Heat and Dust in Active Layers of Protostellar Disks

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Prelude: Hot water at 1 AU

- DR Tau & AS 205 were selected for high accretion rates: $\geq 10^{-7} M_\odot yr^{-1}$
- Inferred physical conditions: $T \approx 1000 K$, $r \approx 1-3$ AU, $N(H_2O) \approx 10^{18}$ cm$^{-2}$

$\Rightarrow \Sigma \geq 0.1$ g cm$^{-2}$
Why study MRI in Protostellar Disks?

- The observational constraints are good
  - The only angularly resolved accretion disks (except galactic disks)
  - Well-determined accretion rates (from boundary-layer emission)
- The conditions for MRI are marginal (perhaps), and marginal cases can be instructive
  - Low electrical conductivity, extremely low $Pm = \nu/\eta$
- The contingency of turbulence may be important to planet formation
  - high densities & dust settling in dead zones
  - gap formation
  - etc.
The importance of dust

- PSDs are detected and characterized by their IR excesses
  - disk mass
    - requires temperature profile, emissivity, dust-to-gas ratio
  - lifetime
    - via IR excess vs. stellar age
  - geometry
    - inner & outer disk radii, gaps, flaring, warps

- Refractory elements in dust are the precursors of planets

- In the solar system, dust (in comets & meteorites) bears a fossil record of the primordial nebula

- Dust controls the coupling of the disk to magnetic fields
Central idea of this work

• The requirements of MHD/MRI give one set of constraints on the dust
• Observed emissions (IR SEDs, silicate & PAH features, molecular lines) give another set
• Let’s try to combine these
• There are some obvious difficulties here
  – uncertainties in grain growth & size distribution
  – uncertainties in ionization rates
  – poor angular resolution (10s of AU at present)
  – immaturity of MRI simulations regarding microphysics, thermodynamics and resolution studies (this is evolving, of course)
Outline of this talk

- Introduction
- Review of required magnetic fields & ionization levels
- (Re)calculation of conductivity & active surface density ($\Sigma_a$) in the presence of grains
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- Summary and Discussion
Observed accretion rates


\[ L_{\text{boundary layer}} \approx \frac{GM^*}{R^*} \dot{M} \]
Minimum Mass Solar Nebula

\[ \Sigma = 1700 r_{\text{AU}}^{-3/2} \text{ g cm}^{-2} \quad T = 280 K r_{\text{AU}}^{-1/2} \quad c_s = 1.0 r_{\text{AU}}^{-1/4} \text{ km s}^{-1} \]

Vertically isothermal: \( \rho(r, z) = \rho_0(r) \exp(-z^2 / 2h^2) \)

\[ \rho_0(r) = \frac{\Sigma}{h \sqrt{2\pi}} = 1.4 \times 10^{-9} r_{\text{AU}}^{-11/4} \text{ g cm}^{-3} \]

Flared disk:

\[ \frac{h}{r} = \frac{c_s}{\Omega r} \approx 0.03 r_{\text{AU}}^{1/4} \]
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Required Magnetic Field

Conservation of angular momentum implies, at \( r \gg r_{\text{in}} \),

\[
\dot{M} \Omega r^2 + 2\pi r^2 \int_{-\infty}^{\infty} \left( \rho v'_r v'_\phi - \overline{B_r B_\phi} / 4\pi \right) dz \approx 0.
\]

But Maxwell stress \( |\overline{B_r B_\phi} / 4\pi| \gg \) Reynolds stress \( \left| \rho v'_r v'_\phi \right| \)

So \( \left| B_r B_\phi \right| \gtrsim \dot{M} \Omega / h_a \)

where \( h_a \approx 0.5 c_s / \Omega \) is the thickness of active layer (one side)

\[
\therefore B = \sqrt{B_r^2 + B_\phi^2 + B_z^2} \gtrsim 3 M_{-7}^{1/2} r_{\text{AU}}^{-9/8} \text{ Gauss}
\]

\[ \Rightarrow \text{Equipartion:} \]

\[
\frac{P_{\text{gas}}}{P_{\text{mag}}} \lesssim 1.0 \Sigma_1 \dot{M}_{-7}^{-1} r_{\text{AU}} ; \quad \Sigma_1 \equiv \Sigma_a / 10 \text{ g cm}^{-2}
\]
Required ionization

An **Elsasser number** \( \Lambda \equiv \frac{V^2}{\eta \Omega} \gtrsim 1 \) is required for MRI.

