LYMAN-ALPHA EMISSION AS A PROBE OF GALAXY ENVIRONMENTS

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ABSTRACT

As a result of resonant scatterings off hydrogen atoms, Lyα emission from star-forming galaxies provides a potential probe of the neutral gas environment around them. In order to determine the utility of Lyα emission as a probe of gas environments we study the effects of environmental anisotropy on the observed Lyα emission by performing radiative transfer calculations for models of neutral hydrogen clouds with prescriptions of spatial and kinematic anisotropies. The environmental anisotropy leads to corresponding anisotropy in the Lyα flux and spectral properties and induces correlations among them. The Lyα flux (or observed luminosity) depends on the viewing angle and shows a correlation with Lyα optical depth along the line of sight of the viewing direction relative to the optical depths in all other directions. The distribution of Lyα flux from a set of randomly oriented clouds is skewed to high values, providing a natural contribution to the Lyα EW distribution seen in observations. A narrower EW distribution is found at larger peak offset of the line, similar to a trend suggested in recent observation. The peak offset appears to correlate with the line shape (a product of the width and the asymmetry of the line) suggesting the possibility of using the Lyα line features alone to determine systemic redshift of galaxies. The study suggests that anisotropies in the spatial and kinematic distributions of neutral hydrogen can be an important ingredient in shaping the observed properties of Lyα emission from star-forming galaxies. We discuss the implications of using Lyα emission to probe the circumgalactic and intergalactic environments of galaxies and introduce an in-progress follow-up study using realistic gas environments from a cosmological galaxy formation simulation.
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1. INTRODUCTION

Despite the great progress that has been made through the centuries in understanding the aspects and structure of the universe there is still much that is not known about how galaxies formed and have evolved. Key to understanding how galaxies have formed and evolved is the gas environment (broadly defined in this work as the density and velocity distributions of the gas both in a galaxy and its surrounding gas halo). This gas environment both provides materials for galaxies to form and evolve and in turn is affected by the formation and evolution of galaxies; therefore, a galaxy’s environment holds valuable information regarding these processes. Current approaches in probing these gas environments require a gas environment to be backlit by background sources (e.g., Steidel et al. 2002). In this approach the spectra of the background sources is imprinted with the absorption features of the intervening gas environment. Since such backlit scenarios require a fortuitous alignment of astronomical objects and the background source only passes through a portion of the gas environment, this approach can only provide a limited picture of gas environments for a limited number of galaxies. A more prolific probe providing a full picture of a gas environment would greatly advance our understanding of galaxy formation and evolution.

Hydrogen Lyman-alpha (Ly\(\alpha\)) photons provide such a probe. Ly\(\alpha\) photons are produced by an electron falling from the first excited to the ground state (\(n = 2\) to \(n = 1\) in orbital notation) in a hydrogen atom. As hydrogen is the most abundant element in the universe and is the majority constituent of galaxy gas environments such photons are not only readily produced but are also highly affected by the neutral hydrogen that makes up much of a galaxy’s environment. Ly\(\alpha\) emission is prevalent in galaxies active in star formation. These galaxies have a large amount of massive stars that emit ionizing photons (with energy above 13.6 eV) to ionize neutral hydrogen atoms in the surrounding interstellar medium (ISM). The subsequent recombination process leads to the formation of Ly\(\alpha\) photons (Partridge & Peebles 1967).
About two-thirds of ionizing photons that ionize the hydrogen gas are converted to Ly$\alpha$ photons. These photons will continue to interact with the ISM and then gas halo that surrounds and contains the galaxy. These interactions allow Ly$\alpha$ photons to probe the gas, encoding information about the gas in the spectral and luminosity properties of the Ly$\alpha$ photons that escape. This is due to radiative transfer effects, which will be described later. These photons then provide us an opportunity to map out the full gas distribution around galaxies with Ly$\alpha$ emission (Zheng et al. 2010; Zheng et al. 2011a; Zheng et al. 2011b).

Galaxies with prevalent Ly$\alpha$ emission features are called Lyman-alpha emitters (LAE’s). The study of LAE’s has been an active field of research since their discovery about 15 years ago (Hu & McMahon 1996; Cowie & Hu 1998; Dey et al. 1998; Hu et al. 1998). Large scale studies of LAE’s have been and even now are being carried out (e.g., Kodaira et al. 2003; Taniguchi et al. 2005; Chonis et al. 2013). With such a large amount of Ly$\alpha$ observations it is now important to unlock the data contained in the observed Ly$\alpha$ photons regarding the environmental properties of the host galaxies. A radiative transfer calculation needs to be performed to understand what effects the gas environment have on the observed Ly$\alpha$ emission properties. This is because every time a Ly$\alpha$ photon is absorbed and re-emitted by a hydrogen atom the propagation direction and frequency (in the observer’s frame of reference) of the photon can be changed, which then affects the photon’s likelihood of undergoing a subsequent interaction with a hydrogen atom. Due to the large number of interactions each photon will undergo and the complex nature of the gas environment in a galaxy and its halo a Monte-Carlo simulation of the radiative transfer processes the Ly$\alpha$ photons experience is the best method to perform the necessary calculations. The simulation used for this study is based on Zheng & Miralda-Escudé (2002). The focus of this study is to investigate the connection between observed Ly$\alpha$ emission properties and the gas environments that produced the Ly$\alpha$ emission in an effort to provide a guide for observational study. Specifically we examine the anisotropic effects that the gas environments have on Ly$\alpha$ emission properties. Verhamme et al. (2006) show in their theoretical study
that Ly\(\alpha\) photons preferentially escape in directions perpendicular to the disk of a galaxy as opposed to parallel the disk of the galaxy, leading to a Ly\(\alpha\) flux anisotropy. Such flux and other anisotropies in the Ly\(\alpha\) emission and their dependence on the galaxy environment is the focus of this study. In order to focus on the anisotropic effects of the gas on the emission we examine analytic distributions of neutral hydrogen gas with simple prescriptions of anisotropies so that the anisotropies of the emission and the anisotropies of the gas distribution can be easily related. In Section 2 we introduce the Monte Carlo simulation and analytic gas environments used, in Section 3 we examine the results of our study, in Section 4 we introduce in-progress follow-up studies necessary to better understanding the physical relatability of our results, and a summary is presented in Section 5.
2. METHODS

2.1. The Simulation

The Monte Carlo algorithm used to simulate Lyα emission from a galaxy works as follows: a photon is created in the rest frame of the gas at the center of the gas distribution with a random propagation direction and an initial frequency following a Gaussian profile centered at the Lyα center with profile width being set by the temperature of the gas. An optical depth is chosen from a decaying exponential distribution and the path length is determined based on the distribution of gas density, temperature, and velocity along the propagation direction. A scattering with a hydrogen atom then occurs after the photon has traveled the propagation length. The photon is Lorentz-transformed into the rest frame of the hydrogen atom off of which it is scattering. A new propagation direction is chosen based on the Compton scattering formula. The recoil of the atom after photon emission is taken into account and the photon is Lorentz-transformed back into the rest frame of the galaxy. A new propagation length is then chosen and the process is repeated. When the photon escapes the gas cloud defined in the simulation its final propagation direction and frequency are recorded. By repeating this process with a large number of photons we can get good statistics on the Lyα emission.

2.2. Analytic Gas Clouds

We apply the simulation to simple spherical distributions of neutral hydrogen gas. These distributions are spherically symmetric in geometry. There are four categories of these distributions that are studied: a “density gradient” distribution, a “velocity gradient” distribution, a “bipolar wind” distribution, and a “velocity gradient plus expansion” distribution. For all the distributions, the temperature of the neutral hydrogen gas is fixed at $2 \times 10^4$ K. The source of photons for the simulation is fixed to be the point at the center of the gas cloud.
We first define the “density gradient” distribution. A density gradient is introduced on top of an otherwise uniform gas density distribution along the $z$ axis (or axis of symmetry) of the gas cloud. The gas as a whole is static, so the only anisotropy in the gas distribution is the density gradient. The density gradient distribution is defined by

$$n(z) = n_0 \left(1 - 2A \frac{z}{R}\right)$$

(1)

where $n(z)$ is the number density of the neutral hydrogen gas, $n_0$ is a set base number density of the cloud, $R$ is the radius of the gas cloud, and $A$ is a parameter setting the magnitude of the density gradient. Thus, gas further along the $+z$ direction has a lower number density while gas further along the $-z$ direction has a higher number density, with $z = 0$ (the equator of the spherical gas distribution) having $n(0) = n_0$, the set base number density of the cloud (which is also the mean number density of the cloud due to the symmetric nature of the density distribution).

