Geometric optics & aberrations

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http://www.northerneye.co.uk/

Outline

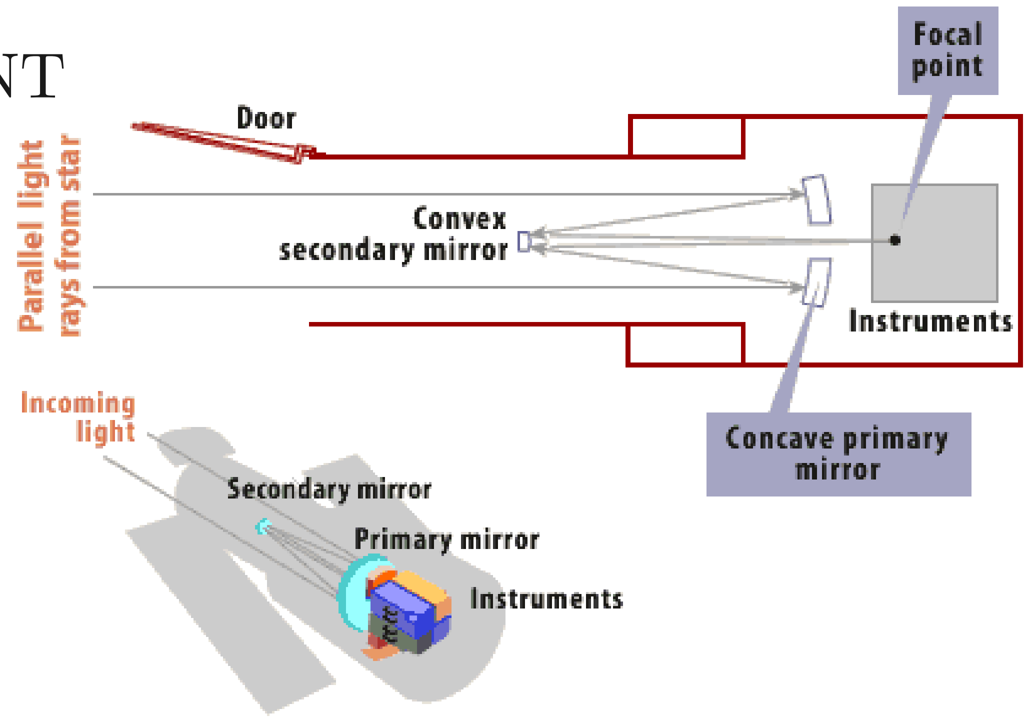
- Introduction: Optics in astronomy
- Basics of geometric optics
- Paraxial approximation
- Seidel aberrations (third-order)
- Aberration correction
- What happened to HST?
- References

How does it all work?

Astronomical sources are very FAINT

We need to COLLECT LIGHT,
we use optical systems

We design curved surfaces that
BEND the trajectories of light rays



http://amazing-space.stsci.edu

Design of optical system using first-order optics → Ray tracing numerically → Optimize to reduce the effect of aberrations

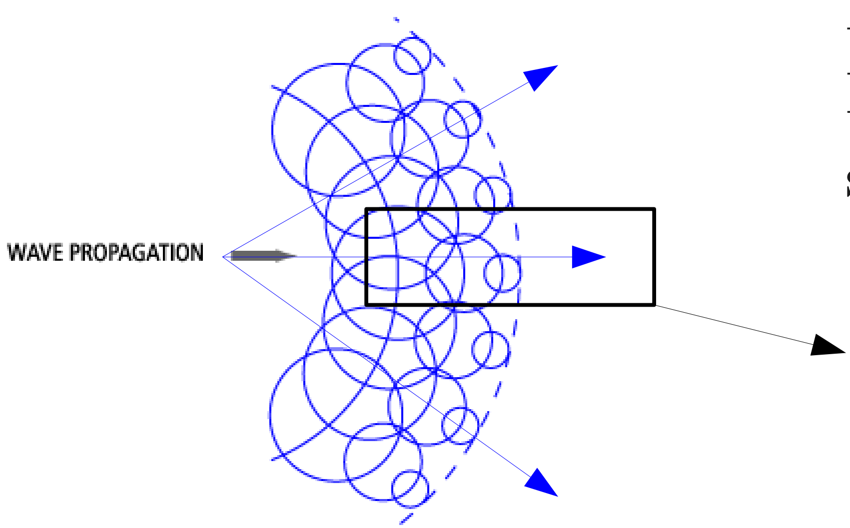
Geometric optics

Geometric optics describes *how light behaves*, not *what light is*.

Gives an accurate description for *wavelengths* that are *short* compared to dimensions of the elements we use to study the behavior of light. (vs. Physical optics - *Simone's talk*)

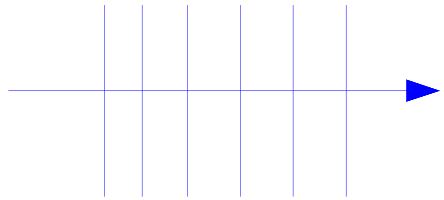
Huygens' principle

Extended object: array of point sources. But in astronomy, we deal with *far* away sources.



http://www.cvimellesgriot.com/

Source at infinity

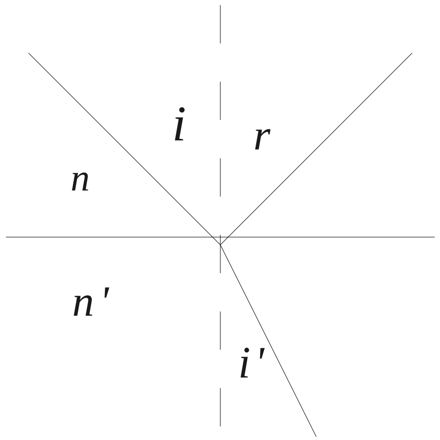


Geometric optics

Snell's laws (empirical)

