AST 554: Irreversible Processes in Plasmas

Spring 2025
Lectures: MW(F) 9:00-10:20am, Jadwin 111
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> The tendency for entropy to increase in isolated systems is expressed in the second law of thermodynamics — perhaps the most pessimistic and amoral formulation in all human thought.
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> — Gregory Hill and Kerry Thornley
> *Principia Discordia* (1965)

> The law that entropy always increases holds, I think, the supreme position among the laws of Nature. If someone points out to you that your pet theory of the universe is in disagreement with Maxwell's equations — then so much the worse for Maxwell's equations. If it is found to be contradicted by observation — well, these experimentalists do bungle things sometimes. But if your theory is found to be against the second law of thermodynamics I can give you no hope; there is nothing for it but to collapse in deepest humiliation.
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> — Sir Arthur Stanley Eddington
> *The Nature of the Physical World* (1927)

Official Description:
Introduction to theory of fluctuations and transport in plasma. Origins of irreversibility, random walks, Brownian motion and diffusion, Langevin and Fokker-Planck theory. Fluctuation-dissipation theorem; test-particle superposition principle. Statistical closure problem. Derivation of kinetic equations from BBGKY hierarchy and Klimontovich formalism; properties of plasma collision operators. Classical transport coefficients in magnetized plasmas; Onsager symmetry. Introduction to plasma turbulence, including quasilinear theory. Applications to current problems in plasma research.

Prerequisites:
This course is nominally intended for second-year plasma-physics graduate students, although some first-years have taken the course successfully. *If you don't know Landau damping or ideal single-fluid magnetohydrodynamics, you should not be taking this course!* It is also assumed that you are familiar with things like Fourier and Laplace transforms, Cauchy's residue theorem, and how to use Green's functions to solve linear ODEs. If you are not comfortable with these techniques, please let us know; you should learn them as soon as possible.

Reference resources:
There is no single textbook that will cover everything in this course. The closest you can get is then an amalgam of textbooks, many of which are out-of-print and/or very expensive (though your colleagues surely have pdf copies of some of these… make friends with some more senior graduate students in the program). In the meantime, here are **recommended textbooks**:

R. D. Hazeltine and F. L. Waelbroeck — *The Framework of Plasma Physics*

P. Helander and D. J. Sigmar	*Collisional Transport in Magnetized Plasmas*
S. Ichimaru	*Statistical Plasma Physics, Volume I: Basic Principles*
	Basic Principles Of Plasma Physics: A Statistical Approach
Yu. L. Klimontovich[1]	*The Statistical Theory of Non-Equilibrium Processes in a Plasma*
N. A. Krall and A. W. Trivelpiece	*Principles of Plasma Physics*
D. Montgomery	*Theory Of Unmagnetized Plasma*
D. R. Nicholson	*Introduction to Plasma Theory*
N. G. van Kampen	*Stochastic Processes in Physics and Chemistry*

Here are also some **recommend articles** to supplement the above textbooks:

S. I. Braginskii	*Transport Processes in Plasma*
T. H. Dupree	*Dynamics of Ionized Gases*
F. L. Hinton	*Collisional Transport in Plasma*
R. M. Kulsrud	*MHD Description of Plasma*

These are made available to you on the course website, as are **Prof. Kunz's LaTeX'd lecture notes**. The latter will be the main reference point for the material taught in this course.

Grading Scheme:
100% ~bi-weekly problem sets

Course Schedule:
Note #1: I have front-loaded the course at a relatively quick pace. There are two reasons for this. First, we need ample time in April to discuss Braginskii's equations in detail and I would also like to survey some advanced (but non-examinable) topics. Second, the plasma generals. I know you'll be studying hard for them, and so I've reduced the homework load in the last part of the semester, and there is no written final exam for this course. The bargain we are striking here is that you'll work extra hard in this course for the first half in order to earn some respite in the second half. If all goes well, the homework load in April will be much lighter than in February and March, we should finish all examinable topics four weeks before generals, and we will have time to review material and discuss past generals questions on topics covered in this course.