The density distribution is intended to investigate how hydrogen gas density affects Lyα emission. Volumes of hydrogen with a greater hydrogen gas density have larger optical depths and therefore shorter mean free paths for Lyα photons, meaning the photons will travel shorter distances between interactions with the hydrogen gas than their counterparts would in a volume with lower gas density. It is how this increased optical depth and the connected increased number of interactions affects Lyα emission properties that this distribution helps us to understand.

We next define the “velocity gradient” distribution. A hydrogen gas cloud with a uniform gas density has superimposed upon it a bulk gas velocity field defined by

$$\mathbf{v}(\mathbf{r}) = \frac{z}{R} \Delta V \hat{z}$$

(2)

where $\mathbf{v}$ is the velocity vector of the hydrogen gas, $\Delta V$ is a parameter setting the strength of the gradient, and $\hat{z}$ is a unit vector pointing along the direction of the positive $z$ axis. This velocity field is zero in the equatorial plane of the spherical gas distribution then increases
in magnitude linearly with $z$ away from the equatorial plane in either direction (i.e., Hubble-like expansion along $z$), with the velocity pointed along the $+z$ direction for locations at a positive value of $z$ and pointed along the $-z$ direction for locations at a negative value of $z$.

This distribution is intended to investigate how hydrogen gas velocity affects Ly$\alpha$ emission. Due to Doppler effects, gas in motion will see the Ly$\alpha$ photons at a different frequency than they are in the rest frame of the gas cloud. This in turn will either increase or decrease the optical depth of the gas depending on whether the photon’s frequency is Doppler shifted closer to or farther away from the Ly$\alpha$ line center. Even in a static cloud there is still a Doppler shift effect due to the thermal motion of individual atoms in the gas. This effect has been taken into account in the simulation code.

The “bipolar wind” distribution is defined by

$$v(r) = \begin{cases} \frac{r}{R} V_R \hat{r} & \text{if } |z|/r > \cos \theta_0, \\ 0 & \text{otherwise} \end{cases}$$

where $V_R$ sets the strength of the outflow. The angle $\theta_0$ is used to set the size of the cone in which the bipolar wind exists. The absolute value around $z$ causes the bipolar wind to exist in a double cone, symmetric over the equator of the spherical gas distribution. The axes of both cones are coincident with the $z$-axis of the sphere. This distribution is physically motivated. Galactic winds from star formation typically exhibit bipolar outflow patterns (Bland & Tully 1988; Shopbell & Bland-Hawthorn 1998; Veilleux & Rupke 2002; Rubin et al. 2013).

The final distribution introduces a Hubble-like expansion on top of the velocity gradient distribution. We call this the “velocity gradient plus expansion” distribution. The full equation characterizing the gas velocity field is

$$v(r) = \frac{r}{R} V_R \hat{r} + \frac{z}{R} \Delta V \hat{z}$$

where $\Delta V$ (as in equation 2) sets the strength of the velocity gradient and $V_R$ (as in
equation 3) sets the strength of the Hubble-like expansion of the gas, directed radially outward. Due to its similarity with the velocity and bipolar distributions a detailed analysis of the results from this distribution is not presented here but the results are included in our general analysis of the results from all the distributions.
3. RESULTS

Three neutral hydrogen number densities \( N_{\text{HI}} \) are used in each gas distribution: \( N_{\text{HI}} = 10^{18} \text{ cm}^{-2}, 10^{19} \text{ cm}^{-2}, \) and \( 10^{20} \text{ cm}^{-2} \). For our specific examination of each distribution we will focus on the \( N_{\text{HI}} = 10^{19} \text{ cm}^{-2} \) case since the effects of this density on the \( \text{Ly} \alpha \) spectra are comparable to both \( N_{\text{HI}} = 10^{18} \text{ cm}^{-2} \) and \( 10^{20} \text{ cm}^{-2} \) and also to prevent large amounts of data cluttering a focused and clear analysis of the effects on the emission properties. The results for the other two column densities are included in our general analysis in Section 3.4.

Due to the spherical symmetry of all the analytic gas models all symmetric directions (i.e., those with the same azimuth angle) encounter the same gas properties and thus anisotropy in the \( \text{Ly} \alpha \) emission properties depends only on the polar angle \( \Theta \). There are several ways that \( \Theta \) can be defined, two of which are the angle between the \( z \) axis and the point on the sphere at which a photon escapes and the angle between the \( z \) axis and the final propagation direction of the photon. Since the latter is the \( \Theta \) that a distant observer would measure it is the most observationally relevant one and we adopt it as our definition of \( \Theta \).

An important preliminary question to ask in regard to this choice of \( \Theta \) is whether any informative properties in the \( \text{Ly} \alpha \) emission would be visible to a distant observer at all. This is because a distant observer measuring photons from our gas distributions is able to measure photons emitting from the entire hemisphere of the gas cloud facing them; thus, photons emitted from an entire hemisphere of the gas cloud should be visible to them. If enough photons escape along the distant observer’s line of sight from every point near the surface of the hemisphere visible to them it is possible that no anisotropic effects will be observed, since for some of our distributions (most notably the velocity gradient) the same set of gas properties are exposed in any hemisphere chosen. It is possible that the properties of the \( \text{Ly} \alpha \) emission could still inform the observer of the general gas properties of the gas cloud (since the observer will be collecting photons from many points in the gas cloud) but may not be able to tell direction-specific gas properties along their line of sight. If,
however, the photons preferentially escape the gas cloud with a final propagation direction that, on average, has some relation with the surface normal of the spherical gas cloud at the point of escape then a distant observer will see an abundance of photons escaping from specific points of the portion of the gas cloud visible to them. In particular, if the photons tend to have a final propagation direction that, on average, is more parallel to the surface normal (as opposed to the isotropic case, where a photon is equally likely to escape at any angle between 0° and 90° with respect to the surface normal\textsuperscript{1}) then a distant observer will see an abundance of photons escaping from the region of the gas cloud immediately surrounding the section of the gas cloud closest to the observer. Thus, any anisotropic effects the gas distribution has on the Ly\textsc{a} emission would be apparent to an observer able to probe along multiple lines of sight. Additionally, the Ly\textsc{a} emission properties along a given line of sight would be linked to the gas properties along that line of sight and the observed emission could possibly be used to determine gas properties along a line of sight relative to the properties in other directions.

We define $\mu_{\text{point}}$ to be the cosine of the angle between the final photon escape direction and the surface normal at the point of escape. This is a measure of how close to the radial direction a photon escapes, with $\mu_{\text{point}} = 1$ corresponding to a completely radial escape. Figure 1 shows the probability density function of $\mu_{\text{point}}$ for the density gradient distribution. The $N_{HI} = 10^{19}$ cm$^{-2}$ case is shown on the left with the $N_{HI} = 10^{20}$ cm$^{-2}$ shown on the right, each with three different values of the anisotropy parameter $A$. For all six plotted functions there is a clear peak at $\mu_{\text{point}} = 1$, meaning that there is an abundance of photons escaping along or close to the radial direction. The same behavior is observed in Figure 2 for the velocity gradient distribution. Thus, for these distributions we can expect that if there are any anisotropic Ly\textsc{a} emission properties introduced by an anisotropic distribution of gas then there will be a line of sight dependence of the properties for distant observers, since each observer will receive an abundance of photons from a specific area of the gas

\textsuperscript{1}Angles greater than 90° at the surface of the sphere will cause the photon to be scattered back into the cloud.
Figure 1: The probability density function of $\mu_{point}$ for the density gradient case. $\mu_{point}$ is defined as the cosine of the angle between the propagation direction of the escaping photon and the surface normal at the point of escape. **Left panel:** the distribution for column density $N_{HI} = 10^{19}$ cm$^{-2}$. **Right panel:** the distribution for column density $N_{HI} = 10^{20}$ cm$^{-2}$. Both panels show the distributions for three values of the anisotropy parameter: $A = 0.0, 0.25,$ and 0.50. A hypothetical isotropic case is shown as a dotted line.

cloud which correlates with the portion of the gas cloud through which a distant observer’s line of sight passes through.