Refraction	$n \sin i = n' \sin i'$
Reflection	$i = r$

Index of refraction $n(T, \lambda) = \frac{c}{v}$



Air at 15C : $(n-1) \times 10^8 = 8342.1 + \frac{2{,}406{,}030}{(1-\nu^2)} + \frac{15{,}996}{(38.9-\nu^2)}$

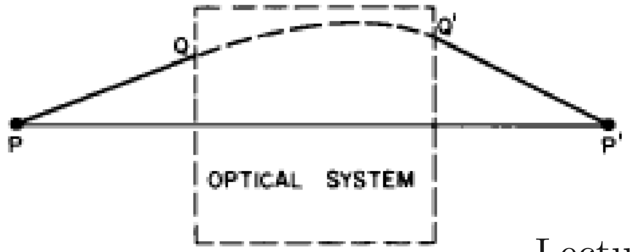
Fermat's principle $\longrightarrow$ a way to think Snell's laws

Variational principle: a ray of light will traverse a medium such that the total optical path will assume an *extreme* value (maximum or minimum).

$$\delta L = \int_A^B n(s)\, ds = 0$$

Geometric optics

Image formation: We could use Snell's laws to trace the rays through a given optical system.



Lectures on Physics, Vol. 1, Feynman

This is ***impractical***! We can find the position of an image very easily if we work in the **Paraxial approximation**

= *small angles with respect to the optical axis in a system with rotational symmetry about that axis.*

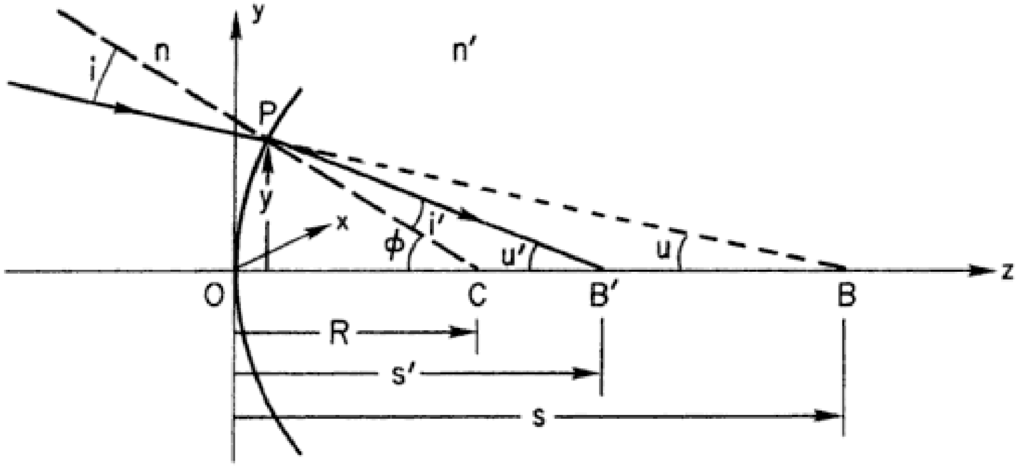
$$\sin\theta \sim \theta$$
$$\tan\theta \sim \theta$$
$$\cos\theta \sim 1$$

first-order geometric optics

In this approximation, the **imaging quality** of an optical system is **ideal**.

Geometric optics

Refraction in spherical surfaces



Astronomical optics, Schroeder

$$\frac{n'}{s'} - \frac{n}{s} = \frac{n'-n}{R}$$

image   object   curvature radius

$$s = \inf, \; f = s'.$$

astronomical source   focal distance

Reflection in spherical surfaces

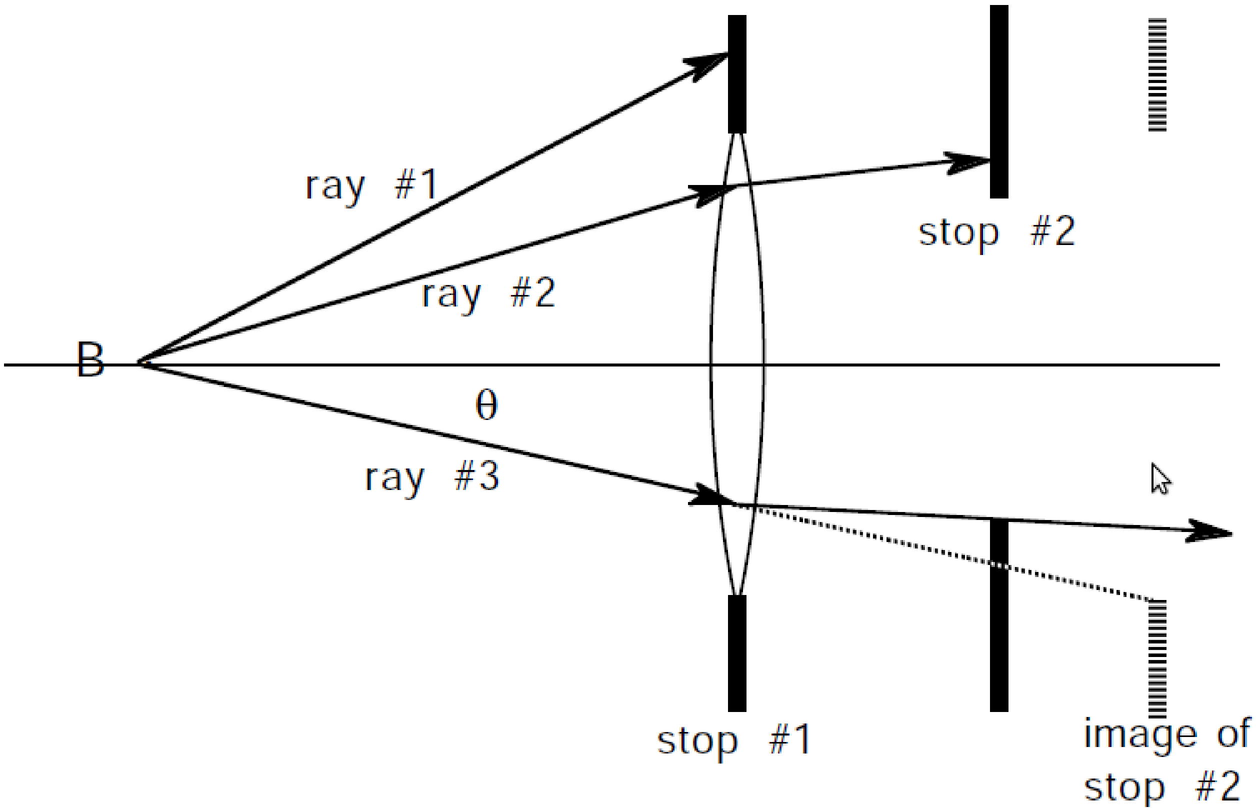
$$n = -n' \longrightarrow \frac{1}{s'} + \frac{1}{s} = \frac{2}{R}$$