Ohm's Law is tensorial: \( \bar{J} = \bar{\sigma} \cdot \bar{E} \) (e.g. Wardle 2007)

Tensorial conductivity \( \bar{\sigma} \) involves ionization \((x_e, x_i)\) but also \( \bar{B} \) via the **Hall parameters**: \( \beta_j \equiv (\text{collision time})/(\text{cyclotron time}) \) for species \( j \).

\[
\partial_t \bar{B} - \nabla \times (\bar{v} \times \bar{B}) = -\frac{c^2}{4\pi} \nabla \times (\bar{\sigma}^{-1} \cdot \nabla \times \bar{B})
\]

\( \Rightarrow \) In Elsasser number, \( \eta \propto \text{largest eigenvalue of } \bar{\sigma}^{-1} \).
Required ionization (continued)

In the extreme Hall regime ($\beta_i \ll 1 \ll \beta_e$),

$$\Lambda \rightarrow \Lambda_H = \frac{en_e B}{\rho \Omega c} \propto x_e B/\Omega$$

$$\beta_e \approx 100 \Sigma^{-1} B_0 r_{AU}^{3/2} \gtrsim 300 \Sigma^{-1} \dot{M}_{-7}^{1/2} r_{AU}^{1/8}$$

$$\beta_i \approx 0.2 \Sigma^{-1} B_0 r_{AU}^{5/4} \gtrsim 0.6 \Sigma^{-1} \dot{M}_{-7}^{1/2} r_{AU}^{-1/8}$$

in the MMSN.

So we're probably in the Hall regime: $\beta_i < 1 \ll \beta_e$

or possibly in the ambipolar regime: $1 < \beta_i \ll \beta_e$

$$\Rightarrow x_e \approx 10^{-10} \Lambda \dot{M}_{-7}^{-1/2} r_{AU}^{-1/4} \text{ (MMSN, Hall regime)}$$
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There have been many calculations of active layers & conductivity

- **Gammie (1996)**
  - cosmic rays, no grains, \( \text{Re}_{M,\text{crit}} = 1 \)
  - \( \Sigma_a(1 \text{ AU}) \approx 100 \text{ g cm}^{-2} \)
- **Glassgold, Najita, & Igea (1997); Igea & Glassgold (1999)**
  - X-rays, no grains
  - \( \Sigma_a(1 \text{ AU}) \approx 40 \text{ g cm}^{-2} \)
  - enlarged chemical reaction network
  - \( a = 0.1 \mu\text{m grains} \)
  - \( \Sigma_a(1 \text{ AU}) \approx 0 \text{ g cm}^{-2} \) at \( f = 10^{-2} \)
  - \( \Sigma_a(1 \text{ AU}) \approx 70 \text{ g cm}^{-2} \) at \( f = 10^{-6} \)
  (\( f \) is dust mass fraction)
- **Fromang, Terquem & Balbus (2002)**
  - metal ions (Mg, Fe.); no grains
  - \( \alpha \) disks not MMSN \( (\alpha = 10^{-2}, \dot{M}_{\text{\tiny -7}} = 1) \)
  - \( \Sigma_a(1 \text{ AU}) \approx 30 \text{ g cm}^{-2} \)
- **Semenov, Wiebe, & Henning (2004)**
  - \( a = 0.1 \mu\text{m grains} \)
  - improved chemistry
  - \( \alpha \) disk (as above)
  - \( \Sigma_a(1 \text{ AU}) \approx 200 \text{ g cm}^{-2} \) (?whole disk)
- **Ilgner & Nelson (2006a) [IN06a]**
  - Much extended network
  - 0.1 \( \mu\text{m grains} \)
  - \( \alpha \) disk (as above)
  - \( \Sigma_a(1 \text{ AU}) \approx 10 \text{ g cm}^{-2} \) for \( f=10^{-6} \)
  ("model7")
Active-layer calculations (continued)

- Ilgner & Nelson (2006b,c; 2008)
  - like IN06a, plus turbulent mixing but no grains
- Wardle (2007)
  - MMSN
  - tensorial conductivity
  - simple chemistry
  - 0.1-3.0 µm grains, \( f = 10^{-2} \approx f_{\text{ISM}} \)
  - \( \Sigma_a(1\text{AU}) \approx 2 \text{ g cm}^{-2} \) for 0.1µm grns.
  - \( \Sigma_a(1\text{AU}) \approx 80 \text{ g cm}^{-2} \) for 3 µm grns.
- Salmeron & Wardle (2008)
  - like Wardle (2007), but only for radii 5 & 10 AU

