SPECTROGRAPHS

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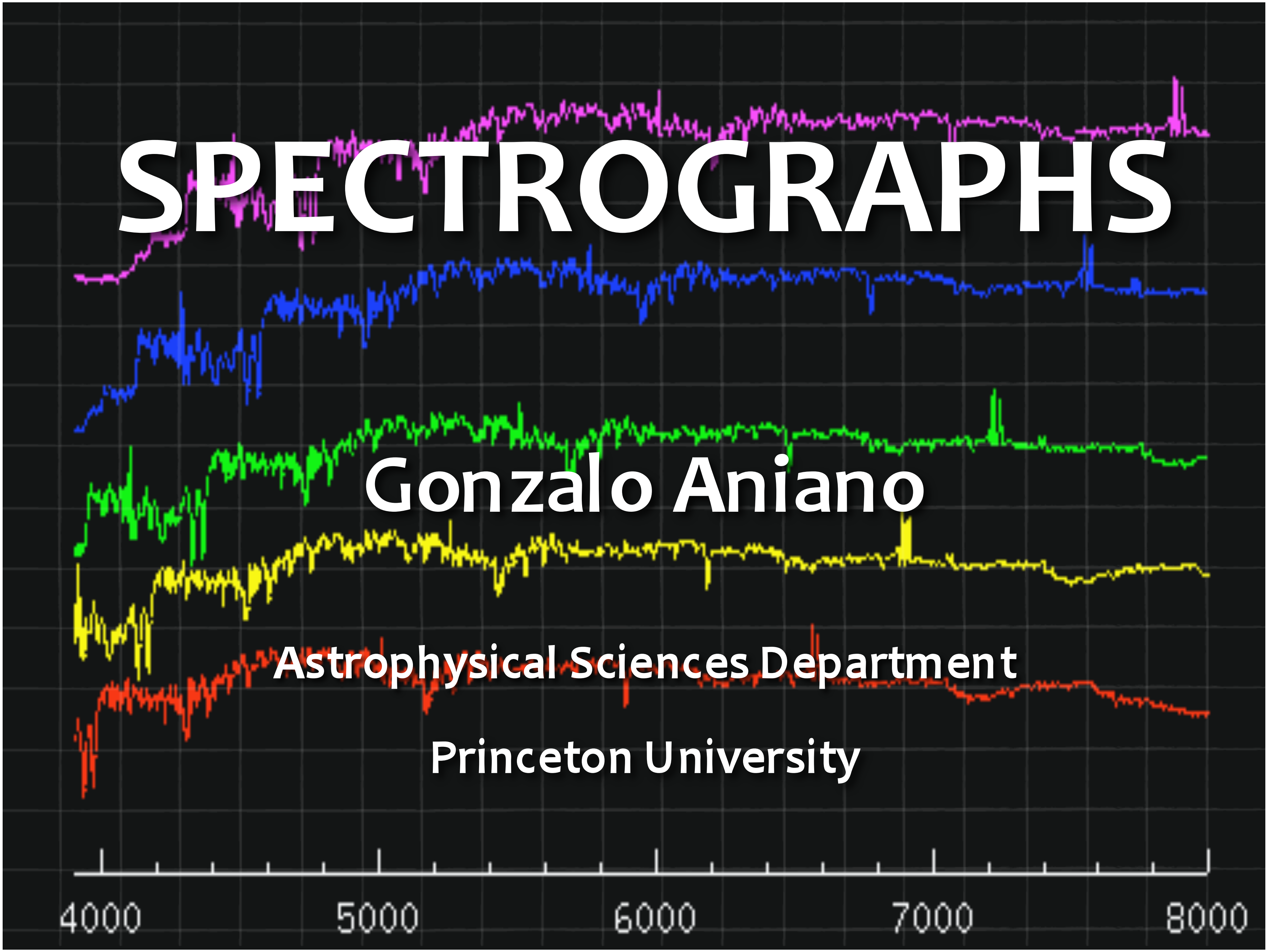

Motivation:

Spectral analysis of celestial objects is probably
the most important means for learning
about the physics of these sources,
with large fraction of telescope time
used to get spectral data...

Scheme of the talk

.Introduction

.Dispersing elements.

.Spectrographs design

.Limiting resolution

.Conclusions

.References

Dispersion elements

We define the angular dispersion of an element as:

$$A = \frac{d\beta}{d\lambda}$$

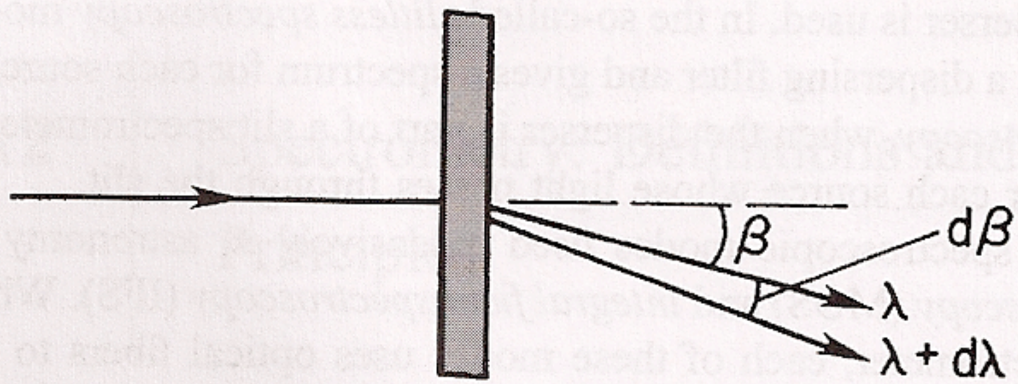

Fig. 12.1. Schematic of dispersive element. Angular dispersion $A = d\beta/d\lambda$.

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Dispersion elements

When we use a dispersing element in an optical system, it gives a linear dispersion in the focal surface of:

$$\frac{dl}{d\lambda} = f\,\frac{d\beta}{d\lambda} = fA$$

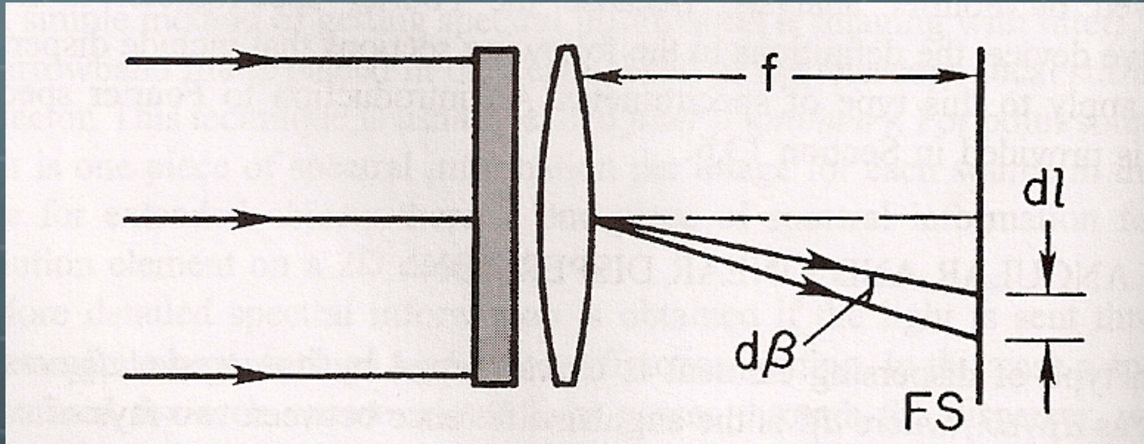

Fig. 12.2. Spectrum in focus on focal surface FS. Linear dispersion $= f\ d\beta/d\lambda$.