Geometric optics

Some basic definitions in an **optical system** (*Colin's talk*)

Aperture stop: the physical stop which limits the cross-section of the image-forming pencil of rays. To determine it, compute size and position of the images of all stops in the system by the preceding elements.

Pupils: The **entrance pupil** is the image of the aperture stop made by the *preceding* optical elements of the system. The **exit pupil** is the image of the aperture stop by the *following* parts of the optical system.



http://www.drdrbill.com/downloads/optics/geometric-optics/Apertures.pdf

Geometric optics

Some basic definitions in an **optical system**

Focal ratio (or f-ratio) : ratio of the focal length to the diameter of the entrance pupil.

→ *Speed of an optical system*
Lower f-ratios require shorter exposure times.

Principal ray: is a ray coming from the object point and passing through the center of the aperture stop.

Field of view: spatial or angular extent imaged, in object space.

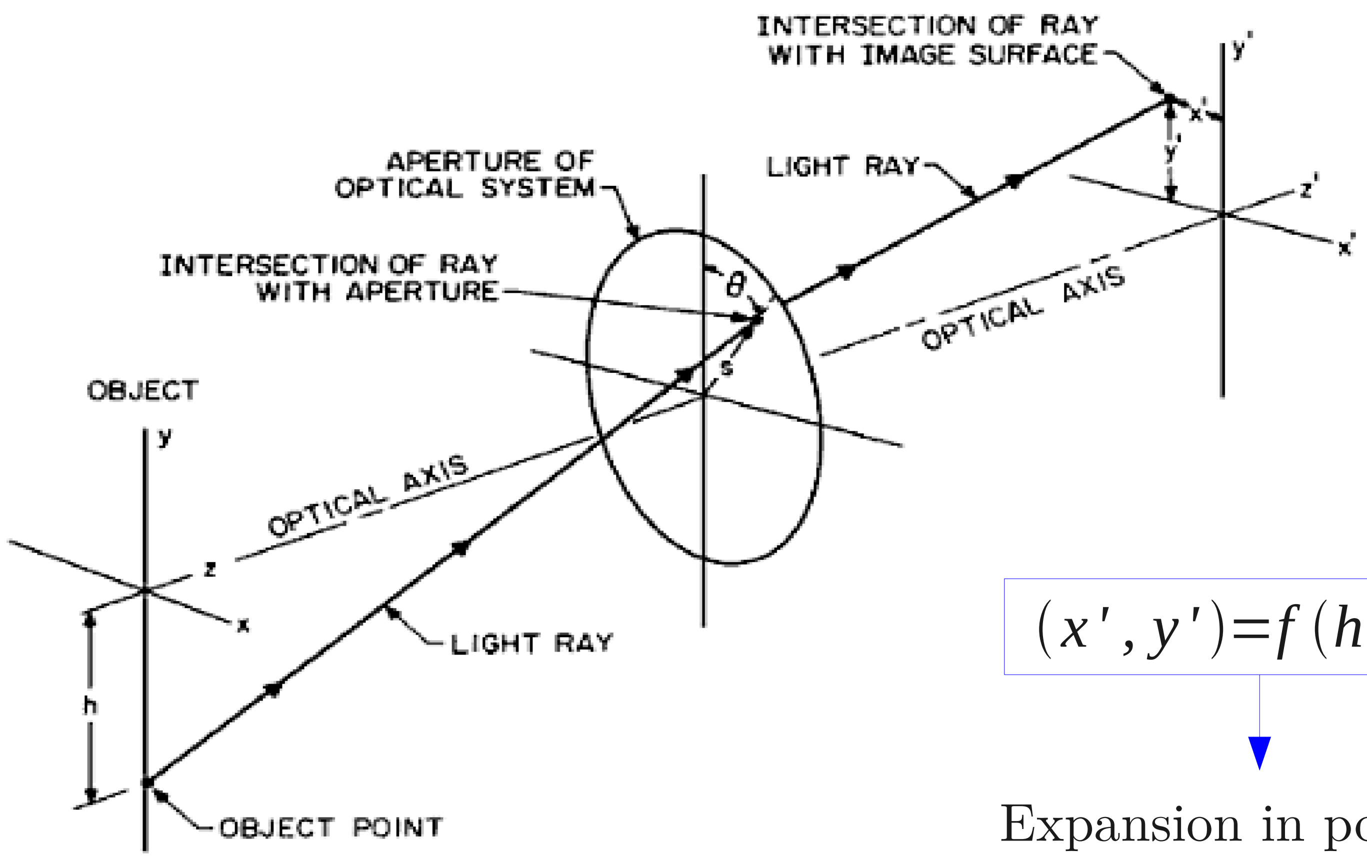
Field stop: limits the **diameter** of the system which can be imaged by an optical system. Smallest ratio of the diameter of the image of stop i and distance between image of stop and entrance pupil: $\tan\phi = \frac{d_i}{2L_i}$

Primary/Seidel aberrations

Optical system *circularly symmetric* with respect to the optical axis.

