Heat and Dust in Active Layers of Protostellar Disks

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  - For much help with dust, CR ionization, etc.
- Martin Ilgner
  - For help with understanding Ilgner & Nelson 2006a
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…and the person who did most of the work

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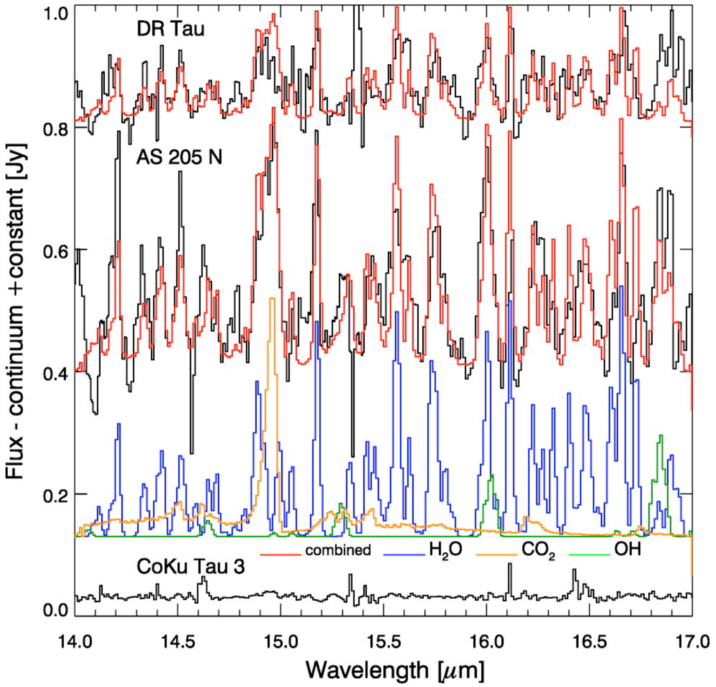
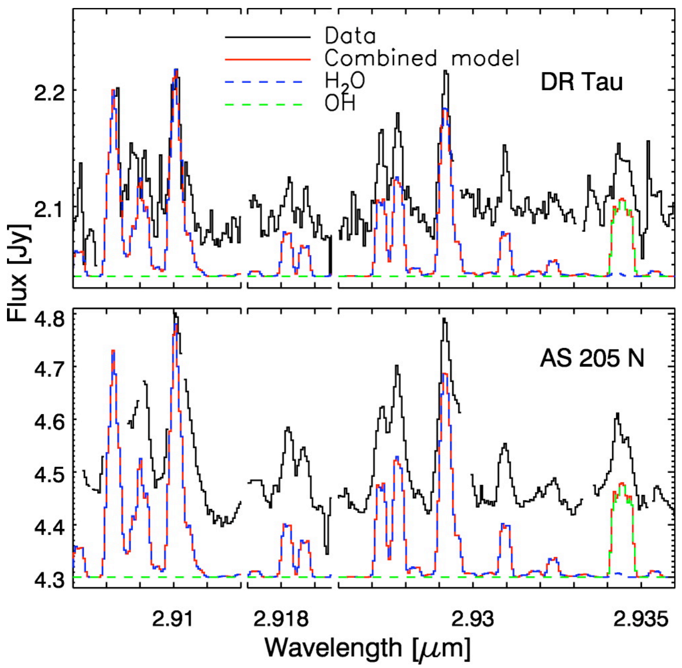
Prelude: Hot water at 1 AU

(Salyk et al. 2008, ApJ 646, L49)





- DR Tau & AS 205 were selected for high accretion rates: $\geq 10^{-7}\ M_{\odot}\mathrm{yr}^{-1}$
- Inferred physical conditions: $T \approx 1000\mathrm{K}$, $r \approx$ 1-3 AU, $N(H_2O) \approx 10^{18}\ \mathrm{cm}^{-2}$

  $\Rightarrow \Sigma \geq 0.1\ \mathrm{g\ cm}^{-2}$

Why study MRI in Protostellar Disks?

- The observational constraints are good
  - The only angularly resolved accretion disks (except galactic disks)
  - Well-determined accretion rates (from boundary-layer emission)
- The conditions for MRI are marginal (perhaps), and marginal cases can be instructive
  - Low electrical conductivity, extremely low $Pm = \nu/\eta$
- The contingency of turbulence may be important to planet formation
  - high densities & dust settling in dead zones
  - gap formation
  - *etc.*

The importance of dust

- PSDs are detected and characterized by their IR excesses
  - disk mass
    - requires temperature profile, emissivity, dust-to-gas ratio
  - lifetime
    - via IR excess vs. stellar age
  - geometry
    - inner & outer disk radii, gaps, flaring, warps
- Refractory elements in dust are the precursors of planets
- In the solar system, dust (in comets & meteorites) bears a fossil record of the primordial nebula
- *Dust controls the coupling of the disk to magnetic fields*

Central idea of this work

- The requirements of MHD/MRI give one set of constraints on the dust
- Observed emissions (IR SEDs, silicate & PAH features, molecular lines) give another set
- **Let's try to combine these**
- There are some obvious difficulties here
  - uncertainties in grain growth & size distribution
  - uncertainties in ionization rates
  - poor angular resolution (10s of AU at present)
  - immaturity of MRI simulations regarding microphysics, thermodynamics and resolution studies (this is evolving, of course)

Outline of this talk

- Introduction
- Review of required magnetic fields & ionization levels
- (Re)calculation of conductivity & active surface density (Σ_a) in the presence of grains
- Implications for the optical depth of active layers to dust
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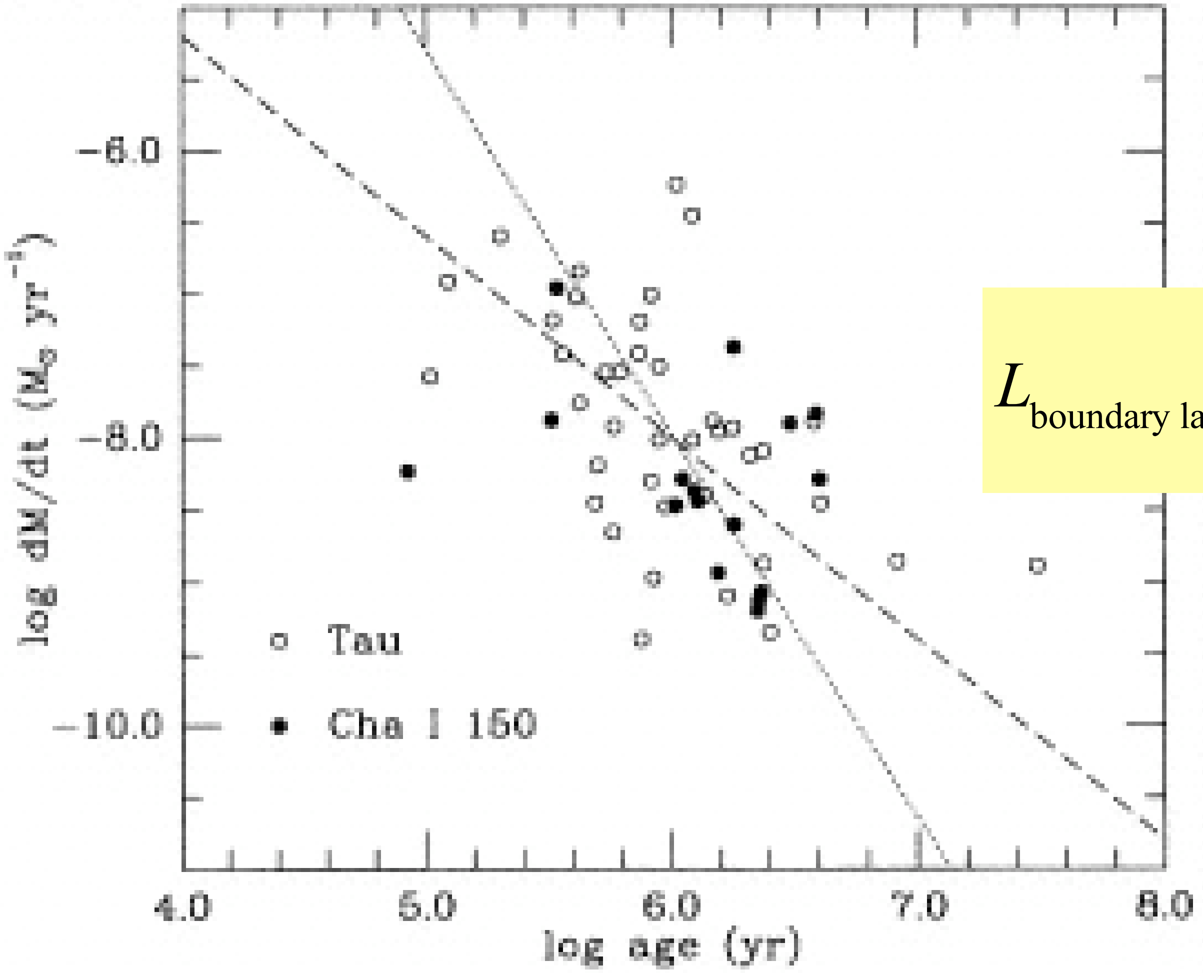
Observed accretion rates



