Homework 1

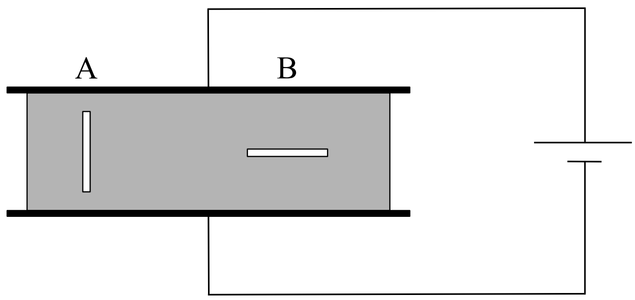
Revision of Merton College HT11 A2 Collection

Read carefully the following instructions:

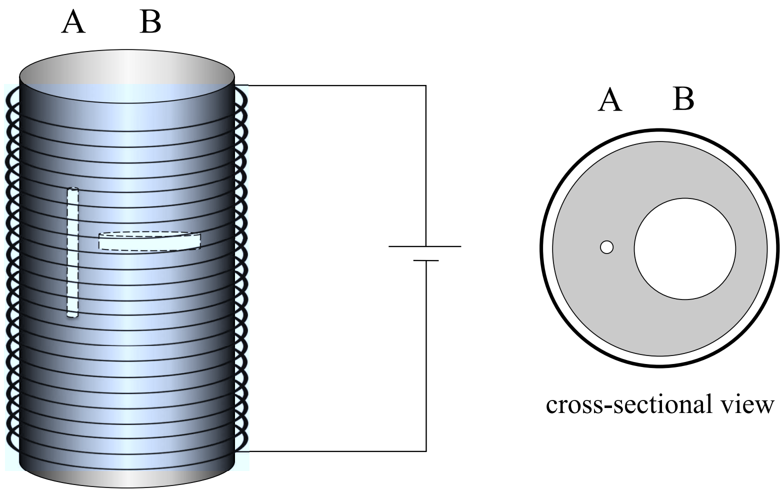
Using my solutions as a guide, make a list of every error you committed whilst sitting this collection, along with a detailed explanation of why you made these errors. For each problem you got wrong, invent a similar problem and solve it. If you attempted and correctly solved the first problem in Section B, do the second problem. Likewise, if you attempted and correctly solved the second problem in Section B, do the first problem.

Section A

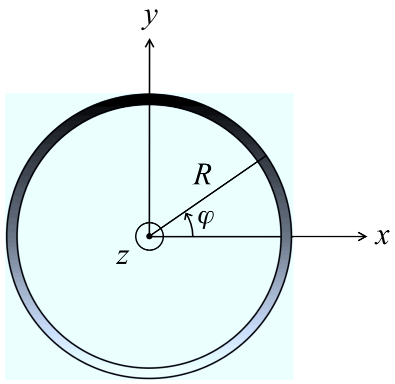
1. A capacitor is filled with a linear dielectric material of constant permittivity $\epsilon > \epsilon_0$, which has two small cracks in it as shown in the figure below. In which of the two cracks (A or B) is the electric field strength greater? Briefly justify your answer. [4]



2. A solenoid is filled with a linear magnetic material of constant permeability $\mu > \mu_0$, which has two small cracks in it as shown in the figure below. In which of the two cracks (A or B) is the magnetic field strength greater? Briefly justify your answer. [4]



3. An insulating circular ring of radius R lies in the x-y plane, centred at the origin. It carries a linear charge density $\lambda = \lambda_0 \sin\phi$, where λ_0 is a constant and ϕ is the usual azimuthal angle measure from the x axis (see figure below). Assume the ring is infinitesimally thin.



a. Using Coulomb's law, calculate the electric field at the point $(x, y, z) = (0, 0, d)$. There are much simpler charge configurations that can reproduce your answer in the limits $d = 0$ and $d \gg R$. What are they? Be quantitative. [4]

b. At $t = 0$, the ring is set spinning at a constant angular velocity $\boldsymbol{\omega} = \omega\,\hat{\boldsymbol{z}}$. Using the Biot-Savart law, calculate the resulting magnetic field at the point $(x, y, z) = (0, 0, d)$ at $t = 0$. You may assume $\omega \ll c/R$ and $\omega \ll c/d$, where c is the speed of light. [3]

Section B

For both questions, the following information may be useful:

$$\int_0^\pi P_\ell(\cos\theta)P_{\ell'}(\cos\theta)\sin\theta\, d\theta = \frac{2}{2\ell+1}\,\delta_{\ell\ell'}$$

$$P_0(\cos\theta) = 1 \qquad P_1(\cos\theta) = \cos\theta \qquad P_2(\cos\theta) = \frac{3}{2}\cos^2\theta - \frac{1}{2}$$

4. A conducting spherical shell of radius R is placed in an otherwise uniform electric field of strength E_0.

a. Show that the electric potential V satisfies Laplace's equation $\nabla^2 V = 0$ inside and outside of the spherical shell. Write down the most general solution for the potential in spherical coordinates (r, θ, ϕ), making use of the Legendre polynomials $P_\ell(\cos\theta)$. [4]

b. Write down the boundary conditions on V at $r = R$ and $r \to \infty$. [2]

c. Using your general solution from (a) and your boundary conditions from (b), solve for the potential outside of the shell V_{out}. [7]

d. Find the charge density $\sigma(\theta)$ on the surface of the shell. Explain your answer physically. [4]

e. Replace the conducting spherical shell with a solid linear dielectric sphere with dielectric constant $\epsilon > \epsilon_0$. Write down the new boundary conditions on V at $r = R$. [3]

5. A charge density $\sigma(\theta) = \sigma_0\cos^2\theta$ is glued over the surface of a spherical shell of radius R.

a. Show that the electric potential V satisfies Laplace's equation $\nabla^2 V = 0$ inside and outside of the spherical shell. Write down the most general solution for the potential in spherical coordinates (r, θ, ϕ), making use of the Legendre polynomials $P_\ell(\cos\theta)$. [4]

b. Write down the boundary conditions on V at $r = 0$, $r = R$, and $r \to \infty$. [4]

c. Using your general solution from (a) and your boundary conditions from (b), solve for the potential inside (V_{in}) and outside (V_{out}) of the shell. [10]

d. Embed the spherical shell in an infinite linear dielectric medium with dielectric constant $\epsilon > \epsilon_0$. Write down the new boundary conditions on V at $r = R$. [2]