Now we can **define an object** by a point at $y{=}h$ from the optical axis.

Terms that involve *off-axis distances in powers higher than 2* in the expansion of the characteristic functions are **geometrical aberrations**.



Modern Optical Engineering, Smith

$$(x', y') = f(h, s, \theta)$$

Expansion in powers of s and h

Primary/Seidel aberrations

$$\begin{aligned} y' &= A_1 s \cos\theta + A_2 h \\ &+ B_1 s^3 \cos\theta + B_2 s^2 h(2 + \cos 2\theta) + (3B_3 + B_4) s h^2 \cos\theta + B_5 h^3 \\ &+ C_1 s^5 \cos\theta + (C_2 + C_3 \cos 2\theta) s^4 h + (C_4 + C_6 \cos^2\theta) s^3 h^2 \cos\theta \\ &+ (C_7 + C_8 \cos 2\theta) s^2 h^3 + C_{10} s h^4 \cos\theta + C_{12} h^5 + D_1 s^7 \cos\theta + \cdots \end{aligned}$$

$$\begin{aligned} x' &= A_1 s \sin\theta \\ &+ B_1 s^3 \sin\theta + B_2 s^2 h \sin 2\theta + (B_3 + B_4) s h^2 \sin\theta \\ &+ C_1 s^5 \sin\theta + C_3 s^4 h \sin 2\theta + (C_5 + C_6 \cos^2\theta) s^3 h^2 \sin\theta \\ &+ C_9 s^2 h^3 \sin 2\theta + C_{11} s h^4 \sin\theta + D_1 s^7 \sin\theta + \cdots \end{aligned}$$

$$\frac{(n+3)(n+5)}{8} - 1$$

Number of nth order terms

A terms are **first-order**, corresponding to the paraxial approximation.

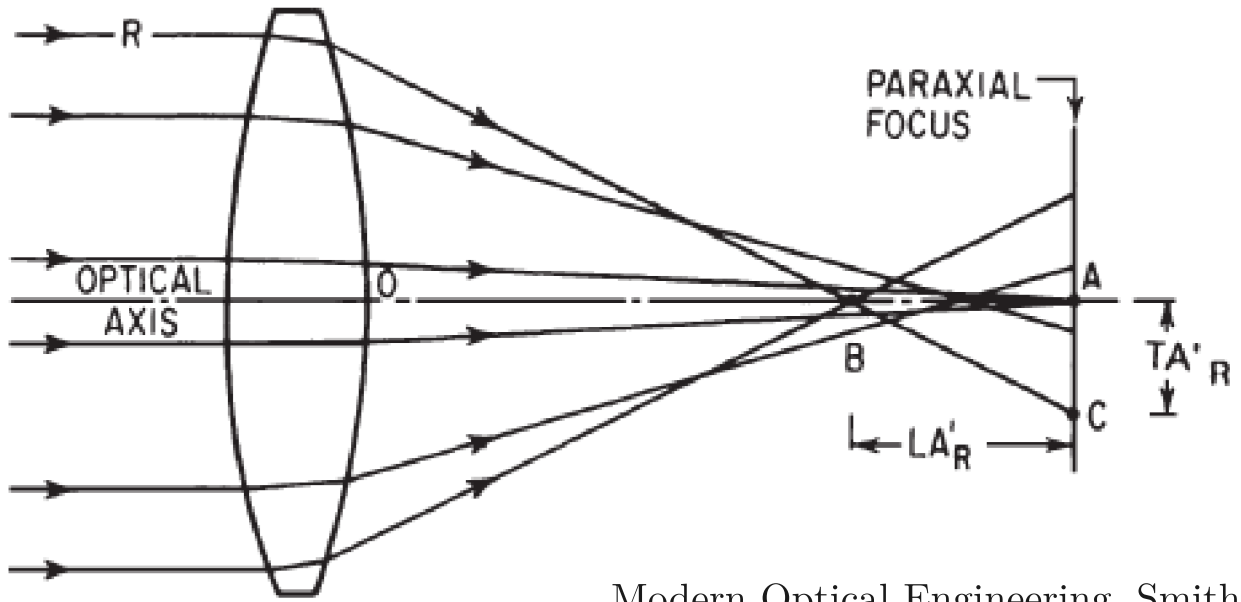
B terms are primary or Seidel **aberrations**: spherical, coma, astigmatism, curvature and distortion.

P. Spherical aberration

Spherical aberration: **variation of focus with aperture**.

Rays close to optical axis come to focus near the paraxial focus position.

As height increases, the focus moves farther.