$$L_{\text{boundary layer}} \approx \frac{GM_*}{R_*}\dot{M}$$

Hartmann *et al*. 1998, ApJ 495, 385

Minimum Mass Solar Nebula

$$\Sigma = 1700\, r_{\mathrm{AU}}^{-3/2}\,\mathrm{g\ cm^{-2}} \qquad T = 280\,\mathrm{K}\, r_{\mathrm{AU}}^{-1/2} \qquad c_s = 1.0\, r_{\mathrm{AU}}^{-1/4}\,\mathrm{km\ s^{-1}}$$

Vertically isothermal: $\rho(r,z) = \rho_0(r)\exp(-z^2/2h^2)$

$$\rho_0(r) = \frac{\Sigma}{h\sqrt{2\pi}} = 1.4\times10^{-9}\, r_{\mathrm{AU}}^{-11/4}\,\mathrm{g\,cm^{-3}}$$

Flared disk:

$$\frac{h}{r} = \frac{c_s}{\Omega r} \approx 0.03\, r_{\mathrm{AU}}^{1/4}$$

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Required Magnetic Field

Conservation of angular momentum implies, at $r \gg r_{\mathrm{in}}$,

$$\dot{M}\Omega r^2 + 2\pi r^2 \int_{-\infty}^{\infty} \left(\rho \overline{v_r' v_\phi'} - \overline{B_r B_\phi} / 4\pi \right) dz \approx 0.$$

But Maxwell stress $\left| \overline{B_r B_\phi} / 4\pi \right| \gg$ Reynolds stress $\left| \rho \overline{v_r' v_\phi'} \right|$

So $\left| \overline{B_r B_\phi} \right| \gtrsim \dot{M}\Omega / h_a$

where $h_a \approx 0.5 c_s / \Omega$ is the thickness of active layer (one side)

$$\therefore B = \sqrt{B_r^2 + B_\phi^2 + B_z^2} \gtrsim 3 M_{-7}^{1/2} r_{\mathrm{AU}}^{-9/8} \text{ Gauss}$$

$\Rightarrow$ **Equipartion :**

$$\frac{P_{\mathrm{gas}}}{P_{\mathrm{mag}}} \lesssim 1.0 \Sigma_1 \dot{M}_{-7}^{-1} r_{\mathrm{AU}} ; \quad \Sigma_1 \equiv \Sigma_a / 10 \text{ g cm}^{-2}$$

Required ionization

An **Elsasser number** $\Lambda \equiv \frac{V_{\mathrm{A}}^2}{\eta\Omega} \gtrsim 1$ is required for MRI.

Ohm's Law is tensorial: $\vec{J} = \overleftrightarrow{\sigma} \bullet \vec{E}$ (e.g. Wardle 2007)

Tensorial conductivity $\overleftrightarrow{\sigma}$ involves ionization (x_e, x_i) but also $\vec{B}$ via the

Hall parameters : $\beta_j \equiv$ (collision time)/(cyclotron time) for species j.

$$\partial_t \vec{B} - \vec{\nabla} \times \left(\vec{v} \times \vec{B} \right) = -\frac{c^2}{4\pi} \vec{\nabla} \times \left(\overleftrightarrow{\sigma}^{-1} \bullet \vec{\nabla} \times \vec{B} \right)$$

$\Rightarrow$ In Elsasser number, $\eta \propto$ largest eigenvalue of $\overleftrightarrow{\sigma}^{-1}$.

Required ionization (continued)

In the extreme Hall regime ($\beta_i \ll 1 \ll \beta_e$), $\Lambda \to \Lambda_{\rm H} = \dfrac{en_e B}{\rho\Omega c} \propto x_e B/\Omega$

$$\beta_e \approx 100\,\Sigma_1^{-1} B_0\, r_{\rm AU}^{3/2} \gtrsim 300\Sigma_1^{-1} \dot{M}_{-7}^{1/2} r_{\rm AU}^{1/8}$$

$$\beta_i \approx 0.2\;\;\Sigma_1^{-1} B_0\, r_{\rm AU}^{5/4} \gtrsim 0.6\;\Sigma_1^{-1} \dot{M}_{-7}^{1/2} r_{\rm AU}^{-1/8}$$ in the MMSN.

So we're probably in the Hall regime: $\beta_i < 1 \ll \beta_e$

or possibly in the ambipolar regime: $1 < \beta_i \ll \beta_e$

$$\Rightarrow x_e \approx 10^{-10}\,\Lambda\,\dot{M}_{-7}^{-1/2} r_{\rm AU}^{-1/4}\;\;\text{(MMSN, Hall regime)}$$

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There have been many calculations of active layers & conductivity

- Gammie (1996)
  - cosmic rays, **no grains**, $\mathrm{Re}_{M,\mathrm{crit}} = 1$
  - $\Sigma_a(1\ \mathrm{AU}) \approx 100\ \mathrm{g\ cm^{-2}}$
- Glassgold, Najita, & Igea (1997); Igea & Glassgold (1999)
  - X-rays, **no grains**
  - $\Sigma_a(1\ \mathrm{AU}) \approx 40\ \mathrm{g\ cm^{-2}}$
- Sano, Miyama, et al. (2000)
  - enlarged chemical reaction network
  - $a =$ **0.1 μm grains**
  - $\Sigma_a(1\mathrm{AU}) \approx 0\ \mathrm{g\ cm^{-2}}$ at $f = 10^{-2}$
  - $\Sigma_a(1\mathrm{AU}) \approx 70\ \mathrm{g\ cm^{-2}}$ at $f = 10^{-6}$

  (***f* is dust mass fraction**)
- Fromang, Terquem &Balbus(2002)
  - metal ions (Mg, Fe,); **no grains**
  - α disks not MMSN $(\alpha = 10^{-2}, \dot{M}_{-7} = 1)$
  - $\Sigma_a(1\mathrm{AU}) \approx 30\ \mathrm{g\ cm^{-2}}$
- Semenov, Wiebe, & Henning(2004)
  - $a =$ **0.1 μm grains**
  - improved chemistry
  - α disk (as above)
  - $\Sigma_a(1\mathrm{AU}) \approx 200\ \mathrm{g\ cm^{-2}}$ (?whole disk)
- Ilgner & Nelson (2006a) [IN06a]
  - Much extended network
  - **0.1 μm grains**
  - α disk (as above)
  - $\Sigma_a(1\mathrm{AU}) \approx 10\ \mathrm{g\ cm^{-2}}$ for f=10^{-6} (“model7”)