For both the density gradient and velocity gradient distributions the height of the peak of the distribution correlates with decreasing column density. This can be understood through examining the hypothetical case of a static gas cloud with extremely high column density. For such a gas cloud, the mean free path of a Ly$\alpha$ photon will be close to zero. As such, the location of the final scattering of the photon before it escapes the gas cloud will effectively be at the surface of the gas cloud. Such a photon will have an equal likelihood of propagating with any angle relative to the surface normal and would produce the isotropic case shown as a dotted line in Figures 1 and 2. In contrast, a gas cloud with very low column density where the mean free path is about the radius of the sphere has two factors that lead to photons escaping more radially. The first is that final scatterings can occur near the center of the sphere, where a photon traveling in any direction will naturally be traveling
Figure 2: The probability density function of $\mu_{\text{point}}$ for the density gradient case. $\mu_{\text{point}}$ is defined as the cosine of the angle between the propagation direction of the escaping photon and the surface normal at the point of escape. The units of the parameter $\Delta V$ shown are km s$^{-1}$. **Left panel:** the distribution for column density $N_{HI} = 10^{19}$ cm$^{-2}$. Distributions are shown for three values of the anisotropy parameter: $\Delta V = 50, 100,$ and $200$ km s$^{-1}$. **Right panel:** the distribution for column density $N_{HI} = 10^{20}$ cm$^{-2}$. Distributions are shown for two values of the anisotropy parameter: $\Delta V = 100$ and $200$ km s$^{-1}$. A hypothetical isotropic case is shown as a dotted line. Please note the vertical scale is different than that of Figure 1.
radially or close to radially. The second is, for scatterings that occur somewhere away from
the center of the gas cloud the photon (in an isotropic gas cloud) is most likely to escape
after the scatter if it propagates in the radial direction (since there is the least amount of gas
before escape in that direction). Any non-radial direction will have an increased chance of
scattering the photon before reaching the edge of the gas cloud and escaping; this effect
also creates an overabundance of photons escaping along radial directions. Interpolating
between these two limiting cases we would expect that lower column density distributions
would have more photons escaping radially than distributions with higher column densi-
ties, which is what is seen in our data. A more thorough analysis of how the second factor
works in non-isotropic distributions (such as our gas clouds) would need to be performed
to be able to exactly say how it works in our study. Whatever the details, we see in our
data that for a fixed column density and anisotropic prescription, the height of the peak at
\( \mu_{\text{point}} = 1 \) increases with increasing anisotropy strength, which may be due to the second
factor or a combination of the factors described above.

3.1. Density Gradient Distribution

We first present the results for the density gradient distribution while simultaneously pre-
senting the analyzation method for the rest of the analytic gas models. For all the models
we first look at the effects of the environment on the Ly\( \alpha \) flux. For this analysis (as well
as the other analyses of the analytic gas clouds) we first calculate the polar angle of the
escape direction of each photon. We then define a parameter \( \mu \equiv \cos \Theta \). For our spherical
distributions, \( \mu \) runs from \(-1\) to \(1\). We then divide our photons into bins of size \( \Delta \mu \). For
a sphere, each \( \Delta \mu \) sized bin corresponds to the same amount of surface area of the sphere,
and thus in the isotropic case each bin should have an equivalent number of photons sorted
into it with random noise effects taken into account.

The flux a distant observer observes at a polar angle \( \Theta \) is proportional to the number
of photons \( \Delta N \) in a narrow angular bin \( \Delta \Theta \) divided by the surface area of the angular bin,
$2\pi R^2 \sin \Theta \Delta \Theta$. This gives the relation

$$F(\Theta) = \frac{\Delta N}{2\pi R^2 \sin \Theta \Delta \Theta}.$$  \hspace{1cm} (5)

To give our analysis a normalized flux that will make comparisons easier we divide Equation 5 by the isotropic flux that would be observed. The isotropic flux is given by the total number of photons $N$ divided by the total surface area of the sphere $4\pi R^2$. The normalized flux then becomes

$$F(\Theta) = \frac{2\Delta N}{N \sin \Theta \Delta \Theta}.$$  \hspace{1cm} (6)

In terms of $\mu$, $\Delta \mu$ can be written as $\sin \Theta \Delta \Theta$, so Equation 6 can be written as

$$F(\mu) = \frac{2\Delta N}{N \Delta \mu}.$$  \hspace{1cm} (7)

The left panel of Figure 3 shows the angular dependence of the flux and also how this dependence changes with increasing density gradient. As can be seen from Equation 1, angles that correspond to directions with a lower neutral hydrogen density (for the density gradient case, this corresponds to angles with $\mu \sim 1$) have more photons escaping in their direction than would be expected in the isotropic case; the inverse holds for angles that correspond to directions with a higher neutral hydrogen density. Since a higher neutral hydrogen density corresponds to a shorter mean free path for scattering Ly$\alpha$ photons it makes sense that photons are more likely to escape along directions with lower densities.

To provide a quantitative measure of the anisotropy in the flux we decompose the flux $F(\mu)$ into its multipole components. These components are expressed by

$$F(\mu) = \sum_{l=0}^{\infty} C_l P_l(\mu)$$  \hspace{1cm} (8)

where $P_l$ is the $l$-th order Legendre polynomial and $C_l$ is a coefficient that determines the strength of the anisotropy associated with the $l$-th order Legendre Polynomial. $C_l$ is deter-
mined using the orthogonal nature of the Legendre polynomials,

\[ C_l = \frac{2l + 1}{2} \int_{-1}^{1} F(\mu)P_l(\mu) d\mu = \frac{2l + 1}{N} \sum_{i=1}^{N} P_l(\mu_i), \]  

(9)

where \( \mu_i \) corresponds to the \( \mu \) of the \( i \)-th photon. The integral is reduced to a sum due to the discrete photon output of our simulation. It is derived using Equation 7 in the limit \( \Delta \mu \) small enough such that each \( \mu \) bin has only 0 or 1 photon in it (i.e., \( \Delta N = 1 \) or 0). The normalization of the Legendre polynomials means \( C_0 = 1 \). The higher order coefficients correspond to the dipole, quadrupole, octupole, etc. moments of the anisotropic flux distribution. A perfectly isotropic case would have \( C_0 = 1 \) with \( C_l = 0 \) for all \( l \geq 1 \), while a perfectly dipolar case would have \( C_0 = 1, C_1 \neq 0 \), and \( C_l = 0 \) for all \( l \geq 2 \).

The right panel of Figure 3 shows the values of the coefficient of multipole expansion for the density gradient case. The dipolar anisotropy in the gas density distribution produces a corresponding dipolar anisotropy in the escaping photon flux. However, the escaping photon flux distribution also has higher order anisotropies, which are not exhibited in the density distribution. These higher order anisotropies are due to the complex nature of the radiative transfer of the photons as they propagate through the gas cloud; due to the large number of scatterings each photon goes through before escaping the gas cloud the anisotropies exhibited by the escaping photons are more complex than would be naively assumed from the gas distribution alone. The strength of these higher order anisotropies increases with increasing density gradient.

This anisotropy in the flux is a very important consideration for observations. It means that observers along different lines of sight may measure different Ly\( \alpha \) fluxes depending on their orientation. It also means that for a single observer two identical gas distributions at different orientations relative to the observer may produce different Ly\( \alpha \) fluxes despite all internal considerations of the gas distributions being equivalent. As far as galaxies are concerned, since the majority of the photons created by stars in a galaxy do not undergo the same radiative transfer effects as Ly\( \alpha \) photons the flux distribution of the continuum
Figure 3: Distribution of Lyα flux observed by distant observers for the density gradient distribution. Column density is $N_{\text{HI}} = 10^{19}$ cm$^{-2}$. Left panel: Lyα flux as a function of the polar angle $\Theta$. The parameter $A$ denotes the strength of the density gradient, as specified by Equation 1. $A=0$ corresponds to the isotropic case. The dotted line represents the expected flux for the noiseless, isotropic case. Right Panel: the multipole expansion coefficients of the anisotropic distribution of flux. This is used as a quantitative measure of the anisotropy of the flux.