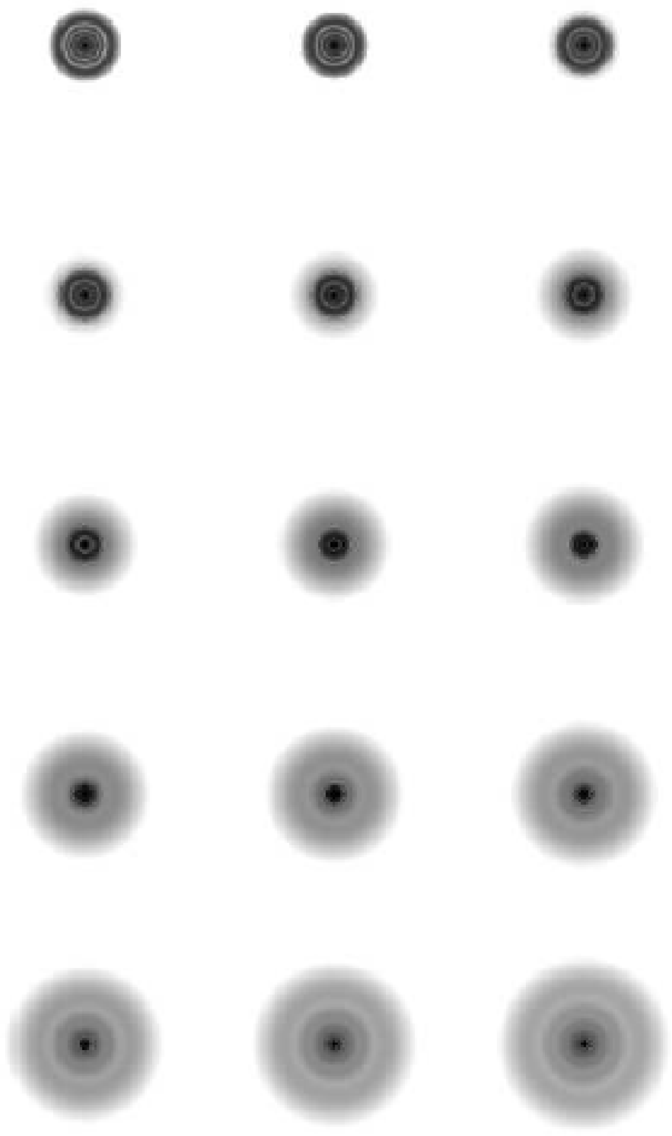


Modern Optical Engineering, Smith

Image of a point: bright dot surrounded by a halo.

Extended image: softened contrast and blur of its details.

P. Spherical aberration

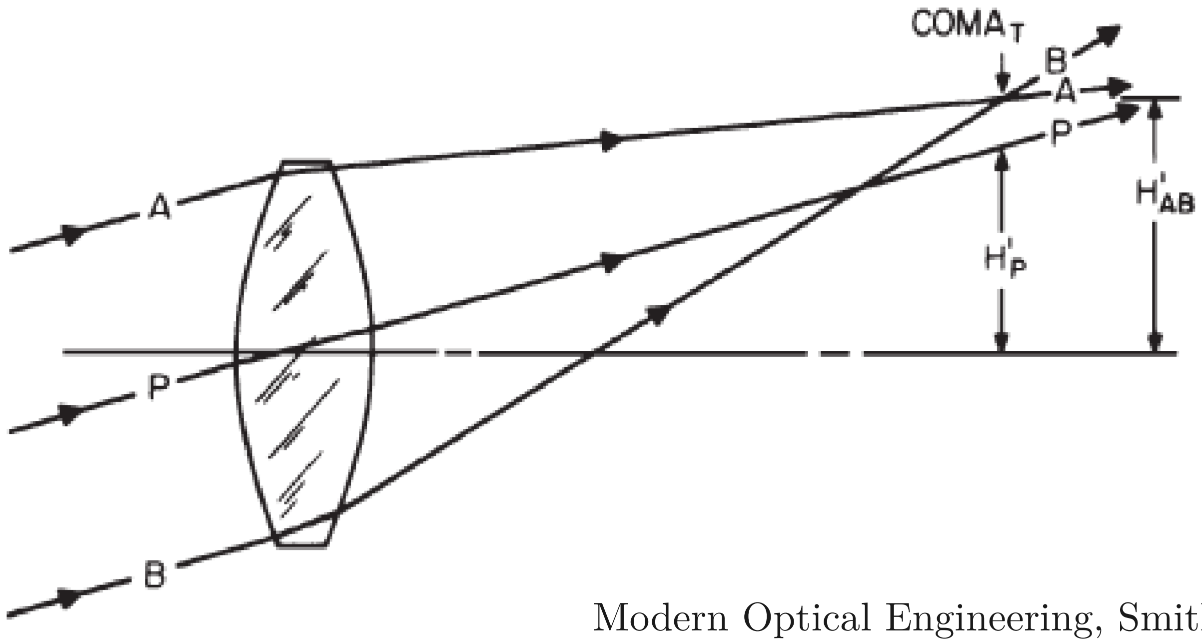


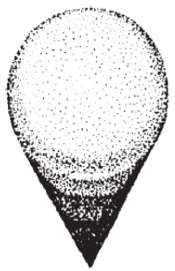
ganymede.nmsu.edu/holtz/a535/ay535notes/

P. Coma

Coma: **variation of magnification with aperture.**

Oblique rays incident on a lens with coma, the rays passing through the edge may be imaged at a different height than those passing through the center.