Note #2: All topics presented in this course are examinable in generals unless otherwise noted. By the end of the course, all students should be able to provide brief definitions of the following terms: Klimontovich distribution; Liouville theorem; Bogoliubov timescale hierarchy; BBGKY hierarchy; Balescu–Lenard collision operator; Landau collision operator; Boltzmann's H theorem; test-particle superposition principle; polarization drag; Bremsstrahlung; Langevin equation; Krook operator; Lenard–Bernstein/Dougherty operator; Lorentz operator; Rosenbluth potentials; runaway electrons; Dreicer field; Spitzer–Härm problem; Chapman–Enskog expansion; parallel, gyro-, and perpendicular viscosity; pressure anisotropy; and, parallel, diamagnetic, and perpendicular heat flux. Also, students should be able to explain which species dominates each transport coefficient and why.

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[1] From a review of Klimontovich's book by J. P. Dougherty: "The book is systematic, and as concise as the material allows. It is, however, written in a very dry, mathematical sytle [*sic*]. There is little mention of any physical motivation for the steps taken, or interpretation of the results obtained; nor is there any indication of the practical circumstances to which any of the calculations are relevant. And readers who are not ashamed to benefit from visual aids will regret that the book does not contain a single diagram."

Overview of course topics:

Introduction to irreversible processes

course preview and discussion of syllabus; Bogoliubov timescale hierarchy; origins of time (ir)reversibility

Statistical mechanics of plasmas

Klimontovich equation; Liouville distribution; BBGKY hierarchy; Mayer cluster expansion; truncating the hierarchy; two-particle correlations

Vlasov kinetics

Vlasov equation; Green's functions and the Vlasov response function; phase mixing and the refinement of velocity space; free-energy conservation; the need for collisions

Plasma collision operator

Bogoliubov's hypothesis; Balescu–Lenard collision operator from BBGKY; properties of Balescu–Lenard operator; H theorem; reduction to Landau operator

Fluctuations and correlations

equilibrium BBGKY hierarchy; Gibbs distribution; two-particle correlations; shielding of moving test charges; test-particle superposition principle (TPSP: weakly coupled plasma can be described as a collection of statistically independent shielded test particles); electric-field fluctuations and the fluctuation-dissipation theorem; Cerenkov radiation and polarization drag; detailed balance of emission and absorption; foreshadowing of recovering Balescu–Lenard and Landau from TPSP

Fokker-Planck theory and Langevin dynamics

Markov processes; Chapman–Kolmogorov equation; Fokker–Planck equation; historical context — Stosszahlansatz as a Markov assumption; recovery of Balescu–Lenard from TPSP; Langevin approach; relationship between Fokker–Planck and Langevin

Approximate collision operators

Krook and Lenard–Bernstein operators; Rosenbluth potentials; mass-ratio expansions; Lorentz operator; collisions on a Maxwellian background; runaway electrons; linearized collision operators; slowing-down distribution

Classical transport theory

Electric conductivity: the Spitzer–Härm problem; Chapman–Enskog expansion

Classical transport theory in a magnetized plasma

Braginskii equations; field-aligned transport; cross-field transport; pressure anisotropy and gyroviscosity

possible additional (non-examinable) topics:

Transport in partially ionized plasmas

conductivity tensor; charge exchange; interaction potentials due to polarization of neutrals by charges

Basic Gyrokinetics

derivation of slab gyrokinetics; Hasegawa–Mima equation; free-energy conservation in gyrokinetics; nonlinear phase mixing and field-perpendicular cascades

Preview of neoclassical transport

Pfirsch–Schlüter regime from Braginskii

Klimontovich and its relation to particle-in-cell methods

coarse-graining, irreversibility, and collision operators in PIC codes

Lecture and homework schedule (subject to change):

M Jan 27 — Introduction to irreversible processes
course preview and discussion of syllabus; time (ir)reversibility; Bogoliubov timescale hierarchy
HW01 assigned (7 days)

W Jan 29 — Statistical mechanics of plasmas
Klimontovich equation; Liouville distribution; reduced distribution functions