- **This study**: like Ilgner & Nelson (2006), with some improvements:
  - Enlarged & updated chemistry based on UMIST06 vs. UMIST95
  - MMSN vs. \( \alpha \) disk
  - H ionization & H\(_2\) formation on grains
  - Two grain populations with variable sizes & mass fractions
    - 10\(^{-2}\) µm \( \leq a_1 \leq a_2 \leq 10 \) µm
    - 0 \( \leq f_1 + f_2 \leq 10^{-2} = f_{\text{ISM}} \)
  - Variation of X-ray flux, X-ray temperature, & CR ionization parameter
Magnetic Reynolds number

We actually use the criterion

\[ \text{Re}_M \equiv \frac{c_s^2}{\eta \Omega} \geq 100 \]

for active zones,

where \( \eta = \eta_{\text{Ohmic}} \approx 230 T^{1/2} x_e^{-1} \text{ cm}^2 \text{ s}^{-1} \).

\[ \Rightarrow x_e \gtrsim 10^{-11} r_{\text{AU}}^{-5/4} \left( \text{cf. } 10^{-10} M_{-7}^{-1/2} r_{\text{AU}}^{-1/4} \text{ for Hall regime} \right) \]

This doesn't involve \( \vec{B} \) or \( \dot{M} \) explicitly.

However, \( \eta_{\text{Hall}} \approx \beta_e \eta_{\text{Hall}} \) in the Hall regime, and \( \beta_e \gtrsim 10^2 \).

The conductivity required to support accretion at high rates is larger than for linear stability, because the tensorial diffusivity increases with magnetic field strength.
X-ray & CR ionization rates

X-rays only: $L_X = 5 \times 10^{29} \text{ erg s}^{-1}$, $T_X = 3 \text{ keV}$

Cosmic rays only: $\zeta_{CR} = 10^{-17} \text{ s}^{-1} \text{ molecule}^{-1}$

Both X-rays & CR
**Electron abundance**

- **Complex chemical network**

\[ \rho \approx 10^{-3} \text{ g cm}^{-3} \text{ at 1 AU} \]

**Standard parameters:**
- Grain size: \( a = 0.1 \mu \text{m} \)
- Gr. mass fraction: \( f = 10^{-2} \)
- Metal abundance: \( x_M = 1.25 \times 10^{-8} \)

\[ \Sigma_a \approx 10 \text{ g cm}^{-3} \text{ at 1 AU} \]
Electron abundance depends roughly on total grain area

Heavy lines: constant grain area,

\[ f = 10^{-4} \left( \frac{a}{1 \mu m} \right) \]

...hence \( f = 10^{-3} f_{\text{ISM}} \) if \( a = 0.1 \mu m \)
$\Sigma_a$ is very sensitive to grains

Standard, except $\zeta = 10^{-15}\text{s}^{-1}$, $f = 10^{-4}$

Standard, except $\zeta = 10^{-16}\text{s}^{-1}$, $f = 10^{-4}$

``Standard'' means

\begin{align*}
  a &= 0.1\mu\text{m}, f = 10^{-2} \\
  \zeta_{\text{CR}} &= 0 \\
  L_X &= 5 \times 10^{29}\text{erg s}^{-1}, T_X = 3\text{keV}
\end{align*}
Grains hasten recombination when gas is “mixed” from shallow to deep

Run to equilibrium @ $\Sigma_{\text{above}} = 0.2 \ \text{g cm}^{-2}$, then plunge to $\Sigma_{\text{above}} = 135 \ \text{g cm}^{-2}$

Without grains

With grains
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The heat of accretion

The heat of accretion has to be radiated: at $r \gg r_{\text{in}}$, 

$$F = \sigma T_{\text{eff}}^4 \approx \frac{3\dot{M}\Omega^2}{8\pi} \implies T_{\text{eff}} \approx 150 \dot{M}^{1/4} r_{\text{AU}}^{-3/4} \text{K}.$$ 