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Dispersion elements

A dispersing element in an optical system can also produce an anamorphic magnification:

$$r = \frac{d1}{d2}$$

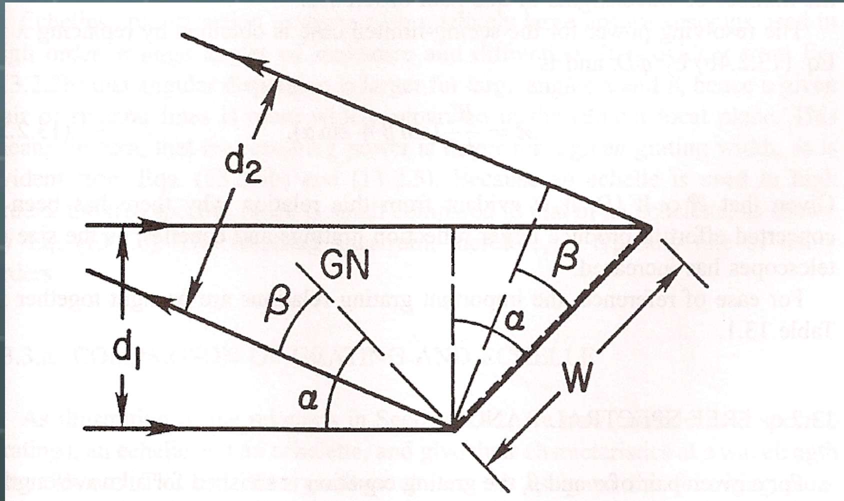

Fig. 13.3. Change in beam width due to anamorphic magnification of grating. See Eq. (13.2.3).

Prisms

Glass has a wavelength-dependant index of refraction:

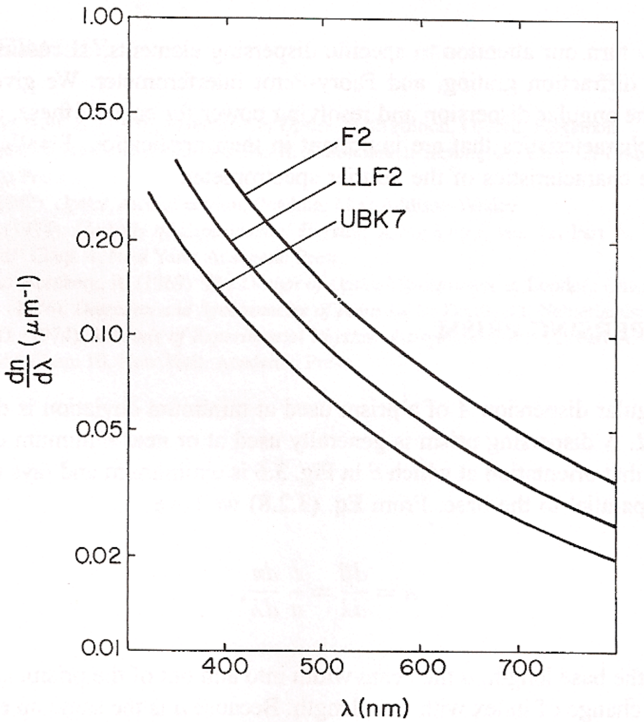

Fig. 13.1. Dispersion curves for three glasses from Schott glass catalog.

$$n = A + \frac{B}{\lambda + C}$$

	A	*B*	*C*
Crown glass	1.477	3.2×10^{-8}	-2.1×10^{-7}
Dense flint glass	1.603	2.08×10^{-8}	1.43×10^{-7}

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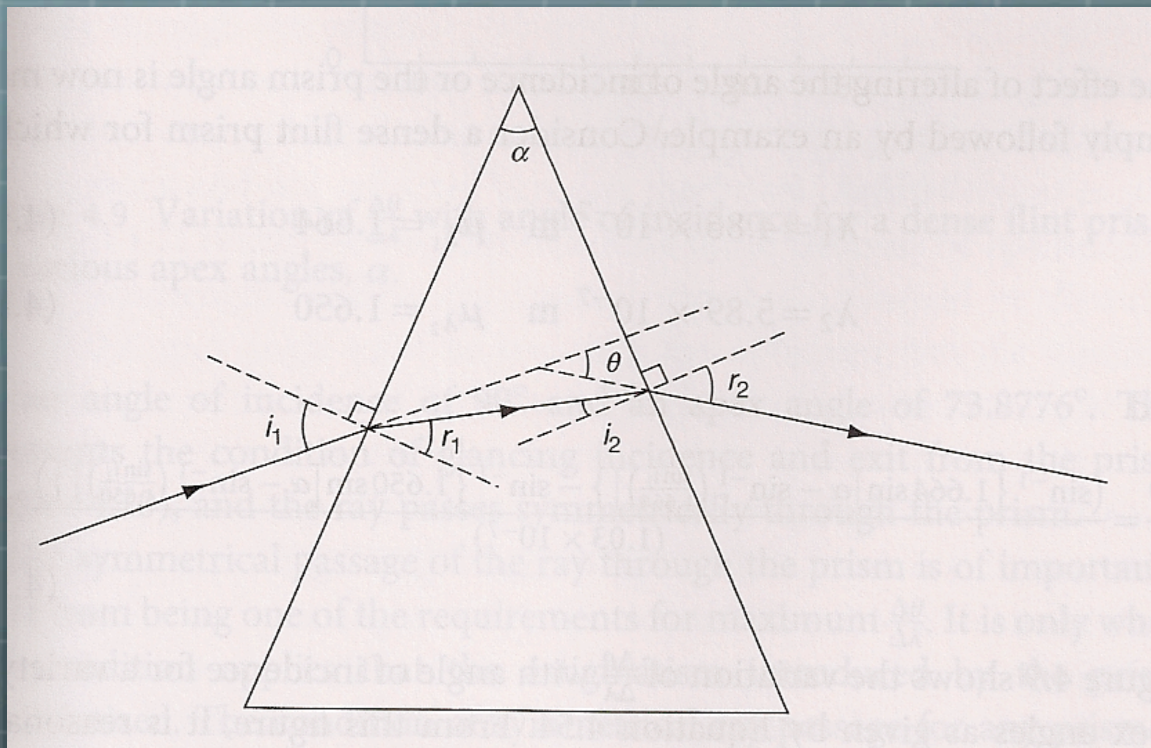
Prisms

Prism are generally used in minimum deviation configuration (minimum θ), where the rays are parallel to the base inside it:



Prisms

At minimum deviation configuration, we have:

$$A = \frac{d\beta}{d\lambda} = \frac{\text{Prism base length}}{\text{beam width}} \frac{dn}{d\lambda}$$

$$r = 1$$

In general, we have →

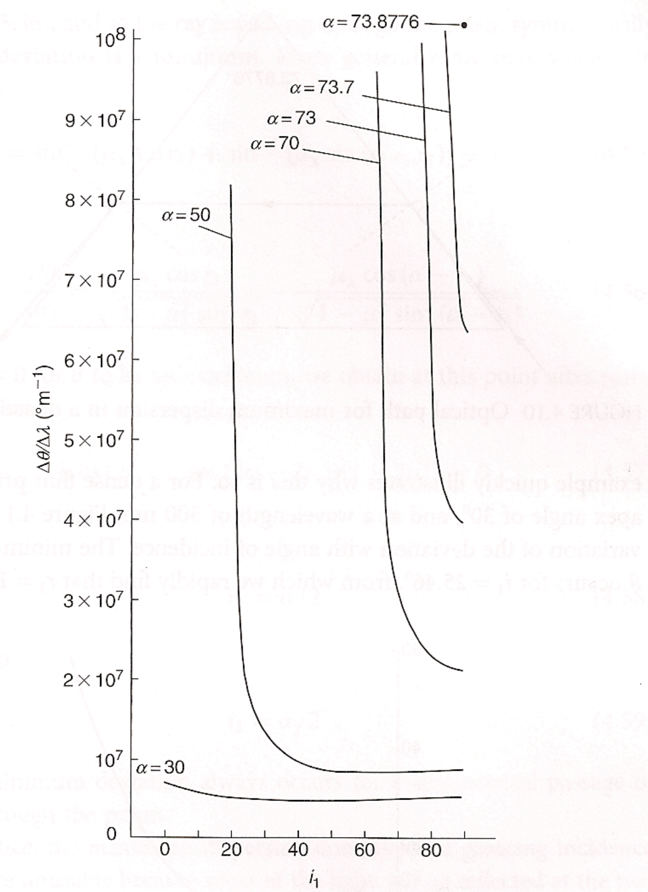
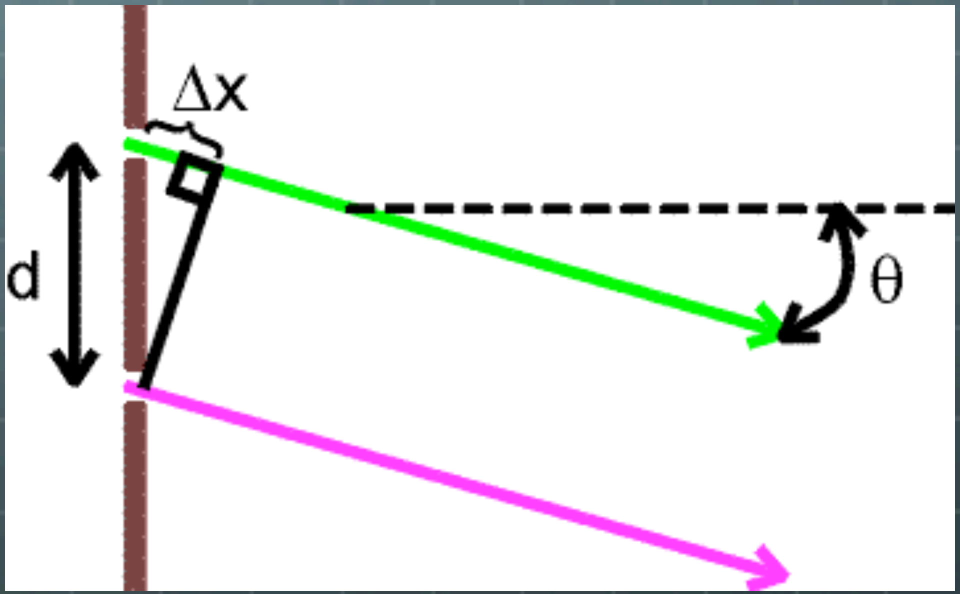


FIGURE 4.9 Variation of $\frac{\Delta\theta}{\Delta\lambda}$ with angle of incidence for a dense flint prism, for various apex angles, α.

Diffraction Gratings

In a simple two-aperture grating, the angular distance from the central maximum to the first fringe is λ/d, where d is the separation of the apertures.



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Diffraction Gratings

In general, as we increase the number of apertures:

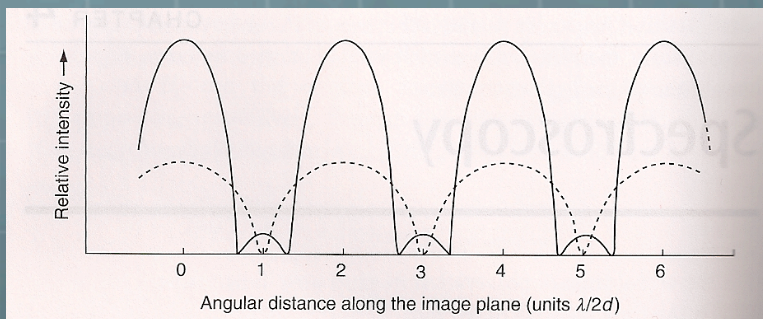
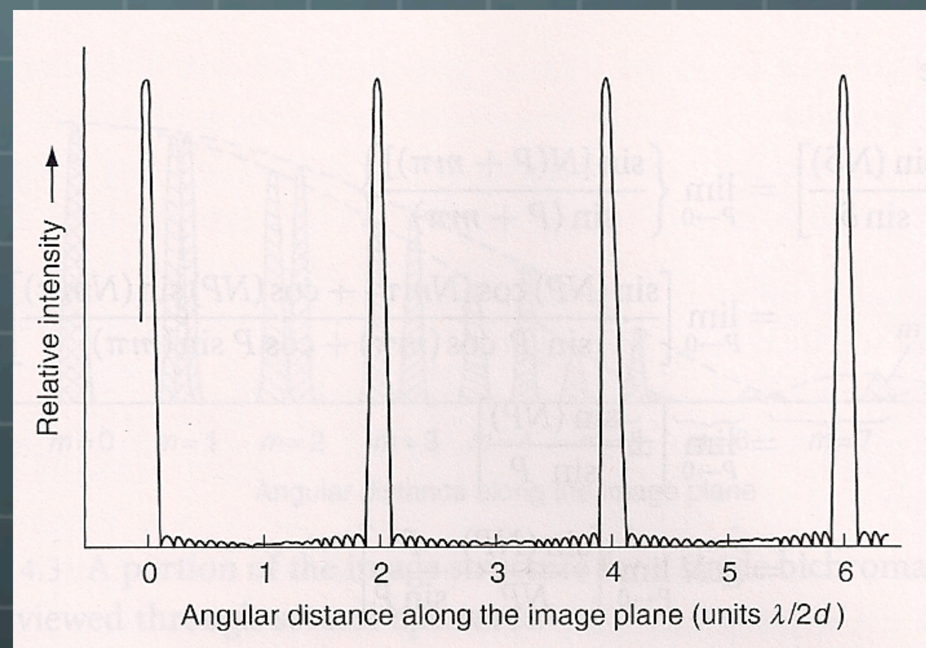


FIGURE 4.1 Small portion of the image structure for a single point source viewed through two apertures (broken curve) and three apertures (full curve).