radiation may not undergo a similar increase or decrease in flux as the Lyα photons escaping along the same line of sight. Thus observers may infer a different Lyα line strength (through a quantitative measure such as equivalent width) than observers along a different line of sight. To investigate this effect in our gas models we plot the probability density of the ratio of the measured Lyα flux and the intrinsic flux, $L/L_0$. The intrinsic flux is the isotropic flux that observers along all lines of sight would measure if there was no intervening gas. The flux distribution is computed using the flux distribution of the left panel of Figure 3. To reduce the effects of noise, we fit the data with the polynomial provided by the multipole expansion up to $l = 4$. This $L/L_0$ distribution is shown in the left panel of Figure 4. In order to give the unitless $L/L_0$ units that are comparable with those used in observation, the corresponding equivalent width (EW) is shown on the upper axis of both panels in Figure 4. $L/L_0$ is proportional to the Lyα EW if we ignore other contributions to the EW, such as dust effects on the continuum and Lyα emission (Verhamme et al. 2012).
Figure 4: Distribution of apparent (observed) Ly$\alpha$ luminosity from observations along random directions of a static spherical cloud with anisotropic density distribution (the density gradient distribution with column density $10^{19}$ cm$^{-2}$). The luminosity $L$ is in units of the intrinsic luminosity ($L_0$). It can be put in terms of the Ly$\alpha$ EW. In the top axis of each panel, the values of EW are marked by assuming the intrinsic EW to be 100 Å (corresponding to a stellar population with Salpeter IMF and 1/20 solar metallicity with age above 10 Myr; see the text). The left panel shows the original distributions, while the right panel shows the distributions smoothed with a Gaussian kernel of standard deviation of 10 Å to mimic the effect of measurement errors.

and observed Ly$\alpha$ emission being only a fraction of the total Ly$\alpha$ emission (Zheng et al. 2011b). For the convenience of comparison, we assume the intrinsic Ly$\alpha$ EW to be 100 Å, corresponding to a stellar population with Salpeter initial stellar mass function and 1/20 solar metallicity with age above 100 Myr (Malhotra & Rhoads 2002).

The case with an isotropic gas distribution has very little spread from the intrinsic luminosity and appears to be a delta function spike. The anisotropic cases show markedly different distributions with a peak on the low $L/L_0$ side of the distribution and a tail on the right side of the distribution which, for $A = 0.50$, is quite extended. The width of the distribution increases with increasing $A$, which corresponds to increasing anisotropy.

The hard edges of the luminosity distributions are not seen in observations and are
a natural consequence of our simple model. In order to make these distributions more relevant to observations, the distributions are smoothed with a Gaussian kernel of $\sigma = 10 \text{ Å}$ to mimic the effect of measurement errors. This value corresponds with the measurement errors of Ouchi et al. (2008). The smoothed distributions are shown in the right panel of Figure 4. These distributions are similar to those shown in Nilsson et al. (2009) and Ouchi et al. (2008) for LAE’s. This implies that the line of sight dependence of Ly$\alpha$ flux due to radiative transfer effects is a possible reason for the observed Ly$\alpha$ EW distributions. Because of the random orientation of LAE’s relative to earth observations thus probe a random sampling of lines of sight on the observed galaxies. Although there is Ly$\alpha$ flux variation between the galaxies themselves (which certainly plays a significant role in the observed Ly$\alpha$ EW distributions) for galaxies that are similar to each other this line of sight dependence on the Ly$\alpha$ flux likely plays a dominant role in producing the observed distributions.

Besides the flux being dependent on viewing angle the Ly$\alpha$ spectra are also dependent on viewing angle because of the system anisotropy. The left panel of Figure 5 shows the normalized Ly$\alpha$ spectra for viewers along the $+z$ ($\Theta = 0$ or $\mu = +1$) and the $-z$ ($\Theta = \pi$ or $\mu = -1$) directions for the $A = 0.50$ case. The isotropic ($A = 0$) case is shown for reference. The bottom axis shows the frequency offset from the Ly$\alpha$ center in units of the Doppler frequency $\Delta \nu_D \equiv (v_p/c)\nu_0$, which corresponds to the frequency offset of a line-center photon (with frequency $\nu_0$) seen by an atom moving with the most probable thermal velocity $v_p = \sqrt{2kT/m_H}$. The top axis marks the offset in velocity units. The reason that the photons upon escape have shifted out of the line center is because photons further from the line center see small optical depths in the hydrogen gas and thus have larger mean free paths, making them more likely to escape. For the density gradient case, the spectra are double peaked, as expected from a static gas cloud (Neufeld 1990; Zheng & Miralda-Escudé 2002).

The two directions shown in the left panel of Figure 5 are the directions of minimum
Figure 5: Normalized Lyα spectra from a static spherical cloud with anisotropic density distribution (for the density gradient distribution with column density $10^{19}$ cm$^{-2}$). Left panel: comparison of spectra observed along the two pole directions ($\mu = \cos \Theta = \pm 1$) for the $A = 0.50$ model. The directions with the lowest and highest column density are $\mu = +1$ and $\mu = -1$, respectively (see Equation 1). Right panel: comparison of spectra observed along one polar direction for clouds with different anisotropic parameter $A$. In both panels, the spectra from the uniform case ($A = 0$) are shown for reference. The frequency offset in units of the Doppler parameter $\Delta v_D$ and in units of velocity are shown in the bottom and top axes, respectively.

and maximum density, with the $+z$ ($\mu \sim +1$) direction having the minimum density and the $-z$ ($\mu \sim -1$) direction having the maximum density. Photons generally diffuse more in frequency space in regions with higher density since a larger shift in frequency is needed to see a lower optical depth and escape than for regions with lower density. This is represented in the figure with a larger spread in the peaks for the $\mu \sim -1$ case than for the $\mu \sim +1$ case.

The right panel of Figure 5 focuses on the $\mu \sim +1$ direction but shows the spectra for both nonzero values of the anisotropy parameter. The spread of the peaks decreases with increasing anisotropy which (along the specified direction) corresponds to a decrease in density. This further supports the statement of the previous paragraph that photons generally diffuse more in frequency space in regions with a higher density. At first glance, one may
expect to have a greater difference in the peak spread for differing anisotropies/directions. However, the spectra of Figure 5 do not show any great difference between the different anisotropies/directions or even from the isotropic case. The reason for this is that the column density modulation on top of the uniform case is only a factor of $1 - A \cos \Theta$. Even for the $A = 0.5$ case, the modulation is only a factor of three. Because of the random walk Ly$\alpha$ photons take through the gas cloud due to resonant scattering, the photons can be thought to probe the optical depths in all directions before escaping. This global probing of the Ly$\alpha$ photons works to reduce the difference in the effective column densities experience by photons escaping along different directions, leading to only small differences in the spectra.

Because the Ly$\alpha$ spectral properties depend on the environmental properties of the hydrogen gas it is possible that correlations exists between the properties themselves since the properties are tied together by the gas environment. Figure 6 shows the viewing angle dependence of two spectral features: the peak offset $v_{\text{peak}}$ with respect to the Ly$\alpha$ line center at the full width at half maximum (FWHM) $\Delta v_{\text{FWHM}}$ of the line. This is a characterization of the line width and thus a measure of the spread of the escaping photons in frequency space. In order to reduce the effects of noise, a Gaussian fit around the peak in each spectrum is made to determine $v_{\text{peak}}$. Both $v_{\text{peak}}$ and $\Delta v_{\text{FWHM}}$ are computed using photons distributed among ten bins of the viewing angle, with the bottom-left and upper-right points corresponding to $\mu \sim +1$ and $-1$, respectively. Only the red peaks of the spectra are analyzed; the reason for this choice is because in actual observations the blue peak likely becomes insignificant because of scattering in the IGM with Hubble flow (Dijkstra et al. 2007; Zheng et al. 2010). The isotropic ($A = 0$) case gives a sense of the noise of our data. For both cases with nonzero anisotropy parameters $v_{\text{peak}}$ and $\Delta v_{\text{FWHM}}$ show a correlation, meaning a greater peak offset leads to a wider line; line broadening and peak shift are tied together. As photons in the gas diffuse more in frequency space the spectrum as a whole moves further away from the Ly$\alpha$ line center and the spread in emitted photon
Figure 6: Peak offset $v_{\text{peak}}$ and FWHM $\Delta v_{\text{FWHM}}$ of Ly$\alpha$ emission from a static spherical cloud with the density gradient distribution with column density $10^{19}$ cm$^{-2}$. **Left panel:** the correlation between $v_{\text{peak}}$ and $\Delta v_{\text{FWHM}}$. **Right panel:** their correlation with the apparent Ly$\alpha$ luminosity or Ly$\alpha$ EW. Only the red peak is analyzed here.

wavelengths increases.