Active-layer calculations (continued)

- Ilgner & Nelson (2006b,c; 2008)
  - like IN06a, plus turbulent mixing **but no grains**
- Wardle (2007)
  - MMSN
  - tensorial conductivity
  - simple chemistry
  - **0.1-3.0 µm grains**, f = $10^{-2} \approx f_{\mathrm{ISM}}$
  - $\Sigma_a(1\,\mathrm{AU}) \approx 2$ g cm^{-2} for 0.1µm grns.
  - $\Sigma_a(1\,\mathrm{AU}) \approx 80$ g cm^{-2} for 3 µm grns.
- Salmeron & Wardle (2008)
  - like Wardle (2007), but only for radii 5 & 10 AU

- **This study**: like Ilgner & Nelson (2006), with some improvements:
  - Enlarged & updated chemistry based on UMIST06 vs. UMIST95
  - MMSN vs. α disk
  - H ionization & H_2 formation on grains
  - **Two grain populations** with variable sizes & mass fractions
    - 10^{-2} µm $\le a_1 \le a_2 \le 10$ µm
    - $0 \le f_1 + f_2 \le 10^{-2} = f_{\mathrm{ISM}}$
  - Variation of X-ray flux, X-ray temperature, & CR ionization parameter

Magnetic Reynolds number

We actually use the criterion

$$\boxed{\mathrm{Re}_M \equiv \frac{c_s^2}{\eta\Omega} \geq 100}$$

for active zones,

where $\eta = \eta_{\mathrm{Ohmic}} \approx 230 T^{1/2} x_e^{-1}\ \mathrm{cm}^2\ \mathrm{s}^{-1}$.

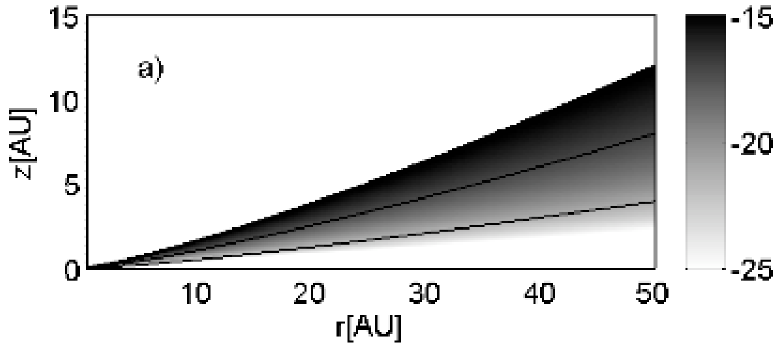
$\Rightarrow x_e \gtrsim 10^{-11} r_{\mathrm{AU}}^{-5/4} \quad \left(\text{cf. } 10^{-10} \Lambda \dot{M}_{-7}^{-1/2} r_{\mathrm{AU}}^{-1/4} \text{ for Hall regime}\right)$

This doesn't involve $\vec{B}$ or $\dot{M}$ explicitly.

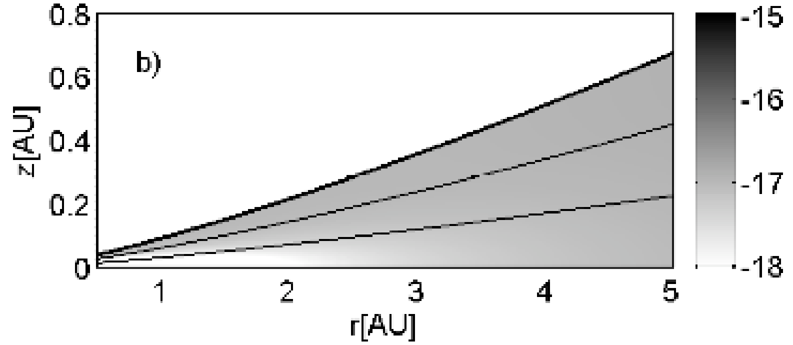
However, $\eta_{\mathrm{Hall}} \approx \beta_e \eta_{\mathrm{Hall}}$ in the Hall regime, and $\beta_e \gtrsim 10^2$.

The conductivity required to support accretion at high rates is larger than for linear stability, because the tensorial diffusivity increases with magnetic field strength.

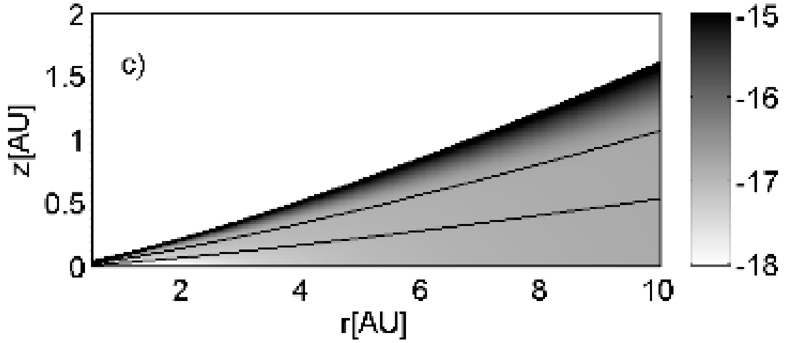
X-ray & CR ionization rates



X-rays only: $L_X = 5\times10^{29}\ \mathrm{erg\ s^{-1}},\ \ T_X = 3\ \mathrm{keV}$