Diffraction Gratings

In general we have:
d distance between apertures
D aperture diameter
N number of apertures

$$I(\theta) = I(0)\left[\frac{\sin^2\left(\frac{\pi D \sin\theta}{\lambda}\right)}{\left(\frac{\pi D \sin\theta}{\lambda}\right)^2}\right]\left[\frac{\sin^2\left(\frac{N\pi d \sin\theta}{\lambda}\right)}{\sin^2\left(\frac{\pi d \sin\theta}{\lambda}\right)}\right]$$

High order maximums overlap:

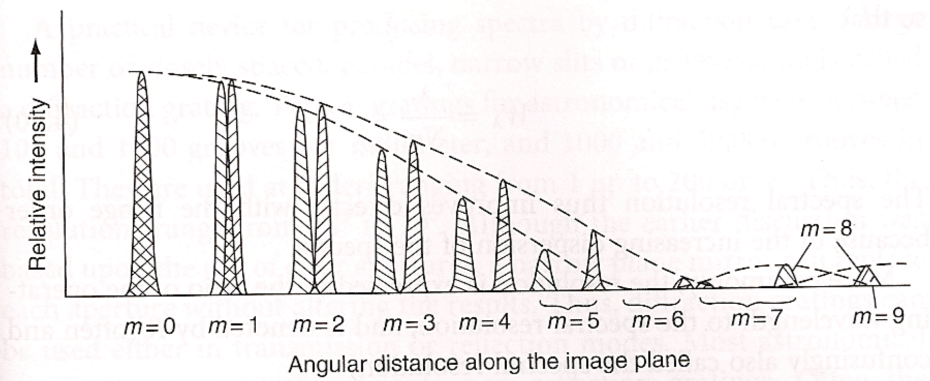


FIGURE 4.3 A portion of the image structure for a single bichromatic point source viewed through several apertures.

Diffraction Gratings

One can easily show that the angular dispersion is given by:

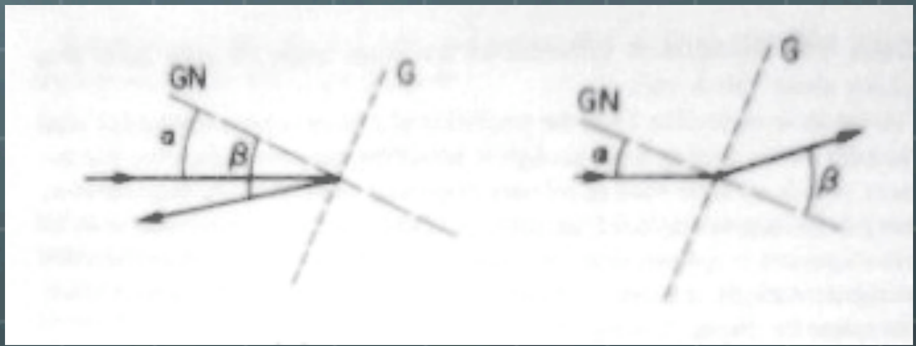
$$A = \frac{d\beta}{d\lambda} = \frac{\text{diffraction order}}{d\ \cos(\beta)}$$

Where β is the diffraction angle.

Diffraction Gratings

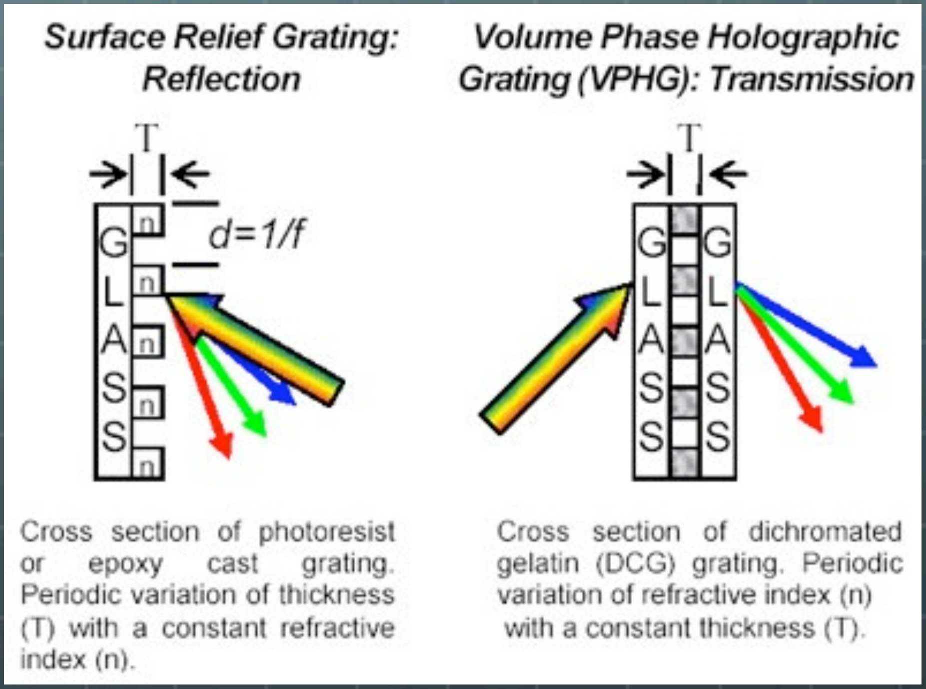
The anamorphic magnification is:

$$r = \frac{|d\beta|}{|d\alpha|} = \frac{\cos(\alpha)}{\cos(\beta)} = \frac{d1}{d2}$$



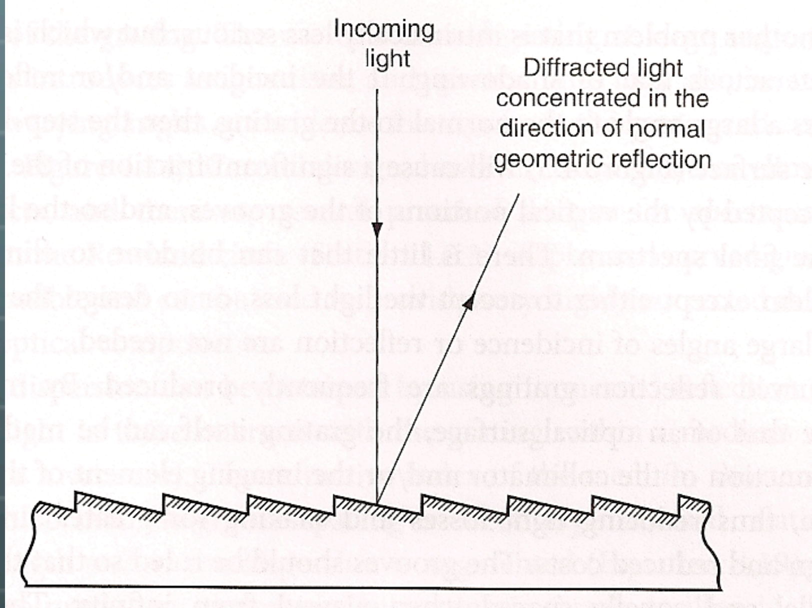
Diffraction Gratings

Volume - phase holographic gratings are currently starting to be used in astronomical spectrographs.
The same diffraction theory applies to reflecting gratings, composed of mirrors in stead of apertures.



Diffraction Gratings

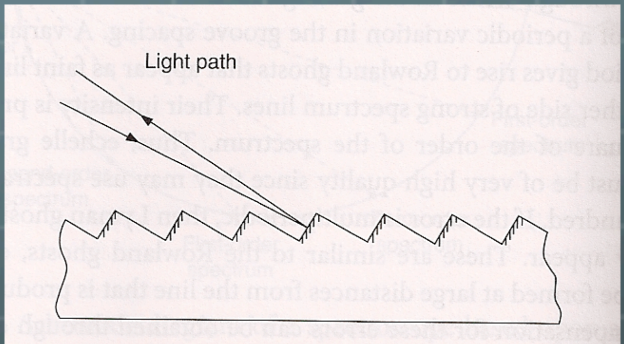
Through blazing the grating, we can concentrate the light in a few interference maxima, thus increasing the efficiency.



Enlarged section through a blazed reflection grating.