The right panel of Figure 6 shows the dependence of $v_{\text{peak}}$ and $\Delta v_{\text{FWHM}}$ on the relative luminosity $L/L_0$ along the line of sight. Both $v_{\text{peak}}$ and $\Delta v_{\text{FWHM}}$ are anticorrelated with $L/L_0$; thus, with increasing luminosity (or EW) the Ly$\alpha$ line lies closer to the Ly$\alpha$ center and there is a smaller spread in emitted Ly$\alpha$ photon wavelengths. These correlations can be understood as follows: lines of sight with higher relative luminosities tend to have lower optical depths, thus allowing photons to escape more easily than photons that escape along lines of sight with lower relative luminosities and higher optical depths, where the photons have to red/blueshift further from the Ly$\alpha$ line center in order to see a large enough mean free path to secure escape. This explains the larger $v_{\text{peak}}$ for lines of sight with lower luminosities; the corresponding larger $\Delta v_{\text{FWHM}}$ can be thought of as an extension of the relationship between $v_{\text{peak}}$ and $\Delta v_{\text{FWHM}}$ as seen in the left panel of Figure 6 or possibly as a product of the increased number of scatterings photons traveling along more optically
thick lines of sight undergo before finally escaping.

In summary, the resonant scatterings of Ly$\alpha$ photons with the neutral hydrogen gas constantly change their propagation directions, enabling the photons to probe optical depths along all directions, preferentially escaping along directions with low optical depth. The anisotropy of the density gradient case leads to a corresponding anisotropy in the Ly$\alpha$ luminosity and spectral properties. This in turn leads to a viewing angle dependence of Ly$\alpha$ flux, spectral peak offset, and line width, with higher fluxes and lower peak offsets and line widths corresponding to viewing angles along lines of sight with lower optical depths. The flux anisotropy leads to a spread in Ly$\alpha$ EW, which can provide a partial explanation for observed EW distributions.

3.2. Velocity Gradient Distribution

Using the same analysis process of the density gradient distribution we now turn to the velocity gradient distribution, given by Equation 2. The environmental properties of this distribution are anisotropic (with reflection symmetry over the equatorial plane) and so we expect corresponding anisotropies in the observed Ly$\alpha$ properties. We also expect these properties to have correlations with the optical depths along each line of sight.

The right panel of Figure 7, similar to Figure 3, shows the distribution of the Ly$\alpha$ flux as a function of the polar angle $\Theta$. For the velocity gradient distribution the gas has a bulk velocity moving in the direction of the poles with a magnitude proportional to $|z|$. $\Delta V$ corresponds to the magnitude of the velocity anisotropy and is the gas velocity at the poles. The polar directions have the largest flux with the equator having the lowest amount of flux. The difference between the polar and equatorial flux increases with increasing velocity anisotropy, with a factor of $\sim 4$ for the $\Delta V = 200$ km s$^{-1}$ distribution. The bulk velocity of the gas causes photons that are near the Ly$\alpha$ center to see a smaller optical depth as compared to the static case of the density gradient distribution due to Doppler
Figure 7: Similar to Figure 3, but for distributions of Lyα flux observed by distant observers and for a uniform density cloud with velocity anisotropy (the velocity gradient distribution with column density $10^{19} \, \text{cm}^{-2}$). The parameter $\Delta V$ denotes the magnitude of the velocity anisotropy. It is the expansion velocity at the cloud poles.

Figure 8: Similar to Figure 4, but for a uniform density cloud with velocity anisotropy (the velocity gradient distribution with column density $10^{19} \, \text{cm}^{-2}$). The parameter $\Delta V$, which is the expansion velocity at the poles, denotes the magnitude of the velocity anisotropy.
shift effects. The relevant (non-relativistic) Doppler shift equation is

$$\Delta \nu = \frac{v}{c} \nu$$

where $\nu$ is the frequency of the photon, $\Delta \nu$ is the frequency shift of the photon, $v$ is the velocity of the gas, and $c$ is the speed of light. Photons with redshifted frequencies (i.e., the gas is moving in the same direction as the photon) will see an even smaller optical depth, while blueshifted photons (those with the gas moving in the opposite direction as the photon) will generally see a larger optical depth to the point that the photon frequency Doppler shifted to the rest frame of the gas corresponds to the Ly$\alpha$ center and then decrease for larger frequencies.

The flux distribution also exhibits the same reflection symmetry over the equatorial plane as the gas velocity distribution. This is quantified in the right panel of Figure 7. For all three values of $\Delta V$ the dipole term ($l = 1$) vanishes, meaning that there is no difference in the flux distribution of the two hemispheres. There is a strong quadrupole ($l = 2$) dependence of the flux distribution, with the value of $C_2$ increasing with increasing anisotropy strength.

Figure 8 shows the apparent luminosity distribution for the velocity gradient distribution. The data of the left and right panels are derived using the same techniques of Figure 4. The $\Delta V = 50$ km s$^{-1}$ case is nearly symmetric while the higher $\Delta V$ cases show similarly skewed distributions as Figure 4, with the peak falling at $L/L_0 < 1$ and an extended tail towards higher $L/L_0$ values. These distributions, like those for the density gradient distribution, have similar features to actual observed distributions (Nilsson et al. 2009; Ouchi et al. 2008). In fact, the velocity gradient distribution creates relative luminosity distributions that closely mimic the observed distribution. This is to be expected because the velocity gradient distribution represents a system closer to actual halos than the density gradient distribution. The velocity gradient distribution approximates the gas outflow seen in galaxies (driven by supernovae and other processes).
Figure 9: Similar to Figure 5, but for a uniform density cloud with velocity anisotropy (the velocity gradient distribution with column density $10^{19}$ cm$^{-2}$).

Figure 10: Similar to Figure 6, but for a uniform density cloud with velocity anisotropy (the velocity gradient distribution with column density $10^{19}$ cm$^{-2}$).
Figure 9 shows the spectra for the photons escaping along the $|\mu| \sim 0$ (equatorial) and $|\mu| \sim 1$ (polar) directions. The equatorial spectrum is shown in the left panel and the polar spectrum is shown in the right panel. The spectra are double peaked, similar to the spectra for the density gradient distribution, but unlike the density gradient distribution the peaks are no longer symmetric about the origin (Ly\textalpha center). The double peak profile can be thought of as being dependent on the density of the gas, with photons on either side of the Ly\textalpha center (with the same value of $|\Delta \nu|$) seeing the same decrease in optical depth whether on the red or blue side of the Ly\textalpha center; the asymmetry between the two peaks can be thought of as being dependent on the bulk velocity of the gas, which, due to the Doppler effect, tends to decrease optical depth for redshifted photons and increase optical depth for blueshifted photons in an expanding gas field (Zheng & Miralda-Escudé 2002). As an example of this, the spectrum of the photons escaping along the equator (where there is a smaller bulk velocity field) has a stronger blue peak than the spectrum of the photons escaping along the poles (where there is a greater velocity field). However, along both directions, the size of the blue peak decreases with increasing anisotropy parameter $\Delta V$, again demonstrating the high dependence of the blue peak on the bulk velocity.

Along the equatorial direction increasing $\Delta V$ tends to move the red peak further redward while increasing the size of the tail redward of the peak. Along the polar directions increasing $\Delta V$ moves the red peak blueward but the tail redward of the peak continues to increase in size. The peak dependence on $\Delta V$ for the polar directions can be understood by noticing that photons escaping along the polar directions escape along the same direction as the gas velocity, and thus a larger $\Delta V$ will correlate directly with a lower optical depth seen by these photons and thus a smaller redshift on average is needed to achieve the mean free path necessary to escape. However, since the size of the red tail increases with increasing $\Delta V$ it appears that a peak with less offset anticorrelates with the red tail size for the velocity gradient distribution. The peak dependence on $\Delta V$ for the equatorial direction can be understood by noticing that a photon traveling outward and being scattered to the
equatorial direction by an atom moving towards one of the pole directions would appear to be redshifted in the restframe of the atom, with a redshift magnitude proportional to the velocity of the atom. Since the scattered direction is perpendicular to the velocity of the atom, the photon keeps the above redshifted frequency after being scattered, as seen in the lab frame. That is, for a higher $\Delta V$ cloud, we expect on average a larger redshift for photons escaping along the equatorial direction, which leads to the larger peak offset.