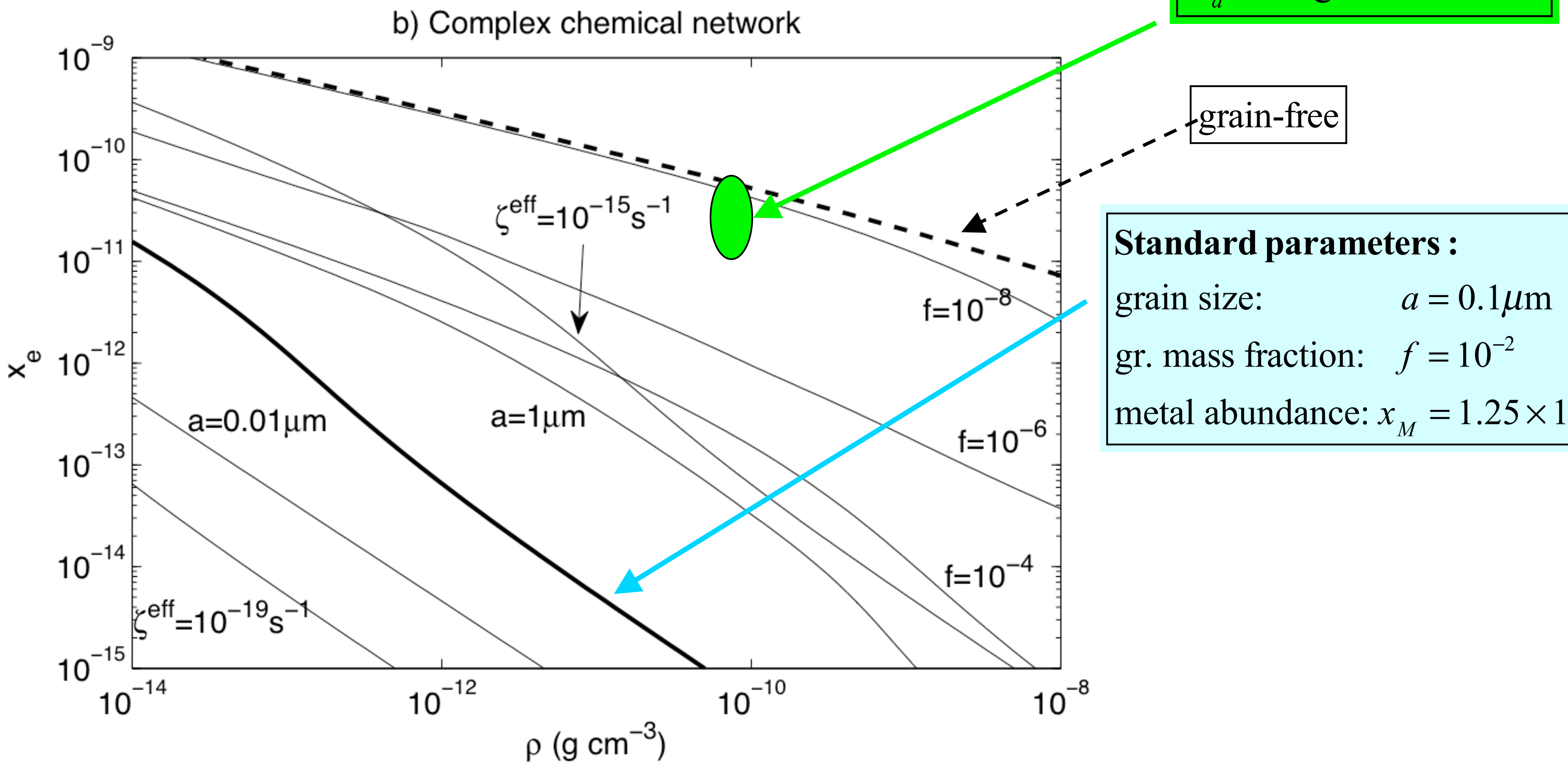


Cosmic rays only: $\zeta_{CR} = 10^{-17}\ \mathrm{s^{-1}\ molecule^{-1}}$



Both X-rays & CR

Electron abundance



$\Sigma_a \approx 10$ g cm^{-3} at 1 AU

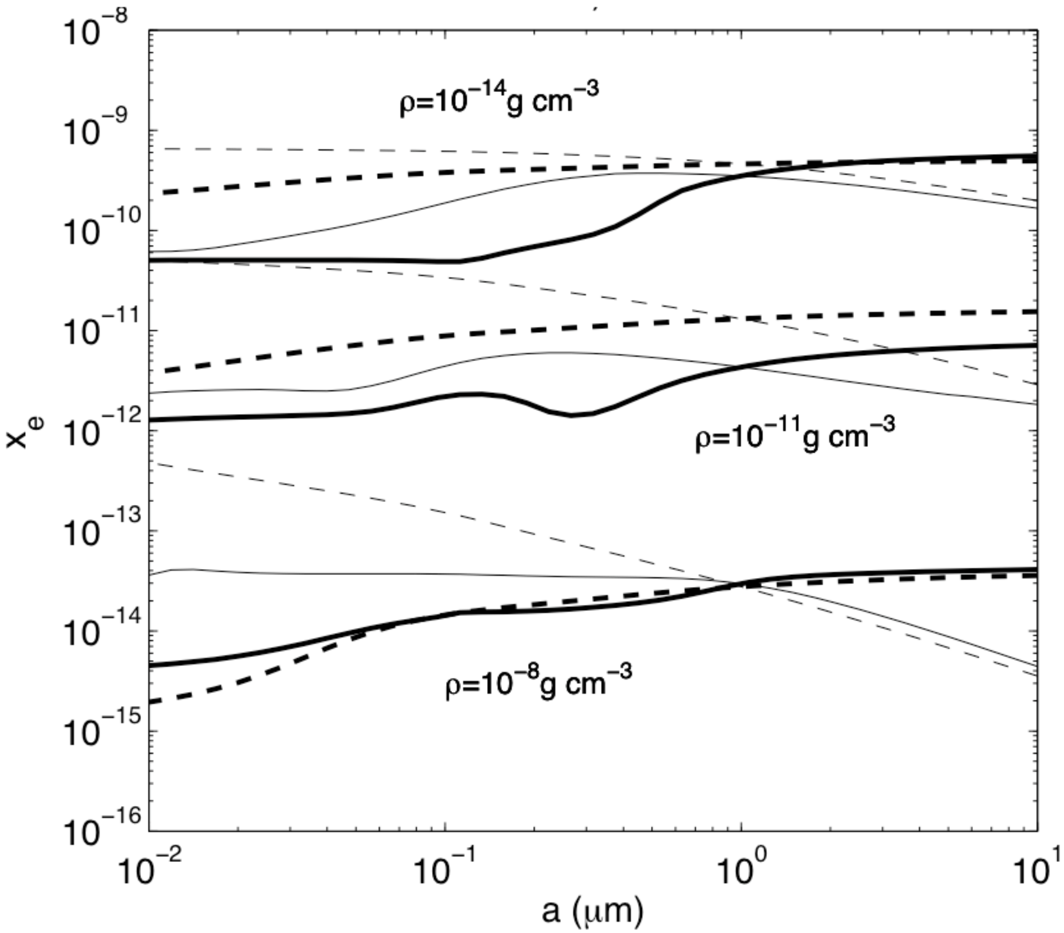
Standard parameters :

grain size: $a = 0.1\mu$m

gr. mass fraction: $f = 10^{-2}$

metal abundance: $x_M = 1.25 \times 10^{-8}$

Electron abundance depends roughly on total grain area



Heavy lines: constant grain area,

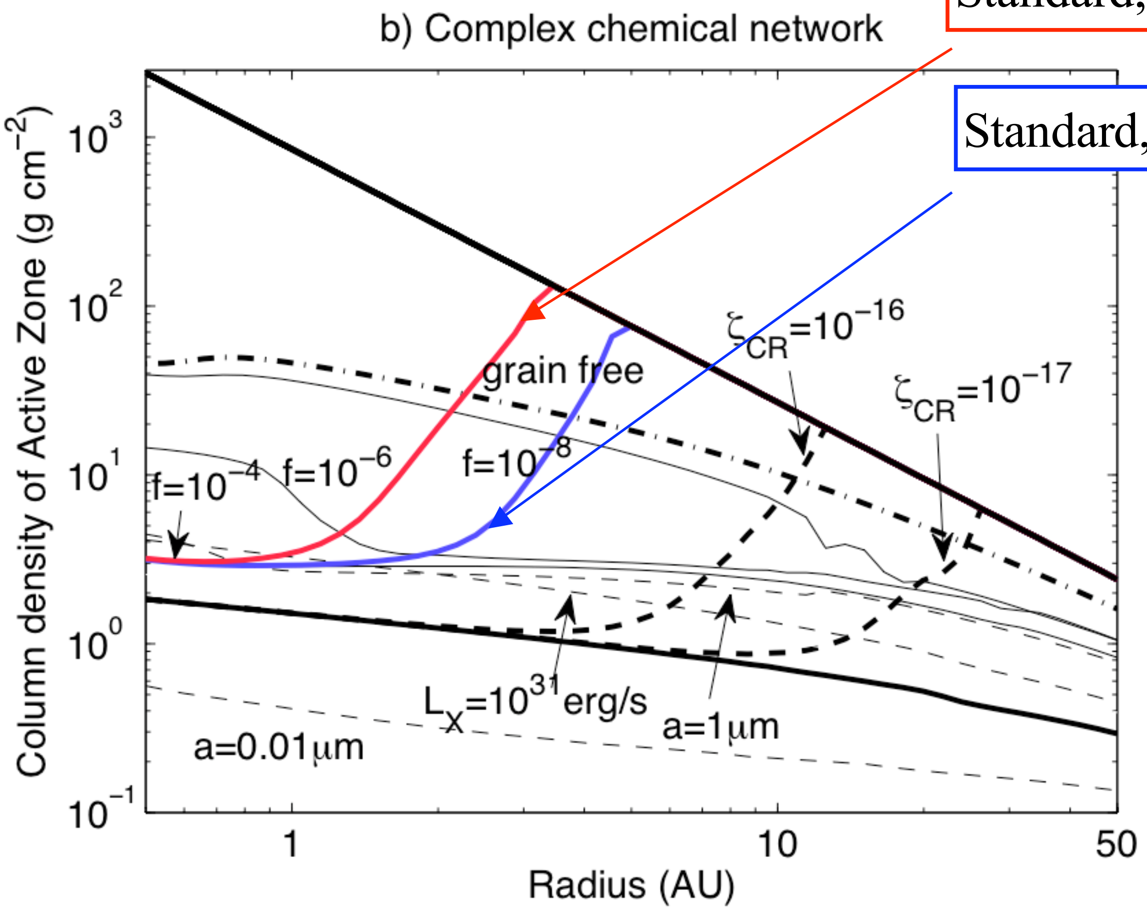
$$f = 10^{-4}\left(\frac{a}{1\mu\text{m}}\right)$$