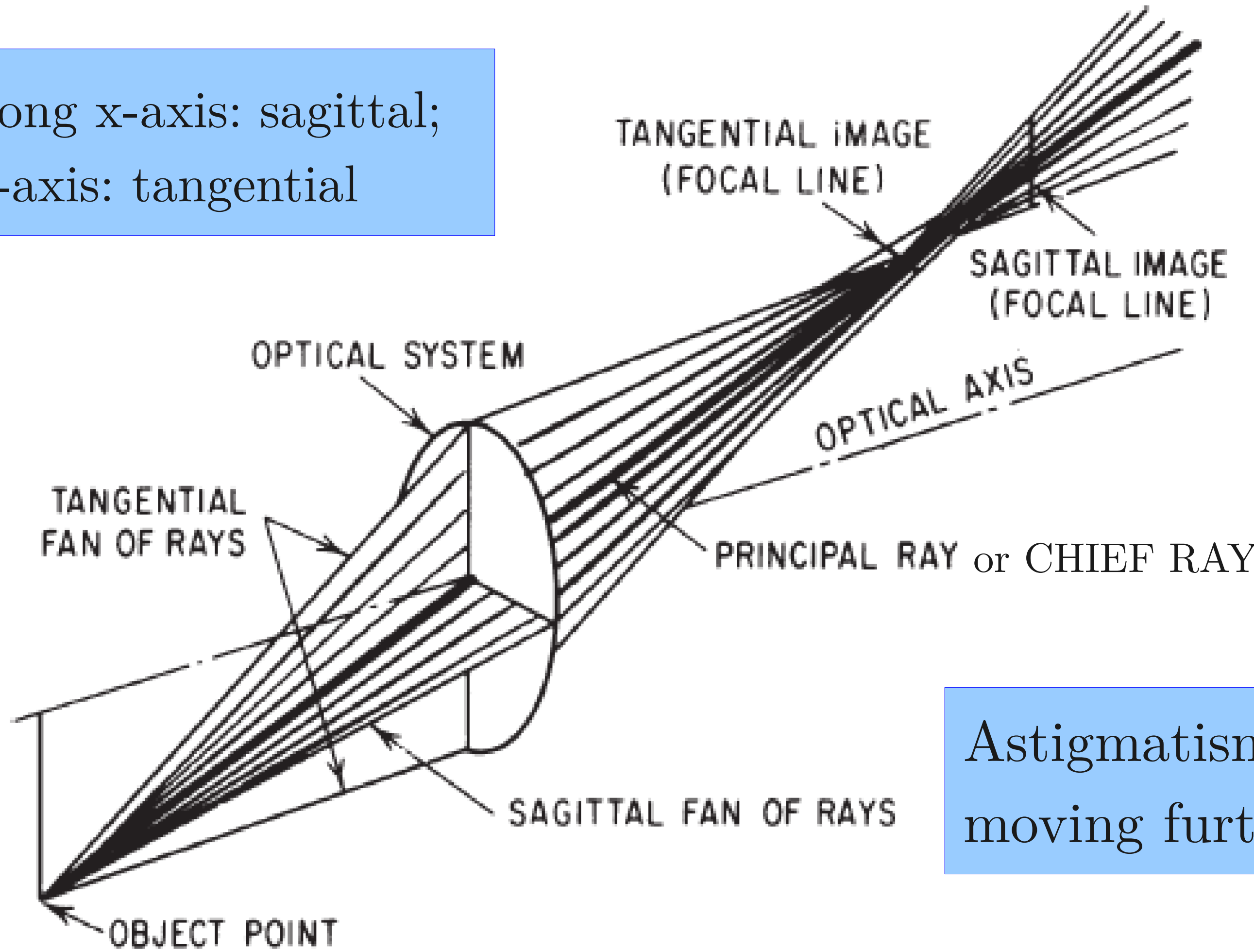
Modern Optical Engineering, Smith

Hard to correct because asymmetrical!

P. Astigmatism and field curvature

Astigmatism occurs when the *tangential and sagittal* images do not coincide. The image of a point turns into **two separate lines** and in between, an elliptical or circular blur.

Rays along x-axis: sagittal; along y-axis: tangential

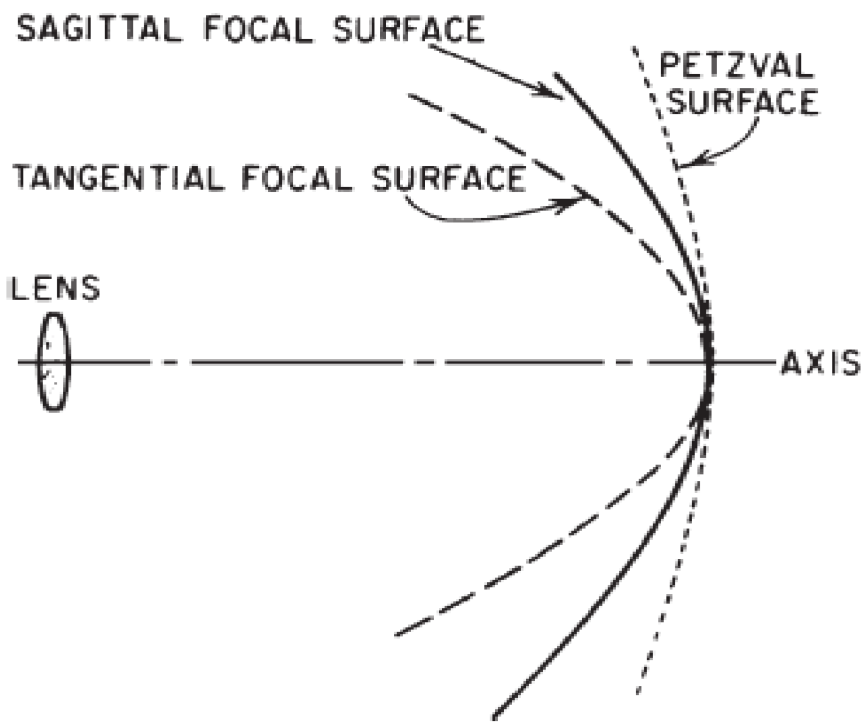


Astigmatism increases when moving further from the axis.