The left panel of Figure 10 shows the relationship between $v_{\text{peak}}$ and $\Delta v_{\text{FWHM}}$. Since both density and velocity distributions affect the emission spectral properties, the relationships between the spectral properties are not as simple as for the density gradient distribution. For both $\Delta V = 50 \text{ km s}^{-1}$ and $\Delta V = 100 \text{ km s}^{-1}$ there is an anticorrelation between $v_{\text{peak}}$ and $\Delta v_{\text{FWHM}}$, while for $\Delta V = 200 \text{ km s}^{-1}$ there exists a correlation, just as for the density gradient distribution. The right panel of Figure 10 shows the peak position and width as a function of the relative luminosity. $\Delta v_{\text{FWHM}}$ shows a less strong relationship with $L/L_0$ than for the density gradient distribution, while $v_{\text{peak}}$ has a tight relationship with $L/L_0$, more so than for the density gradient distribution. The anticorrelation in the case of $v_{\text{peak}}$ can be explained in the same way as for the density gradient distribution: lines of sight with lower optical depths have a greater Ly$\alpha$ flux. For the $\Delta v_{\text{FWHM}}$ data $\Delta V = 50 \text{ km s}^{-1}$ and $\Delta V = 100 \text{ km s}^{-1}$ show a correlation with relative luminosity while $\Delta V = 200 \text{ km s}^{-1}$ has an anticorrelation with relative luminosity, a pattern similar to the data of the left panel of Figure 10, where $\Delta V = 50 \text{ km s}^{-1}$ and $\Delta V = 100 \text{ km s}^{-1}$ exhibit behavior opposite the data of the density gradient distribution (as seen in Figure 6) while $\Delta V = 200 \text{ km s}^{-1}$ exhibits behavior similar to the data of the density gradient distribution.

To summarize, the velocity gradient distributions produce a corresponding anisotropy in the Ly$\alpha$ photon flux and emission properties. Specifically for flux, EW distributions are produced that closely mimic the observed ones. However, the nature of relationships between the properties themselves has high dependence on the value of $\Delta V$. 
3.3. Bipolar Wind Distribution

The bipolar wind distribution is defined as a cloud of uniform density with a Hubble-like expansion (meaning, expanding radially from the center) within a cone defined by $\Theta < 60^\circ$ and $\Theta > 120^\circ$ ($|\mu| < 0.5$) while remaining static outside the cone. Thus, this distribution differs from the velocity gradient distribution by having the velocity directed radially instead of along the $\pm z$ direction and only within a specified volume instead of within the whole cloud. The expansion velocity ($V_R$ in Equation 3) is set to be 200 km s$^{-1}$ at the edge of the gas cloud. We deviate from the previous analyses in that the anisotropy parameter remains fixed and we compare the results from different neutral hydrogen densities $N_{HI} = 10^{18}, 10^{19},$ and $10^{20}$ cm$^{-2}$.

The left panel of Figure 11 shows the viewing angle dependence of the Ly$\alpha$ flux. It has similarities with the velocity gradient distribution, such as the preferential escape of Ly$\alpha$ photons along the polar directions and the lowest flux occurring along the equatorial directions. However, the flux of the bipolar outflow has a distinct transition at ($|\mu| = 0.5$), the boundary between the outflow and the static regions. Within the static region the flux
remains fairly constant along all directions. This makes sense because, in a static cloud of constant density there is no directional variation in the optical depth and the photons can escape isotropically. For $|\mu| > 0.5$ the flux increases linearly with $\mu$. Although the gas properties are isotropic within the bipolar wind cones it appears that interactions with the static portion of the gas cloud creates anisotropy in the escape directions of the Ly$\alpha$ photons. The variation of the flux decreases with increasing column density. As the column density increases it smooths out the anisotropic effects of the velocity field. The right panel of Figure 11 shows the multipole coefficients for the bipolar wind distribution. There is a dominating quadrupole term, similar to the velocity gradient distribution, due to the quadrupole anisotropy of the distribution. This term decreases with increasing column density because of the smoothing effect that an increased column density has on the flux distribution.

Figure 12 shows the apparent luminosity distribution for the bipolar wind distribution. This distribution (related to the EW distribution) shows a peak at $L/L_0 < 1$ with a tail to
higher values, similar to the density gradient and velocity gradient distributions. However, the tail at higher $L/L_0$ exhibits its own peak. The location of the peak at high relative luminosity varies more with column density than the location of the peak at low relative luminosity, so a combination of these distributions taken over a spectrum of column densities could produce distributions more similar to the density and velocity gradient distributions and the observed distributions. It is also important to remember that these analytic gas clouds are specific, idealized cases and that actual systems would be a convoluted and significantly more complex combination of these distributions (and possibly others not tested here) so the anisotropic radiative transfer effect on the observed luminosity distribution will in reality be contributed to by all these and other distributions. Increasing column density tends to decrease the effect of the anisotropic gas properties on the Ly$\alpha$ flux.

Figure 13 shows the spectra along the $|\mu| \sim 0$ (equatorial) and $|\mu| \sim 1$ (polar) directions in the left and right panels, respectively. The spectra are similar in both directions except that for the polar directions the blue peaks are almost entirely suppressed. This is due to
Figure 14: Similar to Figure 10, but for a uniform density cloud with bipolar outflows (the bipolar wind distribution). Here, column density is referred to as $nR$ instead of $N_{HI}$; they are equivalent in meaning.

the same reason that the blue peaks were suppressed for the velocity gradient distributions, with an outflowing gas being more optically thin for photons shifted redward of the line center than for photons shifted blueward. The photons escaping in the equatorial directions mainly scattered through a static gas cloud of constant density. However, unlike the density gradient distribution, which had symmetric spectra, the blue peaks of the bipolar wind distribution are smaller than the red peaks, suggesting that the escaping photons still probed all the lines of sight in some manner and the otherwise “blue/red colorblind” static volume of gas had an overabundance of red photons scattered into it from the gas of the bipolar outflow. It is also worth noting that unlike the flux distribution which tended to be smoothed out with increasing column density the asymmetry of the spectra increases with increasing column density, with the blue peak almost eradicated for $N_{HI} = 10^{20}$ cm$^{-2}$ even in the equatorial directions. The peak shift of the red peaks is less along the polar directions than along the equatorial directions, again suggesting that photons escaping along lines of sight with lower optical depths (which, for red photons, is along the polar directions with the
outflow) diffuse less in frequency space before escaping than photons escaping along lines of sight with higher optical depths.

The left panel of Figure 14 shows the relationship between $v_{\text{peak}}$ and $\Delta v_{\text{FWHM}}$. No clear relationship exists for $N_{HI} = 10^{18} \text{ cm}^{-2}$ but $N_{HI} = 10^{19} \text{ cm}^{-2}$ and $10^{20} \text{ cm}^{-2}$ show a correlation between $v_{\text{peak}}$ and $\Delta v_{\text{FWHM}}$. $v_{\text{peak}}$ and $\Delta v_{\text{FWHM}}$ also tend to increase with increasing column density. The right panel of Figure 14 shows that an anticorrelation exists between $v_{\text{peak}}$ and $L/L_0$ for $N_{HI} = 10^{19} \text{ cm}^{-2}$ and $10^{20} \text{ cm}^{-2}$ (with no clear relationship existing for $N_{HI} = 10^{18} \text{ cm}^{-2}$). The FWHM of the peak (not shown in the panel) also shows a (somewhat weaker) anticorrelation.

3.4. General Results from the Analytic Gas Models

With the three cases examined in detail above, we now perform an analysis of the general results from the analytic gas models. Additional simulation runs are included here. To the cases discussed previously are added additional runs with $N_{HI} = 10^{18} \text{ cm}^{-2}$ and $N_{HI} = 10^{20} \text{ cm}^{-2}$. As explained in subsection 2.2, the results from the velocity gradient plus expansion distribution are also included here, with runs performed at the same three column densities as the other distributions. The runs have $V_R$ (from equation 4) set to 100 km s$^{-1}$ and with $\Delta V$ ranging from -100 to 100 km s$^{-1}$ with a step size of 50 km s$^{-1}$.