...hence $f = 10^{-3} f_{\text{ISM}}$ if $a = 0.1\mu\text{m}$

Σ_a is very sensitive to grains

Standard, except $\zeta = 10^{-15}\,\mathrm{s}^{-1}$, $f = 10^{-4}$

Standard, except $\zeta = 10^{-16}\,\mathrm{s}^{-1}$, $f = 10^{-4}$



b) Complex chemical network

``Standard'' means

$a = 0.1\mu\mathrm{m}$, $f = 10^{-2}$

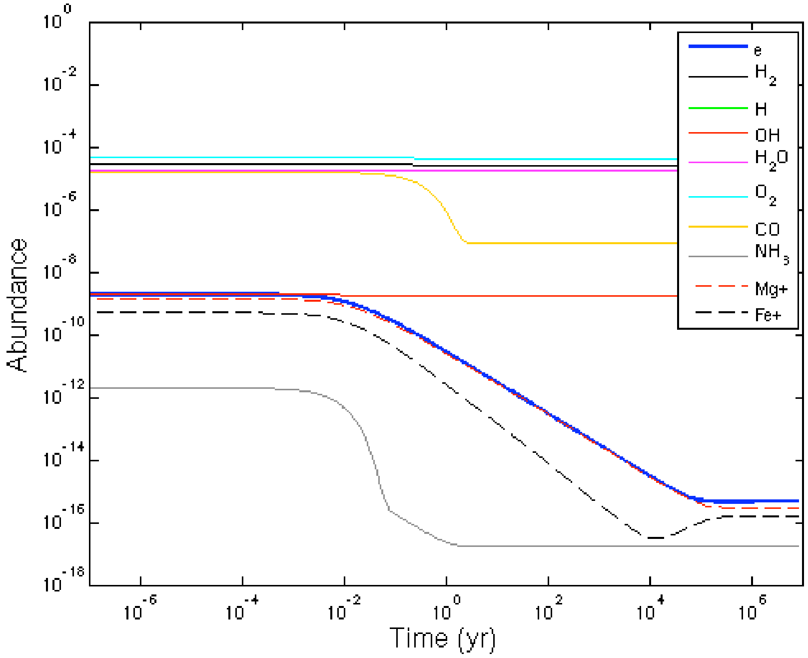
$\zeta_{\mathrm{CR}} = 0$

$L_X = 5\times10^{29}\,\mathrm{erg\ s^{-1}}$, $T_X = 3\ \mathrm{keV}$

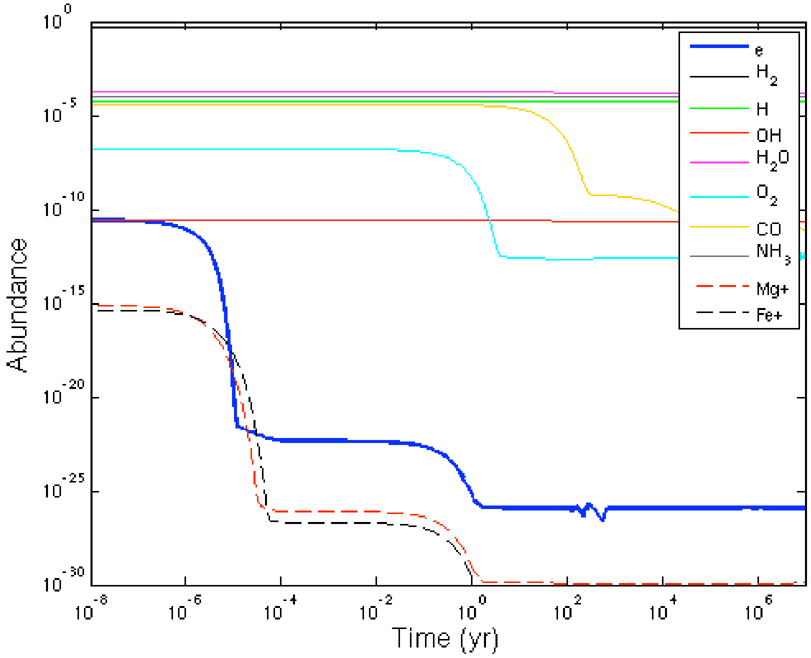
Grains hasten recombination when gas is "mixed" from shallow to deep

Run to equilibrium @ $\Sigma_{\text{above}} = 0.2\ \text{g cm}^{-2}$,

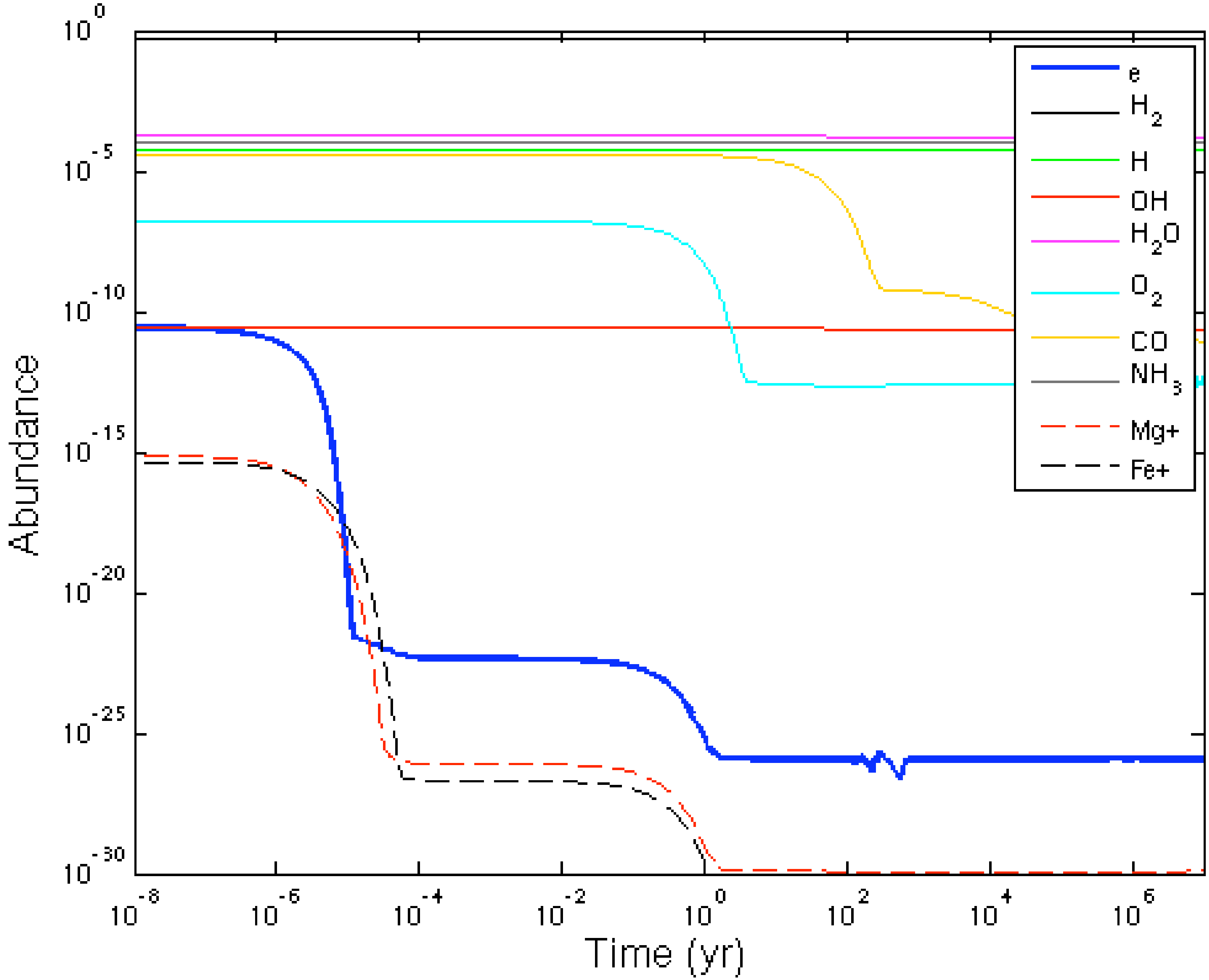
then plunge to $\Sigma_{\text{above}} = 135\ \text{g cm}^{-2}$