Diffraction Gratings

In an echelle configuration, a secondary (orthogonal) grating or prism disperse the light from the high order interference maxima. We can also immerse the grid in a medium of higher refractive index



Enlarged view of an echelle grating.

Fabry-Perot interferometer

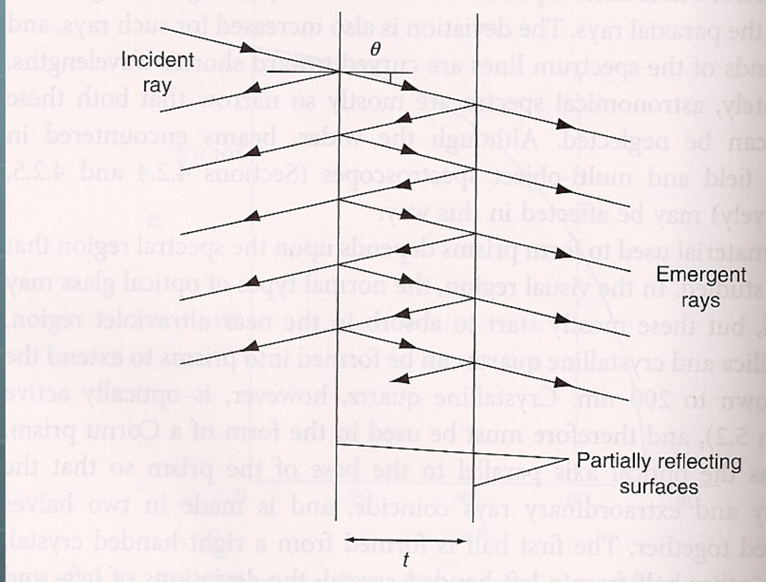


FIGURE 4.15 Optical paths in a Fabry–Perot interferometer.

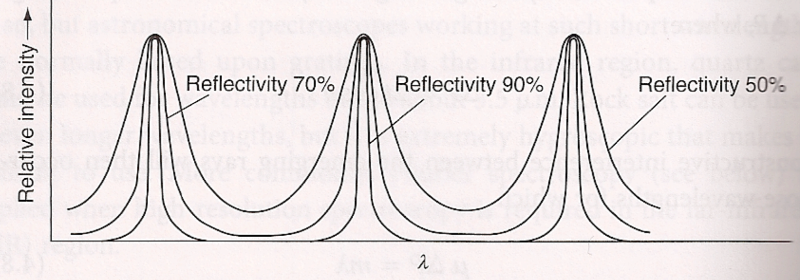


FIGURE 4.17 Intensity versus wavelength in the image of a white-light point source in a Fabry–Perot spectroscope, assuming negligible absorption.

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Specrtometers:

There are several type of spectrometers and narrow band filter configurations.

The basic types of spectrometers are

. Prism or grating spectrometers

. Fabry-Perot interferometers

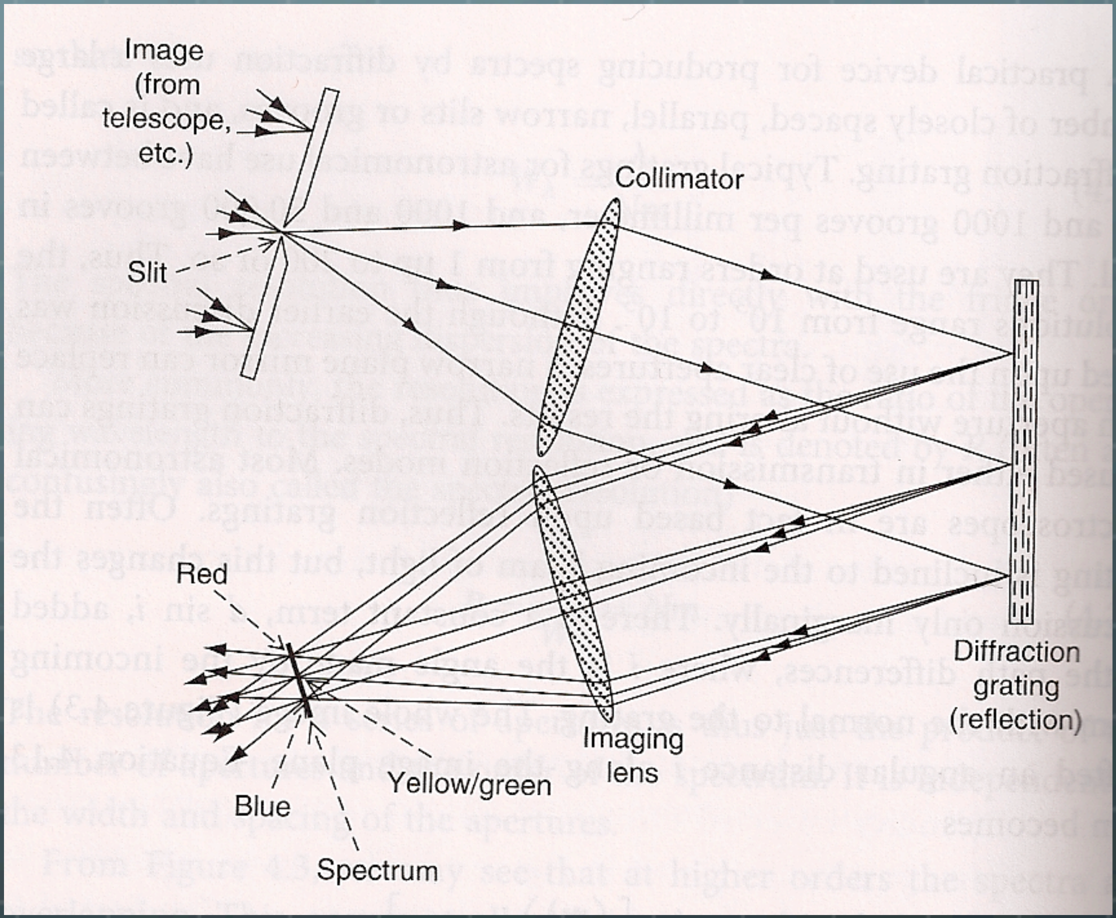
. Fourier transform spectrometer

They can be used in slit or slit-less mode and can also be feed by optical fibers.

Note that most of the detectors are 2D arrays (CCD)

Grating Spectrometers:

The basic design is:



Prism Spectrometers:

The basic design is:

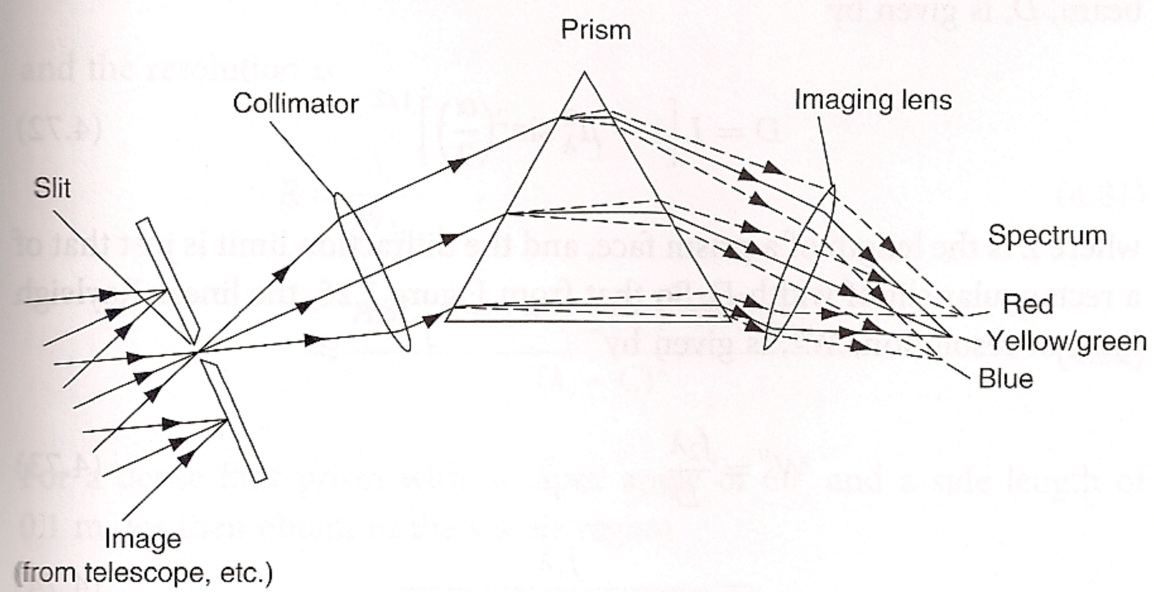


FIGURE 4.12 Basic optical arrangement of a prism spectroscope.

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Fabry-Perot Spectrometers:

The basic design is:

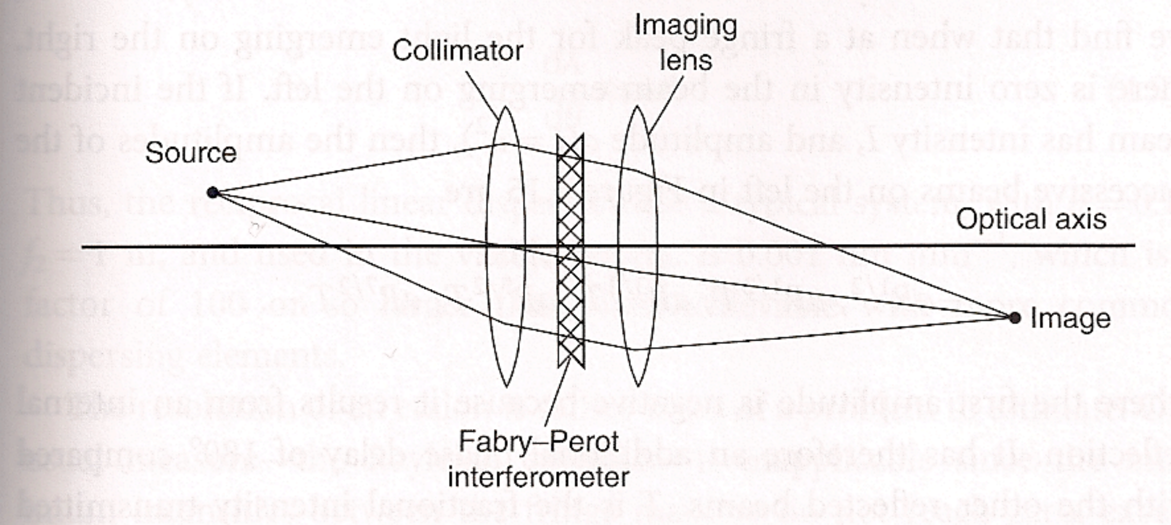

FIGURE 4.16 Optical paths in a Fabry–Perot spectroscope.

Fourier Transform Spectrometers:

The basic design is:

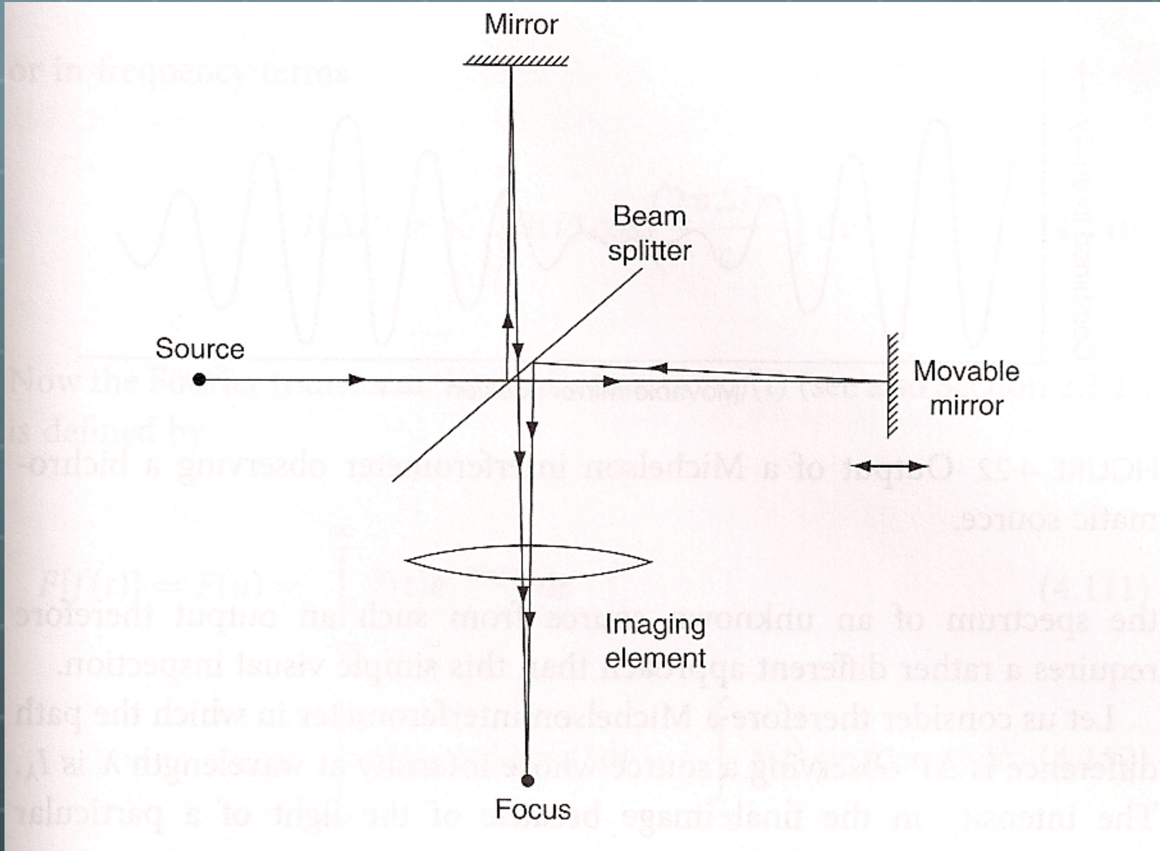

FIGURE 4.20 Optical pathways in a Michelson interferometer.