One would expect the emissivity to be dominated by dust, and that the active layer would be optically thick:

$$\varepsilon_a \approx 1 - \exp\left(-\kappa_{\text{dust}} \Sigma_a\right)$$

$$\kappa_{\text{dust}} \gtrsim 1 \text{ cm}^2\text{g}^{-1} \text{ for ISM dust } (f = 10^{-2}, \bar{a} \sim 0.1\mu\text{m})$$

$$\implies \tau_{\text{dust}} \gtrsim 10^3 f \Sigma_a / (10 \text{ g cm}^{-2}) \equiv 10 f^{-2} \Sigma_1$$

**But $\tau_{\text{dust}}$ may have to be $< 1$ to allow $Re_M > 100$**
IR grain opacities

For $a_{gr} \ll \frac{\hbar c}{kT}$, opacity ($\kappa$) depends on total grain mass;

For $a_{gr} \gg \frac{\hbar c}{kT}$, $\kappa$ depends on total grain area.

Absorption cross section per grain volume, after Draine & Lee (1984)

Dashed: $N(a) \propto a^{-3.5}, 0.005 \leq a \leq 0.25 \mu m$

- $a = 0.01 \mu m$
- $a = 0.1 \mu m$
- $a = 1 \mu m$
- $a = 3 \mu m$
- $a = 10 \mu m$
- $a = 30 \mu m$
**Combined constraints at 1 AU**

- **Typical ISM dust**

- **Maximum d.m.f. (\(f\)) for \(\Sigma_a \geq 10 \text{ g cm}^{-2}\): single-sized grains**

- \(\tau_d = 1\) for single-sized grains and \(T = 150, 300, 600 \text{ K}\) (light dashed)

- \(\tau_d = 1\) for MRN-sized grains, \(T = 150, 300, 600 \text{ K}\) (heavy dashed)

- \(\epsilon_d = \epsilon_{\text{mol}}\)
  - \(T = 600, 300, 150 \text{ K}\)

- Maximum \(f\) for MRN size distribution
  - \(N(a)da \propto a^{-3.5} da\)
  - versus \(a_{\text{max}}\), with
  - \(a_{\text{min}} = 0.01 \mu m\)
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Molecular cooling

- \( \text{H}_2\text{O}, \text{CO}_2 \) are the main opacity sources (at thermal wavelengths) in the Earth’s atmosphere
  - \( \text{O}_3, \text{CH}_4, \text{NO} \) are rarer but also significant
- These (& CO, \( \text{NH}_3 \),…) should be abundant in protostellar disks
  - similar temperatures
- Molecular lines are much narrower in PDs than in the atmosphere, however, because of lower pressures and densities
- This leads to gaps between the lines, and hence lower emissivity
The water molecule: a brief introduction

- Tri-axial rotator:
  - \((I_x, I_y, I_z) \approx (1, 3, 2) \times 10^{-40}\text{ dyn cm}^2\)
  \(\Rightarrow\) richer rotational spectrum than linear molecules (CO, CO\(_2\))
- Lowest vibrational excitation \(\approx 1500\)K
- Partition function at \(<10^3\) K:
  \(Z(T) \approx 180 (T/300\text{K})^{3/2}\)
  - as for classical asymmetric top
  - this is \(\sim\) # excited energy levels
  - lowest vibrational mode \(\approx 1500\) K
- Dipole moment 1.86 Debye

Astrophysical MRI
Ringberg 14-18.04.2009

Heat and Dust in Active Layers
Line broadening

\[ P \approx \sum_a \Omega c_s z_a / h \approx 1 r_{\text{AU}}^{-5/4} \quad \text{dyn cm}^{-2} \approx 1 r_{\text{AU}}^{-5/4} \mu\text{bar} \]

**Doppler broadening dominates by \( > \times 10^4 \)**

(In the atmosphere, \( \Delta \nu_c \sim 100 \Delta \nu_D \))

Collisional broadening: \( \Delta \nu_c = 8 \times 10^{-8} \left( \frac{P}{\mu\text{bar}} \right) \left( \frac{T}{300\text{K}} \right)^{-1/2} \text{ cm}^{-1} \)

Doppler broadening: \( \Delta \nu_D = 1.5 \times 10^{-3} \left( \frac{T}{300\text{K}} \right)^{1/2} \left( \frac{\nu}{1000\text{ cm}^{-1}} \right) \text{ cm}^{-1} \)