Modern Optical Engineering, Smith

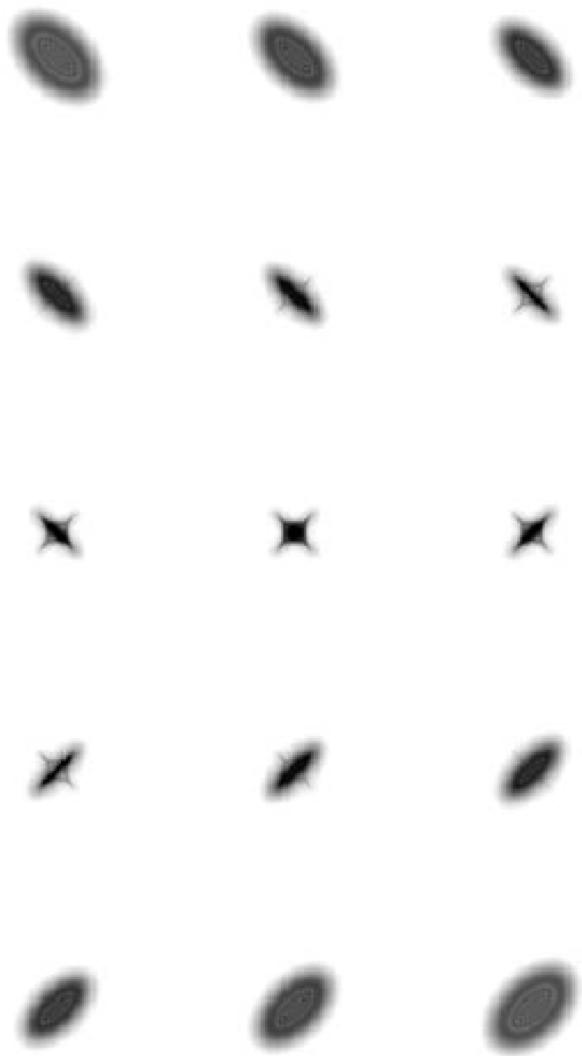
P. Astigmatism and field curvature

Off-axis images do not lie on a plane, rather on a *curved* surface. If there is astigmatism, this surface is a **paraboloid**. In the case with no aberration, the tangential and sagittal images lie on the ***Petzval surface*** (a function of the index of refraction and the surface curvatures of the lens elements).



Modern Optical Engineering, Smith

P. Astigmatism and field curvature



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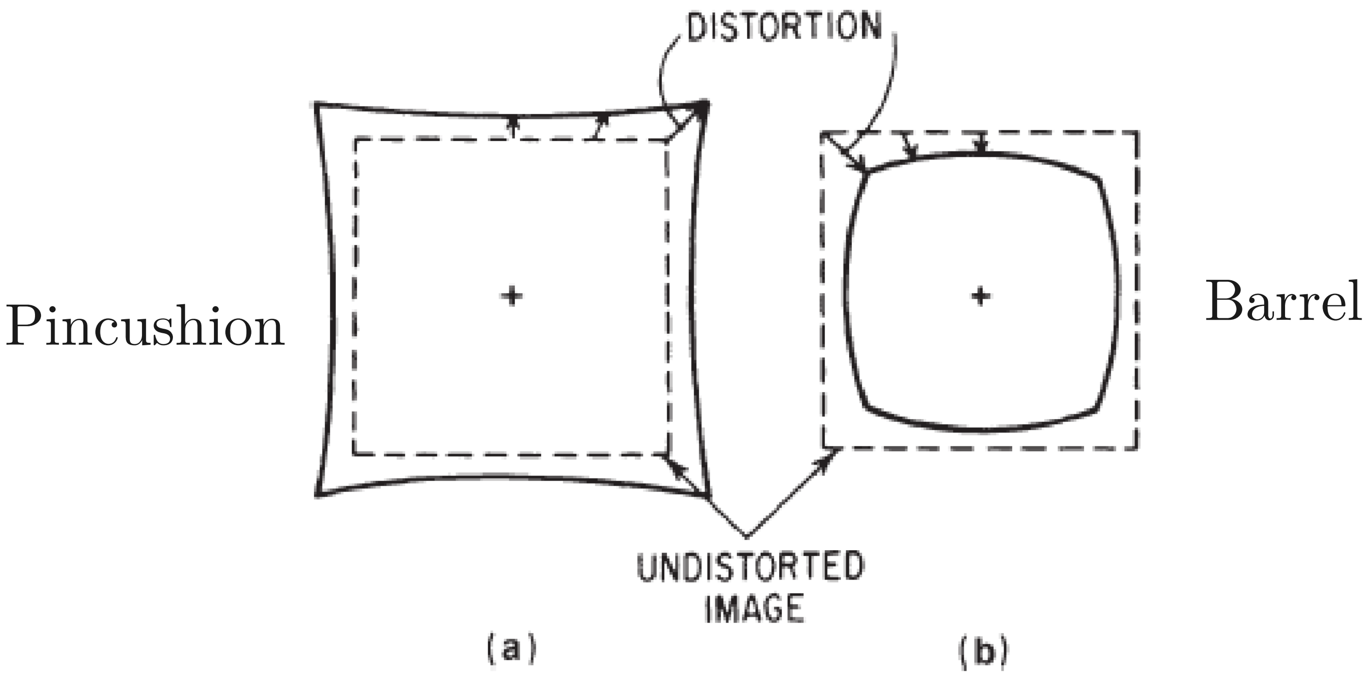
P. Distortion

The image of an *off-axis* point is formed **farther** from the axis or **closer** to the axis than the image height given by the paraxial expression.

