Merton College A2 Hilary Term 2011: Homework 3

1. Griffiths Problem 9.11

2. Griffiths Problem 9.14

3. Griffiths Problem 8.9

4. (after Griffiths 8.12) Suppose you had an electric charge q_e and a magnetic monopole q_m. The field of the electric charge is of course

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{q_e}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}') ,$$

while the field of the magnetic monopole is

$$\vec{B}(\vec{r}) = \frac{\mu_0 c}{4\pi} \frac{q_m}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}') .$$

a. Find the total angular momentum L stored in the fields, if the two charges are separated by a distance d.[1] [Answer: $(\mu_0 c/4\pi) q_e q_m$, which doesn't even depend on d!] When you integrate over angles, you will need the following integral:

$$\int_{-1}^{+1} \frac{1 - u^2}{\left(a^2 - 2bu\right)^{3/2}} du = \frac{2}{3b^3}\left[\left(a^2 - b\right)\sqrt{a^2 + 2b} - \left(a^2 + b\right)\sqrt{a^2 - 2b}\right]$$

A friendly word of advice: Be careful when doing the radial part of the integral. Since $\sqrt{r^2 + d^2 - 2rd} = (r - d)$ for $r > d$, while $\sqrt{r^2 + d^2 - 2rd} = (d - r)$ for $d > r$, you should break up your r integral into two: one from 0 to d plus one from d to infinity.

b. In quantum mechanics angular momentum comes in half-integer multiples of $\hbar$. This led Dirac to suggest in 1931 that if magnetic monopoles exist, electric and magnetic charge must be quantized. If even *one* monopole exists somewhere in the universe, this would "explain" why electric charge comes in discrete units (note that all isolatable particles have charges that are integer multiples of e).

Set your answer from part (a) with $q_e = e$ equal to $m\hbar$ (m being a half-integer) and derive an expression for the magnetic charge q_m. Write your answer solely in terms of e and the fine-structure constant $\alpha = e^2/4\pi\varepsilon_0\hbar c \approx 1/137$. Neat, huh? Give a numerical value for the minimum value of q_m in units of e; this is commonly referred to as g, the "Dirac charge".

c. The fine structure constant α is the coupling constant characterizing the strength of the electromagnetic interaction. What is the corresponding coupling constant for g? What might this imply?

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[1] Hint: You may find it easiest to place the electric charge at the origin and the magnetic monopole at $z = d$, and then to work in spherical coordinates where $\hat{z} = \hat{r}\cos\theta - \hat{\theta}\sin\theta$.