Fourier Transform Spectrometers:

For monochromatic light and a path difference ΔP, we receive an intensity:

$$I'_{\Delta P}(\lambda) = KI(\lambda)\left[1 + \cos\left(\frac{2\pi\Delta P}{\lambda}\right)\right]$$

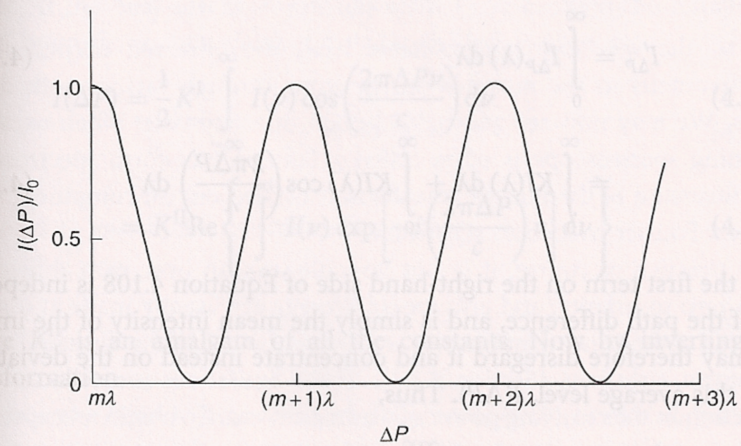

FIGURE 4.21 Variation of fringe intensity with mirror position in a Michelson interferometer.

So, for a given path difference ΔP we get :

$$I(\Delta P) = K\int_0^{\infty} I(\lambda)\cos\left(\frac{2\pi\Delta P}{\lambda}\right)\mathrm{d}\lambda$$

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Basic Properties:

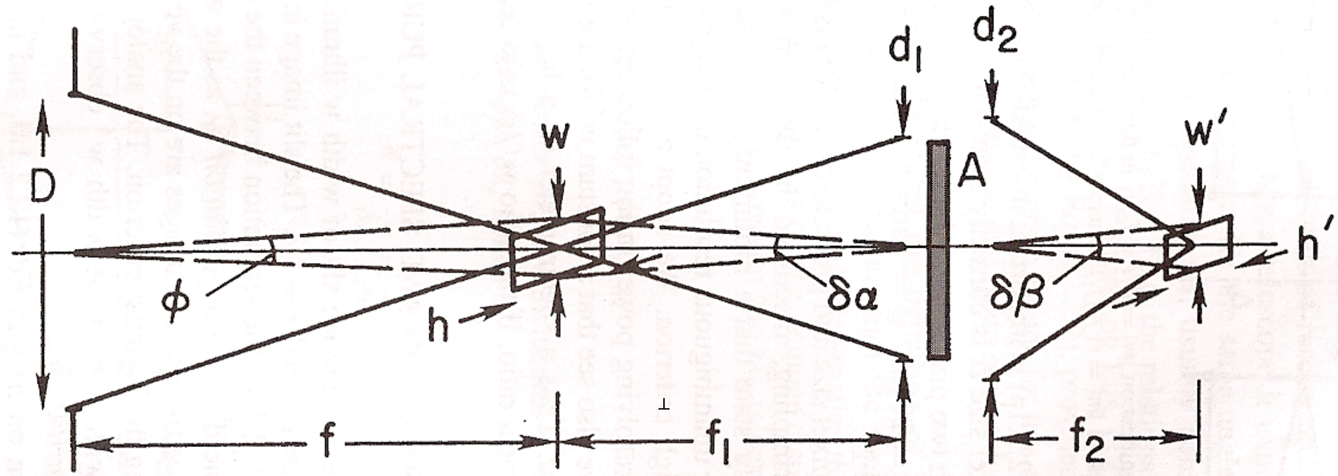


Fig. 12.4. Schematic layout of slit spectrometer with dispersing element of angular dispersion A. See text, Section 12.2, for definitions of parameters.

Clearly:

$$w' = rw\frac{f2}{f1} = r\phi_{\perp}D\frac{f2}{d1}, \quad h' = h\frac{f2}{f1} = \phi_{//}D\frac{f2}{d1}$$

And we should match w' to the detector size…

Limit Resolution

In order to be able to resolve to similar wavelengths, we should get a separation of at least w' in the detector to be able to resolve them.

This condition gives:

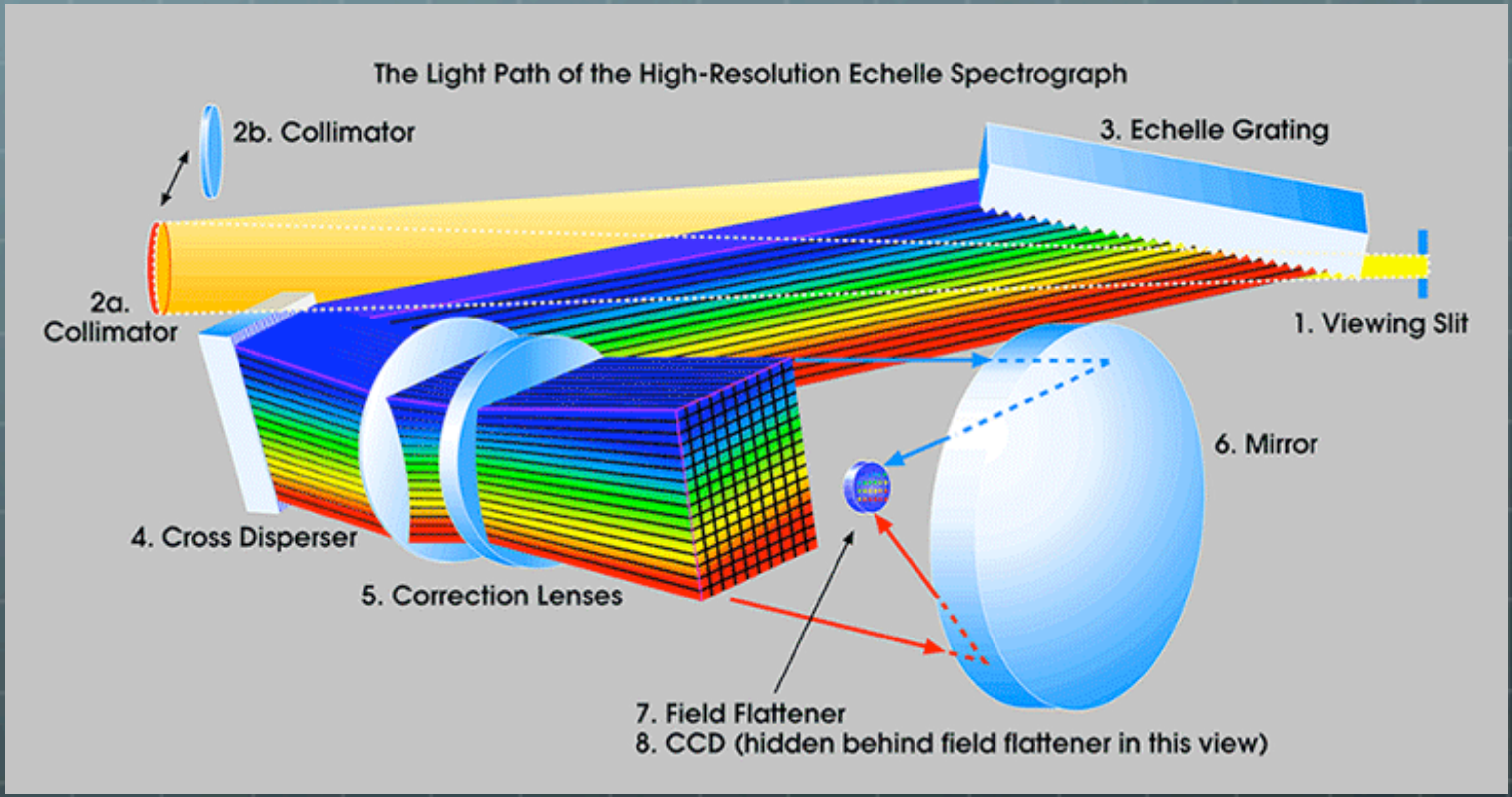
$$\delta\lambda = \frac{d\lambda}{dl}\Delta l = \frac{1}{fA}\Delta l = \frac{\overset{1}{\lambda}}{fA}w' = \frac{r}{A}\phi\frac{D}{d1}$$

Thus, we need larger spectrometers (*d1*) if we want to increase the telescope size and keep the resolution constant...