Overall, we find that anisotropies in the gas density and/or velocity distribution lead to anisotropies in the Ly$\alpha$ flux and spectral property distributions, providing a viewing angle dependence on observed Ly$\alpha$ emission properties. This can be used to in part explain distributions of observed Ly$\alpha$ EW. These anisotropies have some correlation with the optical depths of the gas clouds along the given lines of sight but since photons probe along many lines of sight before escaping the gas cloud and also have a probability of escaping from anywhere in the gas cloud along a specific line of sight the correlation of the photon properties with the line of sight optical depth is not tight. The correlation between emission properties and optical depth along a line of sight is further complicated by de-
dependence on optical depth throughout the gas cloud. As an example, consider the case of a static gas cloud with uniform density. Our results from the density gradient case show that Ly$\alpha$ photons preferentially escape along directions with lower column densities. For gas clouds with a uniform density distribution but differing column densities photons still escape isotropically independent of the value of the column density; thus gas clouds of differing column densities will still be observed to have the same luminosity. Based on this picture a better quantity to use to examine luminosity dependence is relative optical depth, $\tau_{\text{Ly}\alpha}/\langle \tau_{\text{Ly}\alpha} \rangle$. $\tau_{\text{Ly}\alpha}$ is the initial line-center optical depth measured along a specific line of sight. Figure 15 shows the relative luminosity plotted as a function of $\tau_{\text{Ly}\alpha}/\langle \tau_{\text{Ly}\alpha} \rangle$ for all the models. Also shown in Figure 15 are three curves (which are not fits) which are shown as $\exp(-\delta_\tau)$, $\exp(-\delta_\tau/2)$, and $\exp(-\delta_\tau/4)$, where $\delta_\tau \equiv \tau_{\text{Ly}\alpha}/\langle \tau_{\text{Ly}\alpha} \rangle - 1$ is the fractional excess in the initial line-center optical depth. There is a clear anticorrelation between relative luminosity and relative optical depth with the velocity gradient and velocity gradient plus expansion distributions showing the tightest anticorrelation. The stacked points for the bipolar distribution can be understood by recognizing that each bipolar gas distribution has only two different relative optical depths: a low relative optical depth along the bipolar outflows and a high relative optical depth along the static portions of the gas cloud; thus there are only two relative optical depths seen in the data of the bipolar distribution.

The left panel of Figure 16 shows the relationship between Ly$\alpha$ relative luminosity (shown here as EW) and the peak offset $v_{\text{peak}}$ of the line. The points are clustered by column density, which can be understood by remembering previously discussed results that lines of sight with a larger column density in our gas clouds have Ly$\alpha$ line profiles with a larger peak shift from the Ly$\alpha$ center (referred to as $v_{\text{peak}}$). The size of the points correspond with the strength of the anisotropy in each case. An anticorrelation is seen for $N_{\text{HI}} = 10^{19}$ and $10^{20}$ cm$^{-2}$, while the $N_{\text{HI}} = 10^{-18}$ cm$^{-2}$ data show a weak to no relationship. Overall, the velocity gradient, velocity gradient plus expansion, and bipolar wind distributions all show a greater variation in $v_{\text{peak}}$ over a corresponding range of EW than the density gradient.
Figure 15: The relation between the apparent Ly$\alpha$ luminosity and the relative initial line-center optical depth. The value $\langle \tau_{Ly\alpha} \rangle$ is the line-center optical depth for initial Ly$\alpha$ photons, averaged over all directions. The curves show $\exp(-\delta\tau)$, $\exp(-\delta\tau/2)$, and $\exp(-\delta\tau/4)$ to guide the eye, where $\delta\tau$ is the optical depth excess defined as $\delta\tau \equiv \tau_{Ly\alpha}/\langle \tau_{Ly\alpha} \rangle - 1$. The anisotropy models include those caused by density anisotropy (density gradient distribution; black points), by velocity anisotropy (velocity gradient distribution; magenta points) and its extension with an additional isotropic expansion component (velocity gradient plus expansion distribution; red points), and by bipolar outflow (bipolar distribution; blue points). Systems with three different column densities are studied for each case, $10^{18}$ (triangles), $10^{19}$ (squares), and $10^{20}$ cm$^{-2}$ (circles). At a given column density for each distribution, the symbol size indicates the degree of anisotropy of the system, with larger symbols corresponding to a larger anisotropy (e.g., for the density and velocity gradient cases, a larger gradient corresponds to a larger anisotropy). Open and filled symbols denote different setups in the velocity gradient (see the text) Filled symbols denote a positive velocity gradient and open symbols denote a negative velocity gradient.
distributions.

For all the distributions, peak offset is largely determined by column density. For the density gradient distribution and for each column density the spread in relative luminosity/EW for a specific gas cloud increases with increasing anisotropy (characterized by the size of the points). For the velocity gradient and related distributions the spread in EW is largely dependent on column density, with smaller column density corresponding to increased EW spread. However, for a given column density the correlation seen with the density gradient data between EW spread and the degree of anisotropy is again exhibited.

The anticorrelation between relative luminosity/EW and peak offset seen in our data seems to be consistent with recent observations. The right panel of Figure 16 reproduces data points compiled and analyzed by Hashimoto et al. (2013) for LAEs, Lyα blobs (LABs) and Lyman break galaxies (LBGs). LABs are similar to LAEs in that they produce strong Lyα emission features. LBGs may not have strong Lyα emission. They are star-forming galaxies but are detected differently than LAEs. The data of Hashimoto et al. (2013) also show a smaller spread in EW towards larger $v_{\text{peak}}$, similar to our simulated data. It is encouraging to see a similar trend in our simulated data as seen in observation, even with the simplistic cases used. It suggests that the key element in our systems (namely, the effect anisotropic systems have on Lyα emission) could play an important role in influencing the properties of Lyα emission from real systems.

In order to determine $v_{\text{peak}}$, the location of the Lyα line center must be known; to know this the systemic redshift of the Lyα emission source must be determined. Other lines, such as Hα and [O III] can be used (Steidel et al. 2010; McLinden et al. 2011; Hashimoto et al. 2013), but it may be useful to see if any other properties of the Lyα line can inform us about $v_{\text{peak}}$ independent of the systemic redshift, especially for observations and surveys that concentrate on measuring the Lyα emission and may not get spectra on any other lines useful for determining systemic redshift. Based on our results, we see correlations between line properties such as $v_{\text{peak}}$ and the FWHM $\Delta v_{\text{FWHM}}$. The strength of correlation
Figure 16: The relation between the Lyα EW and Lyα line peak offset $v_{\text{peak}}$. **Left panel:** the relation from our models of anisotropic clouds, including those caused by the density gradient distribution (black points), the velocity gradient distribution (magenta points) the velocity gradient plus expansion distribution red points), and by the bipolar distribution (blue points). The three different column densities studied for each case are represented as: $10^{18}$ (triangles), $10^{19}$ (squares), and $10^{20}$ cm$^{-2}$ (circles). At a given column density of each case, the symbol size indicates the degree of anisotropy of the system (larger for stronger anisotropy). Open and filled symbols denote different setups in the velocity gradient. Filled symbols denote a positive velocity gradient and open symbols denote a negative velocity gradient. **Right panel:** the observed relation. The data are taken from those analyzed and compiled in Hashimoto et al. (2013).
and scatter vary between gas distributions. We investigate whether another property of the emission line, asymmetry, also has correlations with the other line properties and perhaps could be used to determine $v_{\text{peak}}$ without knowing the Ly$\alpha$ line center. Similar to Wang et al. (2009), we define an asymmetry parameter related to the ratio of the width of the line blueward of the peak and redward of the peak; formally it is $f_{\text{asym}} = 3W_{\text{blue}}/W_{\text{red}}$, where $W_{\text{blue}}$ is the width at half maximum at the blue side of the peak and $W_{\text{red}}$ is the width at half maximum at the red side of the peak. Figure 17 shown $v_{\text{peak}}$ as a function of $\Delta v_{\text{FWHM}} \times f_{\text{asym}}$. A relatively tight correlation is shown between $v_{\text{peak}}$ and $\Delta v_{\text{FWHM}} \times f_{\text{asym}}$ with the data from the bipolar distributions with $N_{\text{HI}} = 10^{19}$ and $10^{20}$ cm$^{-2}$ providing the largest scatter. Thus, based on our results, $v_{\text{peak}}$ could be determined with some degree of accuracy from $\Delta v_{\text{FWHM}}$ and $f_{\text{asym}}$. This agrees with the observational results of McLinden et al. (2013), which suggest that a correlation exists between line asymmetry and peak offset. Further observations must be made before a secure conclusion can be made on the
nature and strength of this relationship. Such a relationship will not only provide a way to determine peak offset without knowing systemic redshift but also serve as a relation to test theoretical models of environments of star-forming galaxies.
4. FUTURE WORK