Without grains



With grains


Abundance
Time (yr)
10^{0}
10^{-5}
10^{-10}
10^{-15}
10^{-20}
10^{-25}
10^{-30}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
10^{0}
10^{2}
10^{4}
10^{6}
e
H_2
H
OH
H_2O
O_2
CO
NH_3
Mg+
Fe+

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The heat of accretion

The heat of accretion has to be radiated: at $r \gg r_{\text{in}}$,

$$F = \sigma T_{\text{eff}}^4 \approx \frac{3\dot{M}\Omega^2}{8\pi} \quad \Rightarrow \; T_{\text{eff}} \approx 150\, \dot{M}_{-7}^{1/4} r_{\text{AU}}^{-3/4}\ \text{K}.$$

One would expect the emissivity to be dominated by dust, and that the active layer would be optically thick:

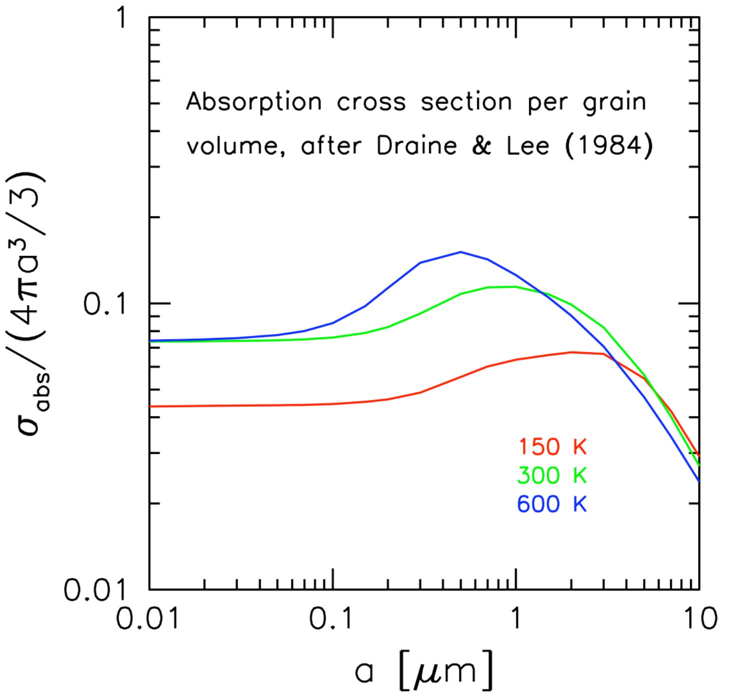
$$\varepsilon_a \approx 1-\exp\left(-\kappa_{\text{dust}}\Sigma_a\right)$$

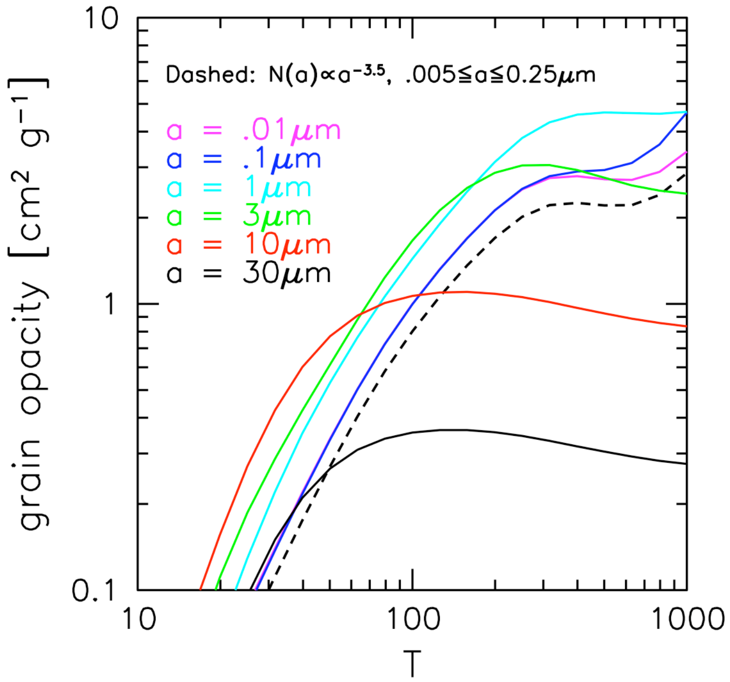
$\kappa_{\text{dust}} \gtrsim 1\ \text{cm}^2\text{g}^{-1}$ for ISM dust ($f = 10^{-2}$, $\bar{a} \sim 0.1\mu\text{m}$

$$\Rightarrow \tau_{\text{dust}} \gtrsim 10^3 f\,\Sigma_a \Big/ \left(10\ \text{g cm}^{-2}\right) \equiv 10 f_{-2}\Sigma_1$$

