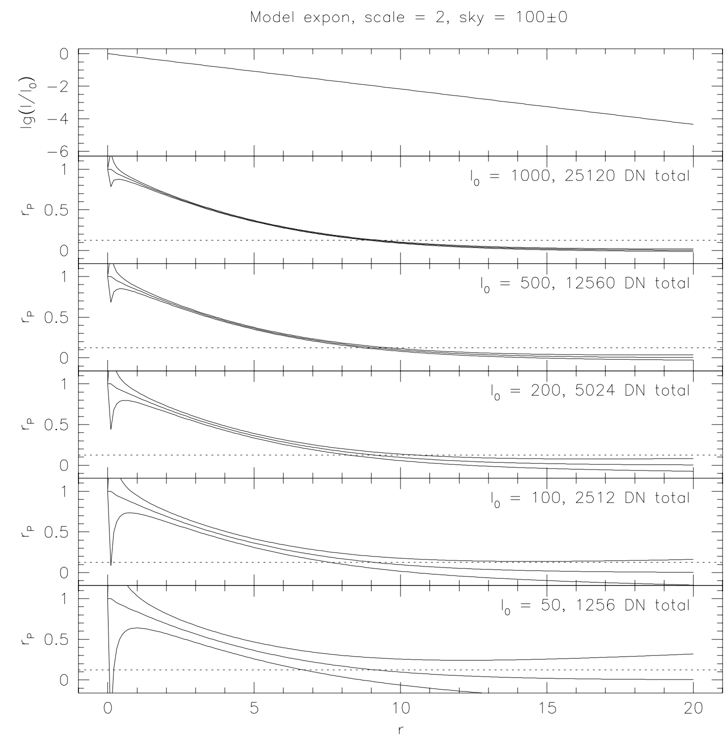

Figure 3: An exponential disk with scale length $\lambda = 2$.

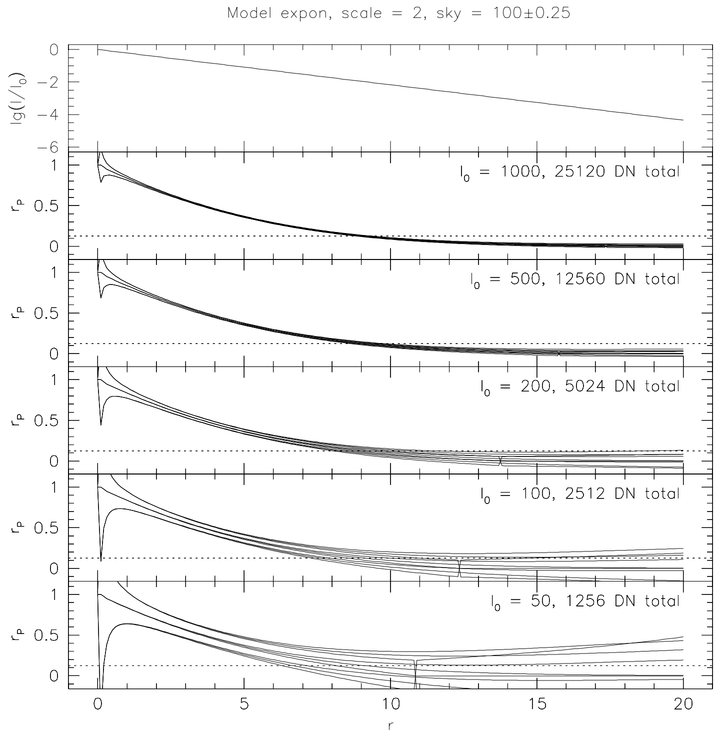

Figure 4: An exponential disk with scale length $\lambda = 2$, and an uncertainty in the sky level of 0.25DN.