F Jan 31 — Statistical mechanics of plasmas (continued)
BBGKY hierarchy; Mayer cluster expansion; truncating the hierarchy; two-particle correlations

M Feb 3 — Statistical mechanics of plasmas (continued)
review BBGKY hierarchy and discussion of two-particle correlations; start…
Vlasov kinetics
Vlasov equation; Green's function
HW01 due; HW02 assigned (14 days)

W Feb 5 — Vlasov kinetics (continued)
finish Vlasov Green's function; phase mixing and the refinement of velocity space; start free-energy conservation

F Feb 7 — Vlasov kinetics (continued)
finish free-energy conservation; prove Landau damping of Langmuir waves conserves free energy and discuss; motivate the need for collisions

M Feb 10 — Solving BBGKY using the Vlasov Green's function
Bogoliubov's hypothesis and solution of the truncated BBGKY hierarchy; Balescu–Lenard operator

W Feb 12 — Balescu–Lenard collision operator
properties of Balescu–Lenard operator; H theorem

F Feb 14 — Landau collision operator
derivation from Balescu–Lenard and discussion

M Feb 17 — Fluctuations and correlations
equilibrium BBGKY hierarchy; Gibbs distribution; two-particle correlations
HW02 due; HW03 assigned (14 days)

W Feb 19 — Fluctuations and correlations (continued)
shielding of moving test charges; test-particle superposition principle (TPSP: weakly coupled plasma can be described as a collection of statistically independent shielded test particles)

F Feb 21 — Fluctuations and correlations (continued)
electric-field fluctuations; polarization drag

M Feb 24 — Fluctuations and correlations (continued)
detailed balance of emission and absorption: fluctuation-dissipation theorem; foreshadowing: recovering Balescu–Lenard from TPSP

W Feb 26 — Fokker–Planck theory
Markov processes; Chapman-Kolmogorov equation; Fokker–Planck equation; Stoßzahlansatz and Bogoliubov hypothesis as Markov assumptions

M Mar 3 — Fokker–Planck theory (continued)
re-capitulation of Fokker–Planck theory; calculation of jump moments
HW03 due; HW04 assigned (14 days + spring break)

W Mar 5 — Fokker–Planck theory (continued)
calculation of jump moments; recovery of Balescu–Lenard from TPSP; Langevin approach

M Mar 10 — SPRING BREAK

M Mar 17	Prospective students visit Princeton — no class
W Mar 19	Langevin theory *Langevin equation; fluctuation-dissipation theorem; correlations (Eulerian vs Lagrangian)*
F Mar 21	Langevin theory (continued) *Taylor formula; Ito vs. Stratonovich; relationship between Fokker–Planck and Langevin*
M Mar 24	Approximate collision operators *Krook and Lenard–Bernstein operators; Rosenbluth potentials; electron-ion collisions and the Lorentz operator* **HW04 due; HW05 assigned (14 days)**
W Mar 26	Approximate collision operators (continued) *finish electron-ion collisions; ion-electron collisions*
M Mar 31	Approximate collision operators (continued) *collisions on a Maxwellian background; runaway electrons*
W Apr 2	Classical transport theory *introduction to classical transport; start Spitzer–Härm problem*
M Apr 7	Classical transport theory (continued) *finish Spitzer–Härm problem* **HW05 due; HW06 assigned (21 days)**
W Apr 9	Classical transport theory in a magnetized plasma *discuss Chapman–Enskog procedure; start derivation of Braginskii MHD*
F Apr 11	Classical transport theory in a magnetized plasma (continued) *finish derivation of Braginskii MHD*
M Apr 14	Classical transport theory in a magnetized plasma (continued) *discussion of field-aligned and cross-field transport; worked example from past generals*
W Apr 16	Classical transport theory in a magnetized plasma (continued) *comment on pressure anisotropy, connection to CGL and drift kinetics*
M Apr 21	Generals prep: presentations of prior Irreversibles problems
W Apr 23	Generals prep: presentations of prior Irreversibles problems
M Apr 28	**HW06 due**
M May 12	General examination begins