Natural width: \( \Delta \nu_0 < 10^{-10}\text{ cm}^{-1} \) for all excitations \(< \left( 3000\text{K} \right) k_B \)
Molecular emissivity of active layer

- We use the H$_2$O line list of Barber et al (2006)
  - ~5x10$^8$ transitions, but only a few thousand matter here
- We calculate the specific intensity emerging from isothermal (T=300K) slabs of 10 g cm$^{-2}$, solar abundance of O, all in H$_2$O
  \[ \rightleftharpoons 3.6 \times 10^{20} \text{ mol g}^{-1} \]

Cumulative emissivity defined by a sum over saturated lines:

\[
\epsilon(< \nu) = \frac{\pi}{\sigma T^4} \int_0^\nu I_{\nu'} d\nu' \approx \frac{\pi}{\sigma T^4} \sum_{\nu_k \leq \nu} B_{\nu_k}(T) \int_0^\nu \left\{ 1 - \exp \left[ -NI_k \phi(\nu' - \nu_k) \right] \right\} d\nu',
\]

- This ignores line overlap: a very good approximation
Emissivities (continued)

Solid curves: $\Sigma_a = 10 \text{ g cm}^{-2}$; Dashed: $1 \text{ g cm}^{-2}$
Top to bottom: $T = 600, 450, 300, 150 \text{ K}$

Hence total emissivity is typically $\sim 10^{-2}$ for $\Sigma_a \sim 1 - 10 \text{ g cm}^{-2}$.
Compare the Planck-averaged opacity: $\kappa_p(\text{H}_2\text{O}) \approx 8 \text{ cm}^2 \text{ g}^{-1}$
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Heat and Dust in Active Layers

Combined constraints at 1 AU

Typical ISM dust

Maximum d.m.f. \( f \) for \( \Sigma_a \geq 10 \text{ g cm}^{-2} \): single-sized grains

\( \tau_d = 1 \) for single-sized grains and \( T=150,300,600 \text{K} \) (light dashed)

\( \tau_d = 1 \) for MRN-sized grains, \( T=150,300,600 \text{K} \) (heavy dashed)

\( \epsilon_d = \epsilon_{\text{mol}} \)
\( T=600,300,150 \text{K} \)

Maximum \( f \) for MRN size distribution
\( N(a)da \propto a^{-3.5} \)
versus \( a_{\text{max}} \), with
\( a_{\text{min}} = 0.01 \mu\text{m} \)
Evidence for small dust

- Flat IR SEDs
  - equilibrium temperature of submicron grains is larger by \( \sim (\epsilon_{\text{vis}}/\epsilon_{\text{IR}})^{1/4} \)
- Silicate features indicate presence of \( \leq 0.1\mu m \) grains
  - especially chrystalline features: enstatite, fosterite
- PAH features in some disks
- Importantly, these signatures of small grains are seen even in “old” PDs
- This is not to deny the evidence for grain growth (e.g. from sub-mm data), but it does suggest that some process replenishes the small-grain population, even as the bulk of the grain mass shifts to larger sizes
- And even a small mass in small grains has an enormous effect on \( \Sigma_a \)
Summary

• Ionization $x_e \geq 10^{-11}$-$10^{-10}$ and magnetic fields $\geq 3$ G are required at 1 AU to sustain accretion rates $\sim 10^{-7} \, M_\odot \, yr^{-1}$ in MMSN
  – Influence of magnetic field on ion-neutral drift is important here

• With standard ionization sources (X-rays, CR), such $x_e$ require small grains to be suppressed by $\sim 10^{-4}$ compared to ISM

• Active layers should be at least marginally optically thin, even if submicron grains are entirely absent, and perhaps so thin that molecular emission lines may dominate the cooling
  – thus molecular line observations may probe physical conditions in active layers
Questions for future research

• Have we overlooked a major source of nonthermal ionization?
  – Only $\sim 10^{-4}$ of the locally dissipated energy would need to be invested in such sources (Particle acceleration in reconnection events? Lightning?)
  – How do we calculate such sources, or how might they be confirmed observationally?

• The thermal structure of turbulent active layers needs to be explicitly modeled in simulations
  – Can temperatures approaching 1000 K occur in the upper parts of the layer (where $\Sigma \leq 0.1$ g cm$^{-2}$)?
  – Does true stratification ($dS/dz > 0$) inhibit mixing?