But τ_{dust} may have to be < 1 to allow Re_M>100

IR grain opacities



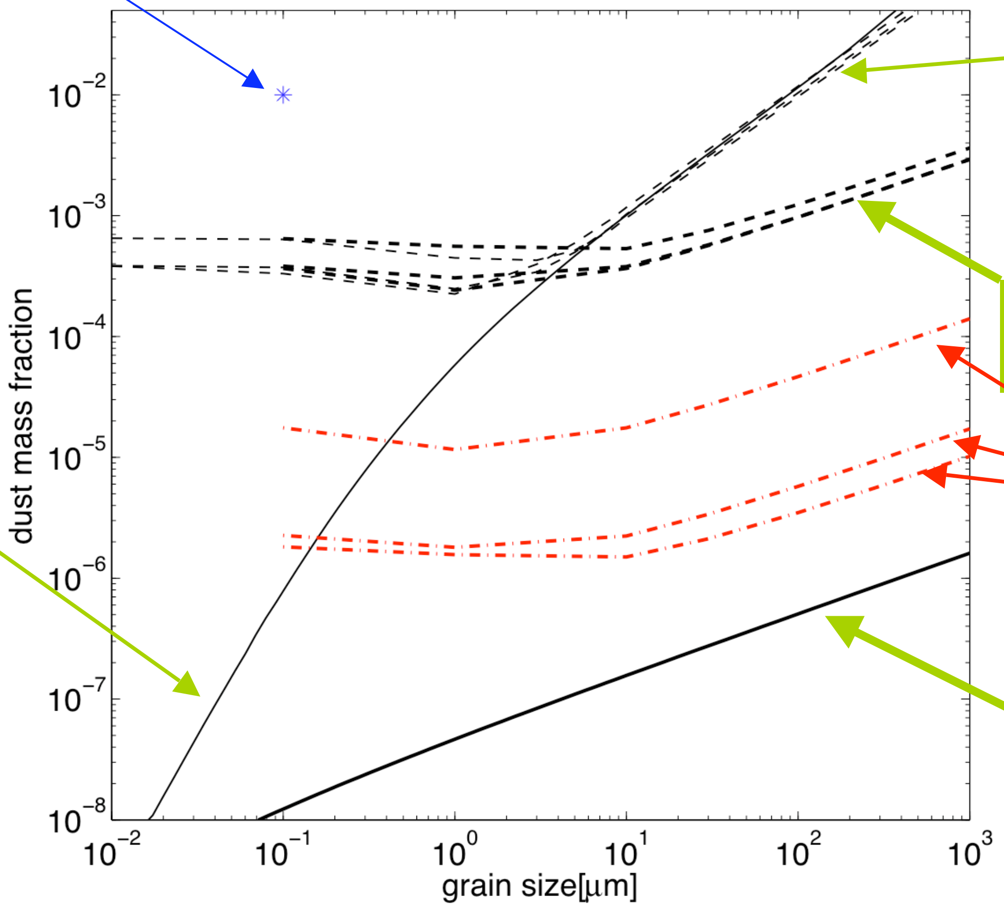


For $a_{gr} \ll \dfrac{\hbar c}{kT}$, opacity ($\kappa$) depends on total grain *mass;*

For $a_{gr} \gg \dfrac{\hbar c}{kT}$, κ depends on total grain area.

Combined constraints at 1 AU

Typical ISM dust



τ_d=1 for single-sized grains and
T=150,300,600K (light dashed)

τ_d=1 for MRN-sized grains,
T=150,300,600K (heavy dashed)

Maximum d.m.f. (*f*)
for $\Sigma_a \geq 10$ g cm^{-2}:
single-sized grains

$\varepsilon_d = \varepsilon_{mol}$
T=600,300,150K

Maximum f for MRN
size distribution
$N(a)da \propto a^{-3.5}da$
versus a_{max}, with
$a_{min} = 0.01\mu m$

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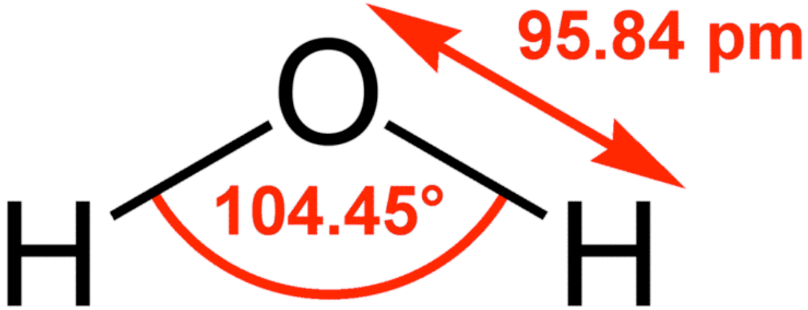
Molecular cooling

- H_2O, CO_2 are the main opacity sources (at thermal wavelengths) in the Earth's atmosphere
  - O_3, CH_4, NO are rarer but also significant
- These (& CO, NH_3,…) should be abundant in protostellar disks
  - similar temperatures
- Molecular lines are much narrower in PDs than in the atmosphere, however, because of lower pressures and densities
- This leads to gaps between the lines, and hence lower emissivity

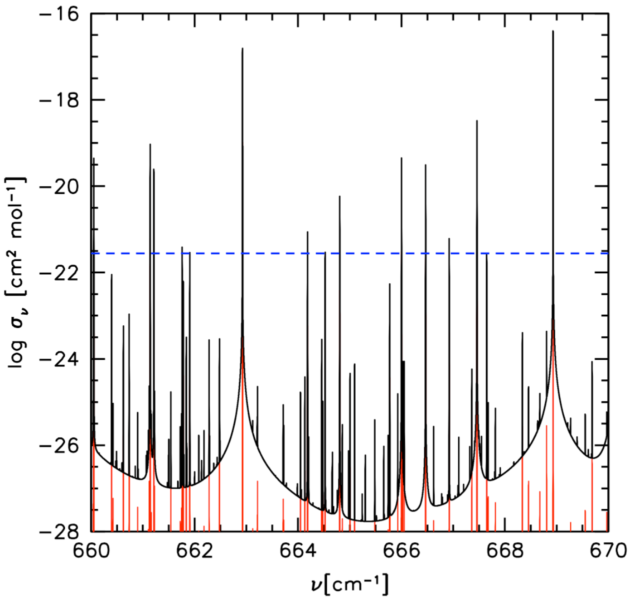
The water molecule: a brief introduction

- Tri-axial rotator:
  - $(I_x, I_y, I_z) \approx (1, 3, 2) \times 10^{-40}$ dyn cm^2
  - $\Rightarrow$ richer rotational spectrum than linear molecules (CO, CO_2)
- Lowest vibrational excitation ≈ 1500K
- Partition function at $<10^3$ K: $Z(T) \approx 180\ (T/300\mathrm{K})^{3/2}$
  - as for classical asymmetric top
  - this is ~ # excited energy levels
  - lowest vibrational mode ≈ 1500 K
- Dipole moment 1.86 Debye



Line broadening

$$P \approx \Sigma_a \Omega c_s z_a / h \approx 1 r_{\rm AU}^{-5/4} \text{ dyn cm}^{-2} \approx 1 r_{\rm AU}^{-5/4} \mu\text{bar}$$



Doppler broadening dominates by $> \times 10^4$

(In the atmosphere, $\Delta\nu_c \sim 100 \Delta\nu_D$)

Collisional broadening: $\Delta\nu_c = 8 \times 10^{-8} \left(\frac{P}{\mu\text{bar}}\right)\left(\frac{T}{300\text{K}}\right)^{-1/2} \text{cm}^{-1}$

Doppler broadening: $\Delta\nu_D = 1.5 \times 10^{-3} \left(\frac{T}{300\,\text{K}}\right)^{1/2} \left(\frac{\nu}{1000\,\text{cm}^{-1}}\right) \text{cm}^{-1}$

Natural width: $\Delta\nu_0 < 10^{-10} \text{cm}^{-1}$ for all excitations $< (3000\text{K}) k_\text{B}$

Molecular emissivity of active layer

- We use the H_2O line list of Barber et al (2006)
  - ~5×10^8 transitions, but only a few thousand matter here
- We calculate the specific intensity emerging from isothermal (T=300K) slabs of 10 g cm^{-2}, solar abundance of O, all in H_2O