Although our simple analytic gas distributions provide a clear and neat analysis of the anisotropic effects of the gas environment on Ly$\alpha$ emission properties a more detailed a physically-motivated analysis needs to be made before our results can reliably be tied to observations. We are working to apply the radiative transfer calculation to more realistic and complex gas distributions that are the output of a high-resolution large-scale galaxy formation simulation (Cen 2011). This simulation runs over a large range of redshifts up to $z = 0$. Star formation and supernova feedback are incorporated into the simulation. We freeze the simulation at a specific redshift ($z = 3.1$ in our study). A three-dimensional box centered on a galaxy is pulled from the simulation. The box is large enough to contain the galaxy and its surrounding gas halo. The size of the box varies slightly between galaxies but each is about 220 kpc on a side. Each box has a resolution of about 700 cells on a side. With this frozen 3-D picture of the galaxy and its environment the Ly$\alpha$ radiative transfer simulation is ran on top of the gas environment extracted from the simulation. The temperature, density, gas velocity magnitude and direction, and star-formation rate of each cell is tracked by the simulation and is all used in our subsequent Ly$\alpha$ radiative transfer calculation. Ly$\alpha$ photons for the radiative transfer simulation are drawn from cells in the galaxy with active star formation and the photons are weighted proportional to the amount of star formation in the cell. Two main sets of data are collected from the simulation runs. The first set of data provides a spatially-extended high-resolution look into the Ly$\alpha$ emission properties across the gas in the simulation box along a specific line of sight. This is accomplished by calculating the probability of the photons at each scattering to escape along a specified line of sight and then adding the photon’s properties (weighted by the escape probability) to a three-dimensional array that stores spatial information in two of its dimensions and photon frequency in the third. The second set of data, much like the analytic gas clouds, records the final escape direction and frequency shift of each photon...
after they escape from the simulation box. In each case the effects of the IGM absorption that occurs between escaping from the simulation box and propagating to $z = 0$ are taken into account. This absorption affects both the spectra and the apparent luminosity of the Ly$\alpha$ photons along a given line of sight.

We first take a close look at one of the galaxies outputted from the cosmological galaxy formation simulation. Figure 18 shows the density and velocity field for a cutout plane from the three-dimensional box of the gas distribution that is pulled from the simulation. The galaxy itself and part of the surrounding halo is shown. The full three-dimensional box of the gas distribution is about 200 kpc on a side. A bipolar outflow of the gas from either side of the galaxy plane is shown while a high-density inflow region is coming in to the galaxy from the $-z$ direction. A full three-dimensional examination of the gas in the box (which is not able to be presented in this work) shows that this inflow region does make contact with the galaxy plane and thus is actively accreting gas onto the galaxy. This and the other galaxies taken from the galaxy formation simulation are much more dynamic and complex systems than the analytic gas clouds previously examined; however, they are much more realistic and it will be interesting to determine whether the anisotropic properties of the Ly$\alpha$ emission seen in the analytic gas clouds continue into these more complex and realistic systems. The important thing to realize from Figure 18 is that the gas velocity and density distribution is clearly anisotropic and so we expect anisotropies in the Ly$\alpha$ emission properties. Figure 19 shows the Ly$\alpha$ luminosity distribution of the galaxy in Figure 18 along the same line of sight. The Ly$\alpha$ emission is not only extended from the source galaxy but also shows a lot of asymmetry, relating to the complex nature of the gas distribution. We have 5000 simulated galaxies in total. Examining the simulated Ly$\alpha$ emission from these more realistic gas distributions will allow us to better make connections between our theoretical study and observations.
Figure 18: A sample galaxy taken from the cosmological galaxy formation simulation at redshift \( z = 3.1 \). The number density of the neutral hydrogen gas is represented by the color in the figure, with the color scale bar on the right mapping specific number densities to specific colors. The white arrows shown are arbitrarily scaled velocity vectors showing the direction and magnitude of the gas velocity at the location of the base of the arrow. The galaxy itself is seen as the high density region in the middle of the figure with the low density halo surrounding it. The \( z \) and \( x \) axes defined by the axis labels are based on the corresponding axis definitions in the full three-dimensional realization of the gas distribution outputted from the simulation. This figure is a two-dimensional slice taken at the middle value of the \( y \)-axis, so the origin of this figure coincides with the origin of the cube of gas taken from the simulation. The galaxy by chance has the normal to its plane oriented more or less along the \( z \)-axis.
Figure 19: The simulated Lyα image seen by a distant observer of the sample galaxy of Figure 18. The color represents the observed log of the surface brightness in units of erg s$^{-1}$ cm$^{-2}$ arcsecond$^{-2}$. The surface brightness has been smoothed with a 1.4 arcsecond smoothing in order to better simulate actual observations. The black line shows the contour corresponding to a surface brightness of $1 \times 10^{-16}$. 
5. DISCUSSION AND SUMMARY

We perform a theoretical study of the effects that anisotropic gas environments have on observed Ly\(\alpha\) emission properties. The motivation of this work is to understand if and how Ly\(\alpha\) emission can be used as a probe of the gas environment in and around galaxies by attempting to connect features of the Ly\(\alpha\) emission properties with specific features of the gas environment. We find in our theoretical study anisotropies in the gas environment lead to anisotropies in the Ly\(\alpha\) emission and that observed Ly\(\alpha\) emission properties have correlations with the line of sight optical depth.

We consider spherical distributions of hydrogen gas with introduced simple prescriptions of anisotropy in the gas density and velocity distribution. The Ly\(\alpha\) source is taken to be the center of the gas cloud. We examine results from four types of anisotropic distributions: a “density gradient” distribution with a modulation in the base gas density as a gradient along the \(z\) axis of a static gas cloud sphere, a “velocity gradient” distribution which gives the gas a velocity parallel to the \(z\) axis proportional to its distance away from the \(x−y\) plane of the sphere, a “bipolar wind” distribution which defines a radial outflow of gas inside a cone (with gas velocity proportional to its distance from the gas cloud center) and a static gas cloud outside the cone, and a “velocity gradient plus expansion” distribution which introduces a Hubble-like radial expansion on top of the velocity gradient distribution. For each distribution, systems of various column densities and anisotropy strengths are set up. We perform Monte Carlo Ly\(\alpha\) radiative transfer calculations in order to determine the emitted Ly\(\alpha\) spectra and analyze the anisotropy of the Ly\(\alpha\) flux and spectral properties.

Because of the nature of the resonant scatterings Ly\(\alpha\) photons undergo with neutral hydrogen these photons take a random walk through the gas, changing direction and frequency as they go. Because of the random walk these photons take before escaping they can be though to probe all lines of sight on their journey out of a gas cloud; however, lines
of sight with low optical depths have a larger Lyα flux, a smaller peak offset from the Lyα center, and a smaller FWHM for the observed Lyα line than lines of sight with higher optical depths. Thus, anisotropies in the gas environment of a galaxy and its associated halo are associated with anisotropies in the Lyα emission properties. More specifically, we find that the Lyα flux along a specific line of sight is correlated with the optical depth along that line of sight relative to the optical depths along all lines of sight. This anisotropic feature of the Lyα flux produces luminosity distributions (derived from probing the simulated emission along many random lines of sight) which mimic those observed in Ouchi et al. (2008), Nilsson et al. (2009), and Ciardullo et al. (2012), especially in the density gradient and velocity gradient distributions.

Because of the dependence of Lyα spectral properties on the gas environment, there are correlations between the spectral properties themselves. We specifically identify correlations between line peak shift and FWHM, peak shift and an asymmetry parameter we define as three times the FWHM and the line width ratio on either side of the peak (specifically, the line width ratio is defined as $W_{\text{blue}}/W_{\text{red}}$) and an anticorrelation between peak shift and luminosity (which relationship has also been observed; see Hashimoto et al. (2013)). The peak shift/luminosity anticorrelation can be understood by noticing that lines of sight with higher relative optical depths will have less photons escaping along them (lower luminosity) and the photons that do escape will on average have shifted further from the line center in order to scatter from the cloud. The relationship between the peak shift and FWHM can be understood as a correlation between FWHM and line of sight optical depth (since the peak shift is found to be correlated with the relative optical depth) with lines of sight having larger relative optical depths producing photons with larger scatter in frequency space. The peak shift correlation with the asymmetry parameter can be thought of as an interplay of the other two relationships discussed in this paragraph. This relationship, if confirmed by observations, can provide a way of determining the redshifted Lyα center in observed galaxies and thus determine the systemic redshift of a galaxy without needing to measure
any other spectral line.

A large ensemble of high-resolution simulated galaxies are necessary for a statistical study that can correspond well with observations. Such a study is currently in progress using galaxies from the output of a cosmological galaxy formation simulation (Cen 2011). From the results and analyses we present in this paper, we expect that a more detailed study with simulated galaxies will greatly advance our understanding of the interactions between Ly$\alpha$ emission and the circumgalactic/intergalactic medium and of galaxy formation and evolution through the circum-galactic/inter-galactic medium probed by Ly$\alpha$ emission.
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6. REFERENCES


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