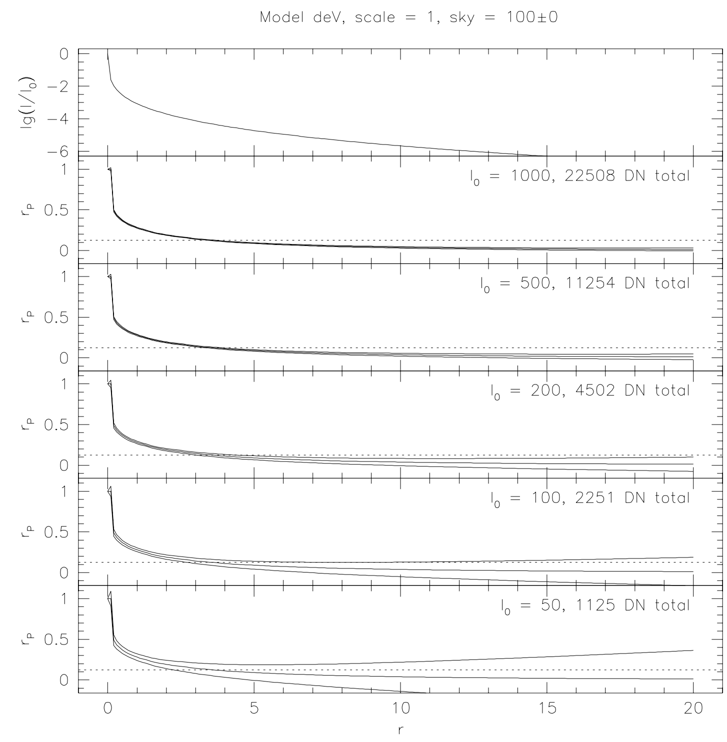

Figure 5: A de Vaucouleurs model with effective radius $r_e = 1$.

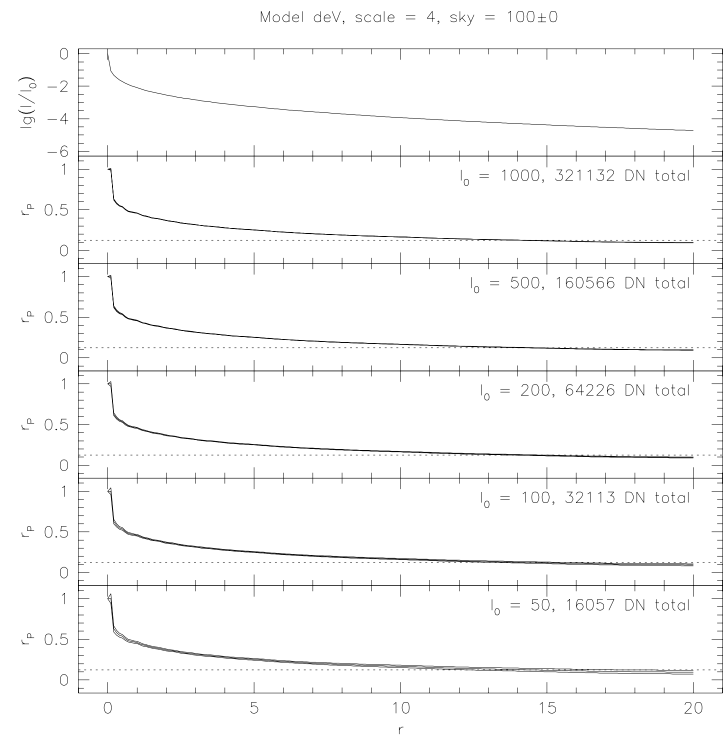

Figure 6: A de Vaucouleurs model with effective radius $r_e = 4$.

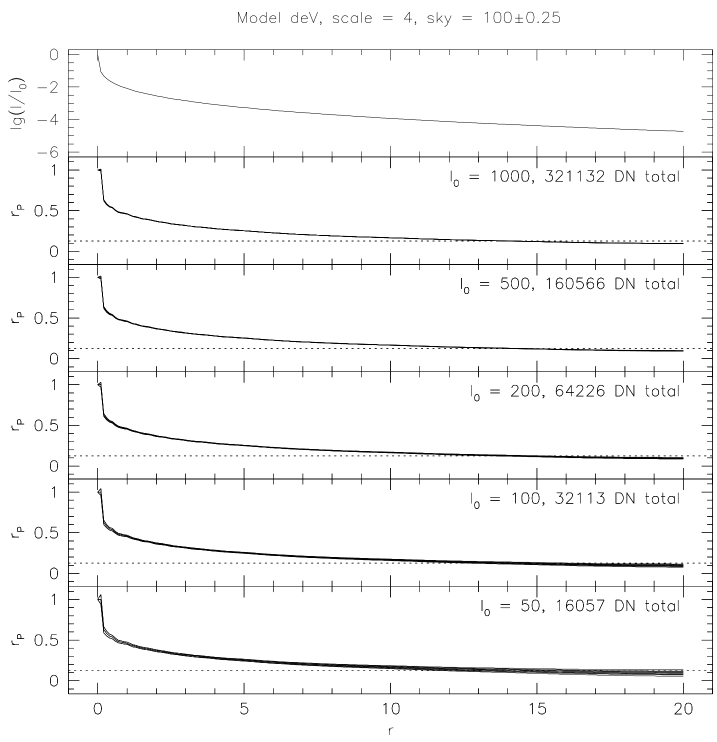

Figure 7: A de Vaucouleurs model with effective radius $r_e = 4$, and an uncertainty in the sky level of 0.25DN.