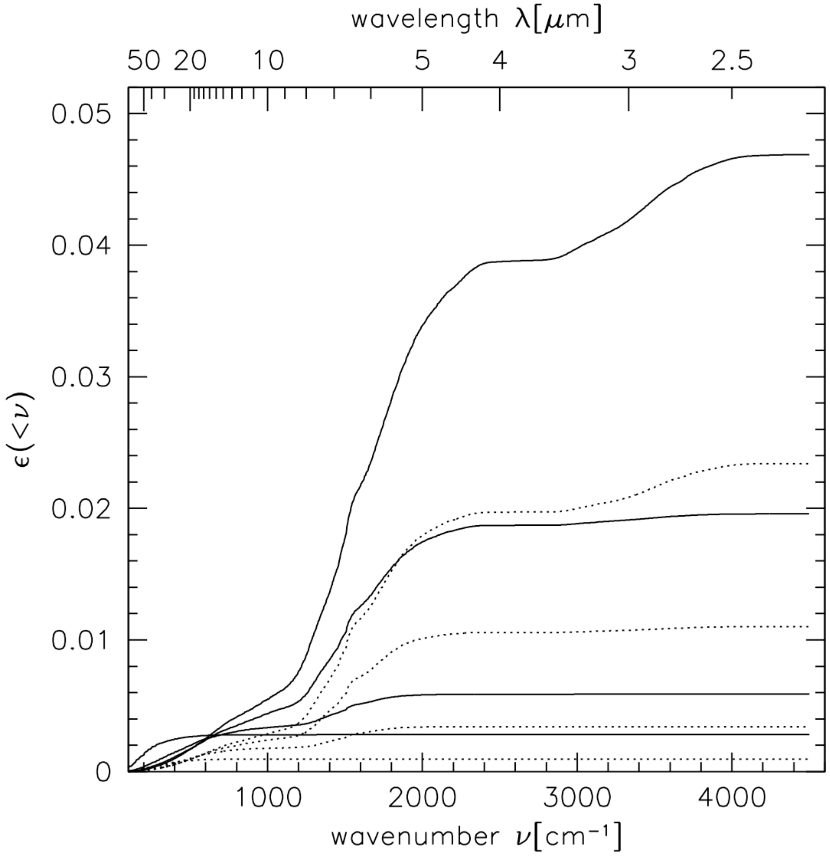
  $\Leftrightarrow 3.6\times10^{20}$ mol g^{-1}

Cumulative emissivity defined by a sum over saturated lines:

$$\epsilon(<\nu) = \frac{\pi}{\sigma T^4}\int_0^{\nu} I_{\nu'}\,d\nu' \approx \frac{\pi}{\sigma T^4}\sum_{\nu_k\le\nu} B_{\nu_k}(T)\int_0^{\nu}\left\{1-\exp\left[-NI_k\phi(\nu'-\nu_k)\right]\right\}d\nu',$$

- This ignores line overlap: a very good approximation

Emissivities (continued)



Solid curves: $\Sigma_a = 10 \text{ g cm}^{-2}$; Dashed: 1 g cm^{-2}

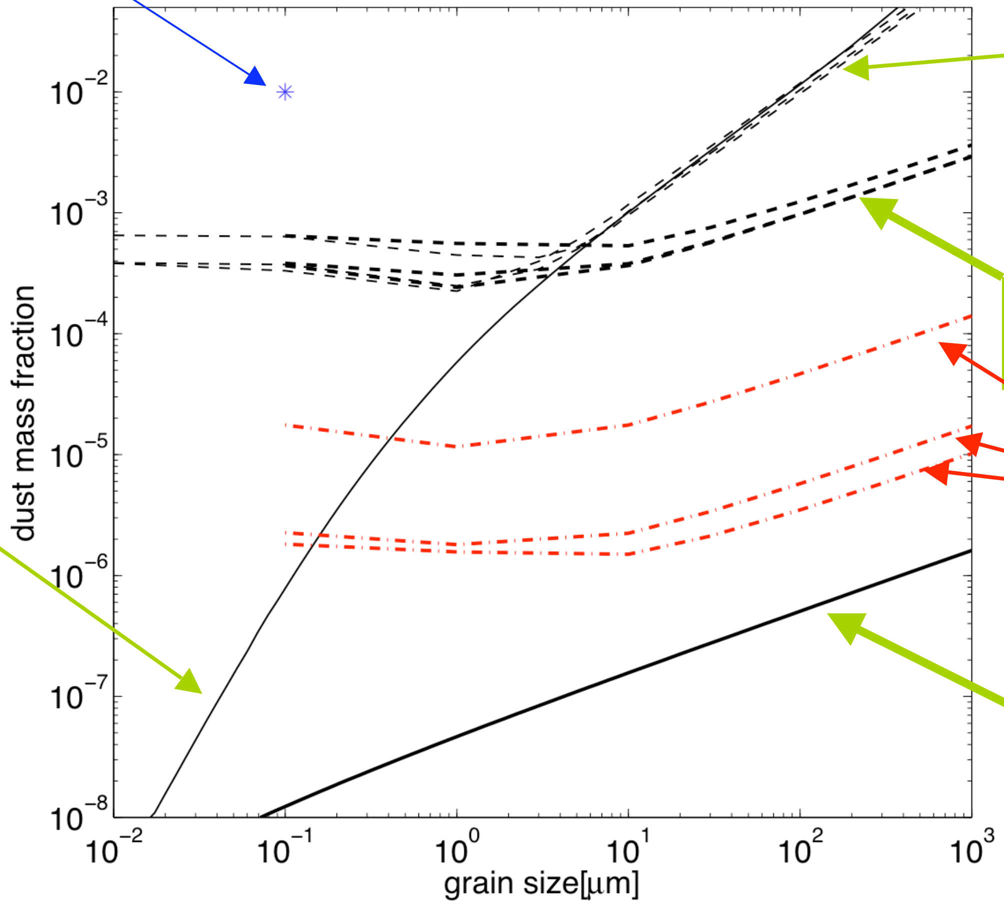
Top to bottom: $T = 600, 450, 300, 150$ K

Hence total emissivity is typically $\sim 10^{-2}$ for $\Sigma_a \sim 1 - 10 \text{ g cm}^{-2}$.

Compare the Planck-averaged opacity: $\kappa_P(H_2O) \approx 8 \text{ cm}^2 \text{ g}^{-1}$

Combined constraints at 1 AU

Typical ISM dust



τ_d=1 for single-sized grains and T=150,300,600K (light dashed)

τ_d=1 for MRN-sized grains, T=150,300,600K (heavy dashed)

Maximum d.m.f. (f) for $\Sigma_a \geq 10$ g cm^{-2} : single-sized grains

$\varepsilon_d = \varepsilon_{mol}$
T=600,300,150K

Maximum f for MRN size distribution $N(a)da \propto a^{-3.5}da$ versus a_{max}, with $a_{min} = 0.01\mu$m

Evidence for small dust

- Flat IR SEDs
  - equilibrium temperature of submicron grains is larger by $\sim(\varepsilon_{vis}/\varepsilon_{IR})^{1/4}$
- Silicate features indicate presence of $\leq 0.1\mu m$ grains
  - especially chrystalline features: enstatite, fosterite
- PAH features in some disks
- Importantly, these signatures of small grains are seen even in "old" PDs
- This is not to deny the evidence for grain growth (e.g. from sub-mm data), but it does suggest that some process replenishes the small-grain population, even as the bulk of the grain mass shifts to larger sizes
- And even a small mass in small grains has an enormous effect on Σ_a

Summary

- Ionization $x_e \geq 10^{-11}$-10^{-10} and magnetic fields $\geq$ 3 G are required at 1 AU to sustain accretion rates ~ $10^{-7}\ M_{\odot}\mathrm{yr}^{-1}$ in MMSN
  - Influence of magnetic field on ion-neutral drift is important here
- With standard ionization sources (X-rays, CR), such x_e require small grains to be suppressed by ~10^{-4} compared to ISM
- Active layers should be at least marginally optically thin, even if submicron grains are entirely absent, and perhaps so thin that molecular emission lines may dominate the cooling
  - thus molecular line observations may probe physical conditions in active layers

Questions for future research

- Have we overlooked a major source of nonthermal ionization?
  - Only ~10^{-4} of the locally dissipated energy would need to be invested in such sources (Particle acceleration in reconnection events? Lightning?)
  - How do we calculate such sources, or how might they be confirmed observationally?
- The thermal structure of turbulent active layers needs to be explicitly modeled in simulations
  - Can temperatures approaching 1000 K occur in the upper parts of the layer (where $\Sigma \leq 0.1$ g cm^{-2}) ?
  - Does true stratification ($dS/dz > 0$) inhibit mixing?