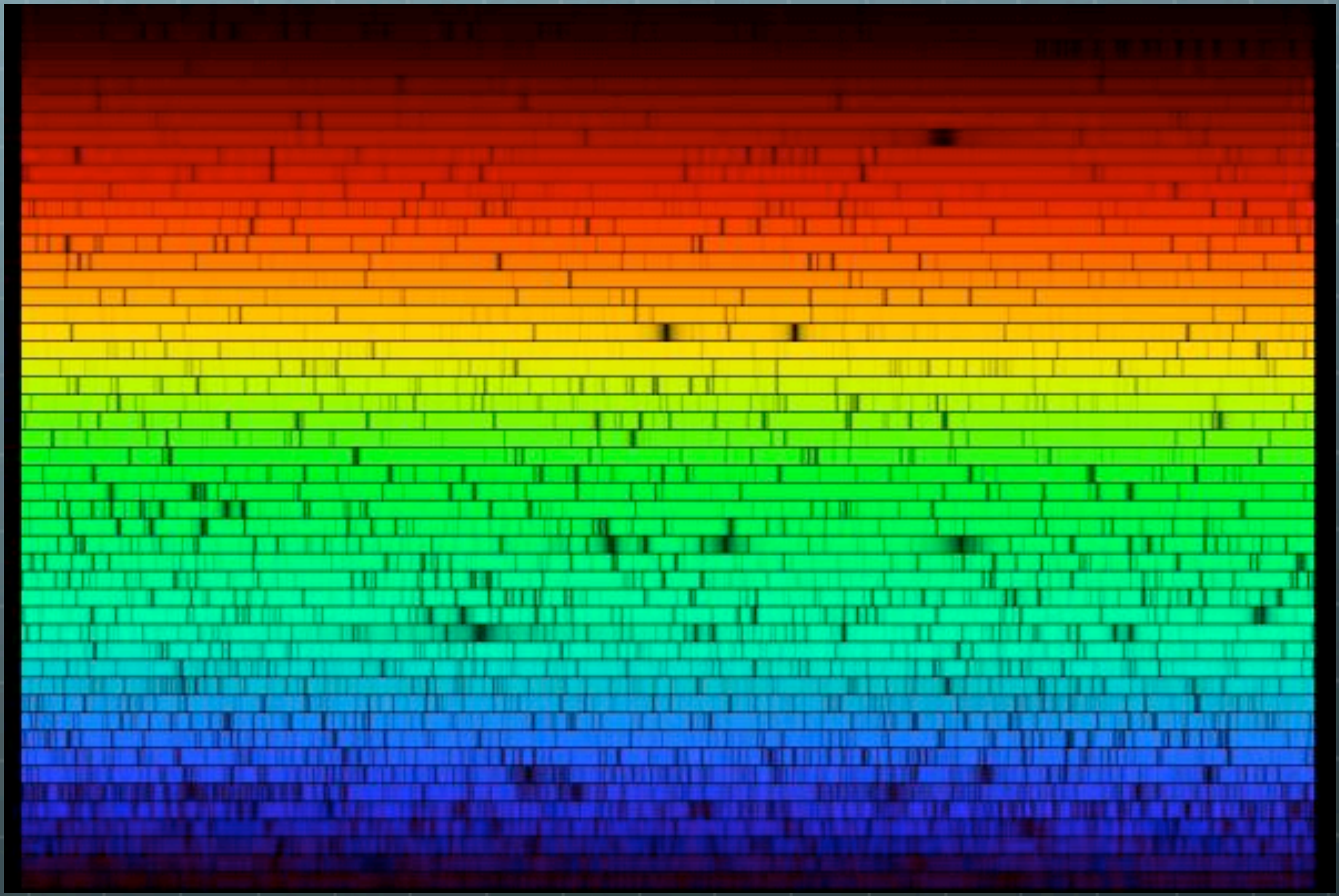
$$R = \frac{\lambda}{d\lambda} = A\; d2\frac{1}{D}\frac{1}{\phi}$$

Limit Resolution

A echelle configuration achieves high values of R:



Limit Resolution



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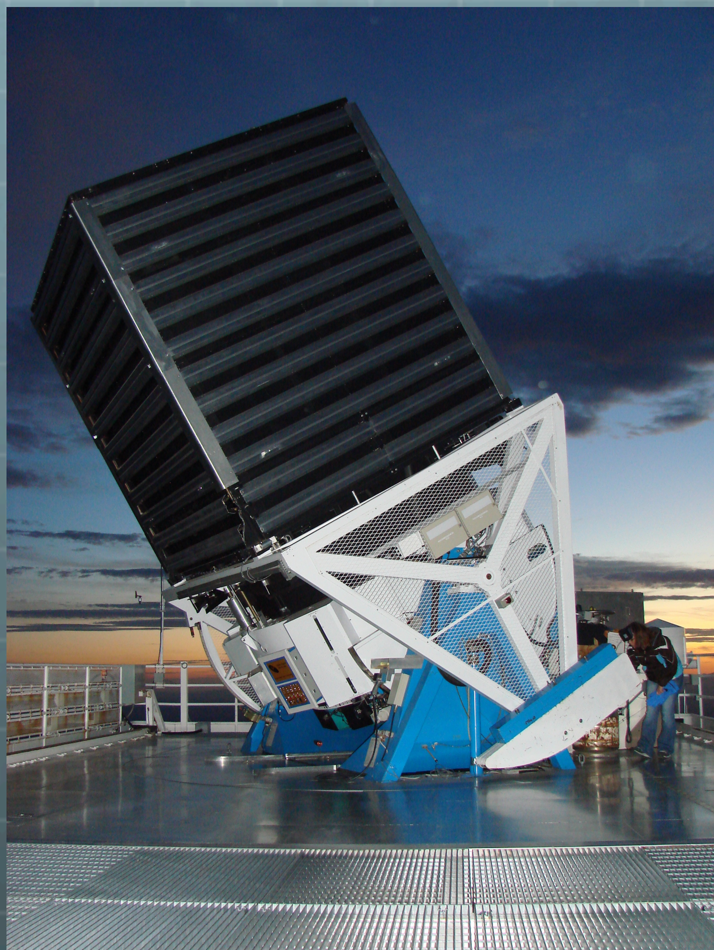
Limit Resolution



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Diffraction limit

If we use a diffraction limit telescope $\phi \approx 1.22 \frac{\lambda}{D}$

We get the maximum resolving power:

$$\delta\lambda = \frac{r}{A}\phi\frac{D}{d1} = \frac{r}{A}\frac{\lambda}{d1} = \frac{\lambda}{Ad2}$$

Thus, we have a theoretical maximum resolving power:

$$R = \frac{\lambda}{\delta\lambda} = A\,d2$$

$$R_{Grating} = \text{diffraction order x Number of slits}$$

$$R_{\mathrm{Pr}\,ism} = \text{Prism base length}\frac{dn}{d\lambda}$$

Conclusion

Construction of spectrographs require careful analysis of several efficiency and quality factors in the design of the optical elements.

Larger telescopes require larger optical instruments.

There is a maximum (diffraction limited) attainable resolution.

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