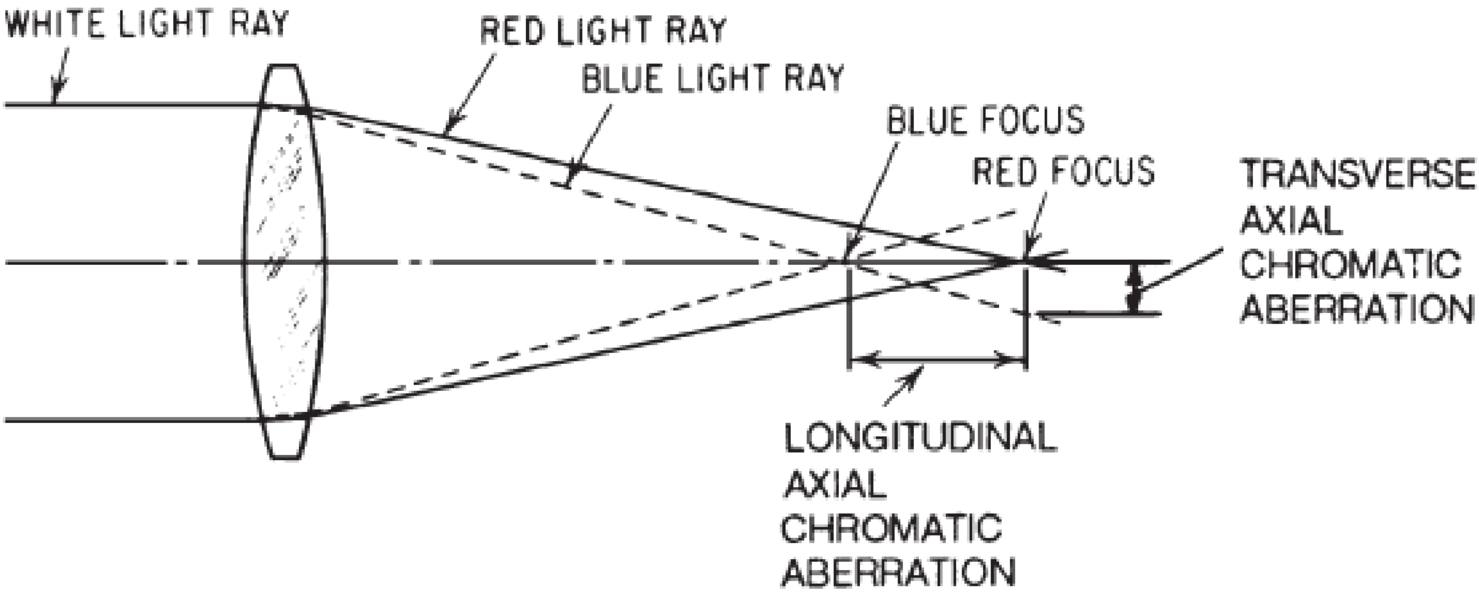
Amount of distortion increases as image size increases.



Modern Optical Engineering, Smith

P. Chromatic aberration

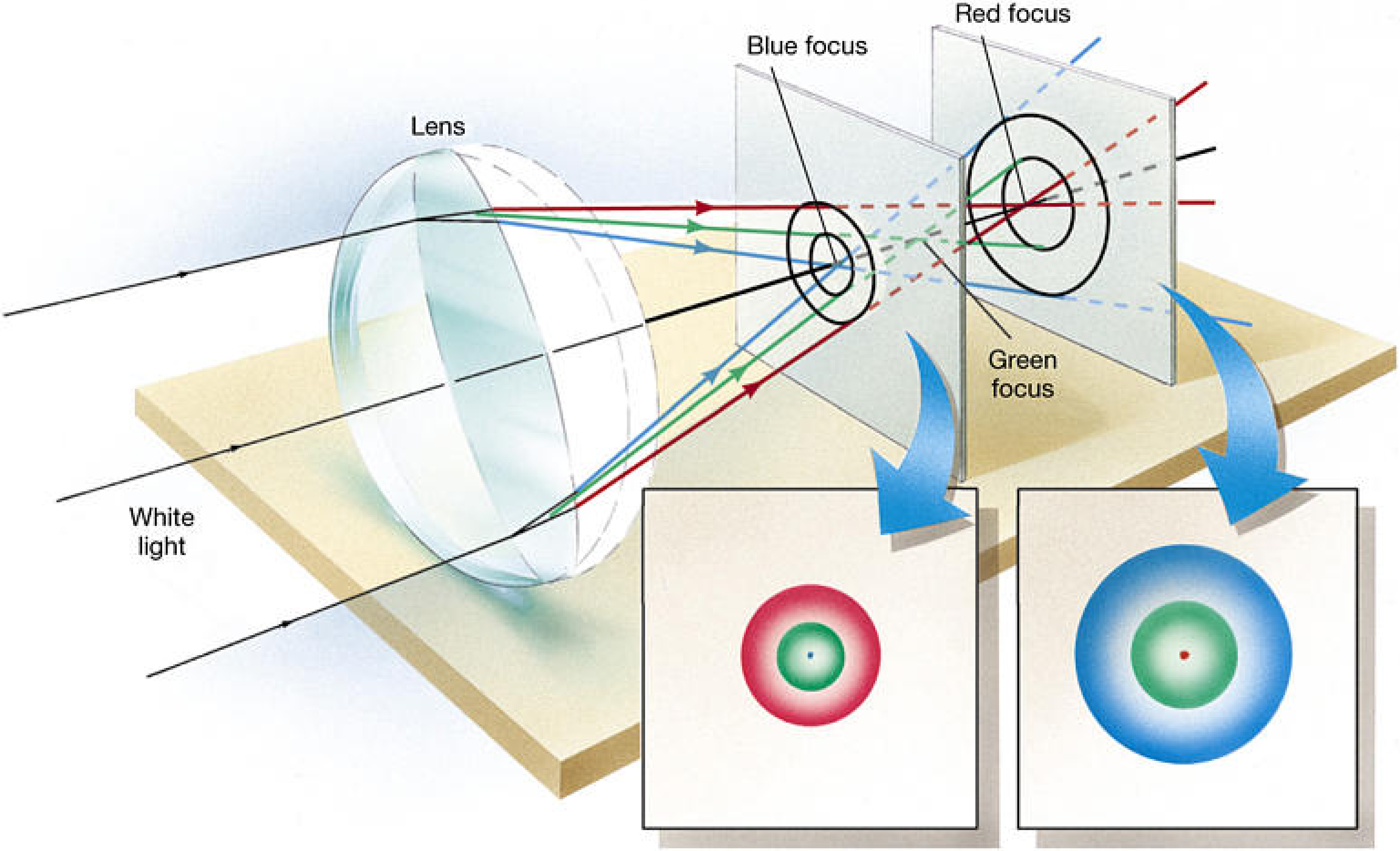
$n(\lambda)$ → Axial chromatic aberration is the longitudinal *variation of focus* (or image position) **with wavelength**. The index of refraction is higher for shorter wavelengths: more strongly refracted.



Modern Optical Engineering, Smith

All aberrations will be different for each color!

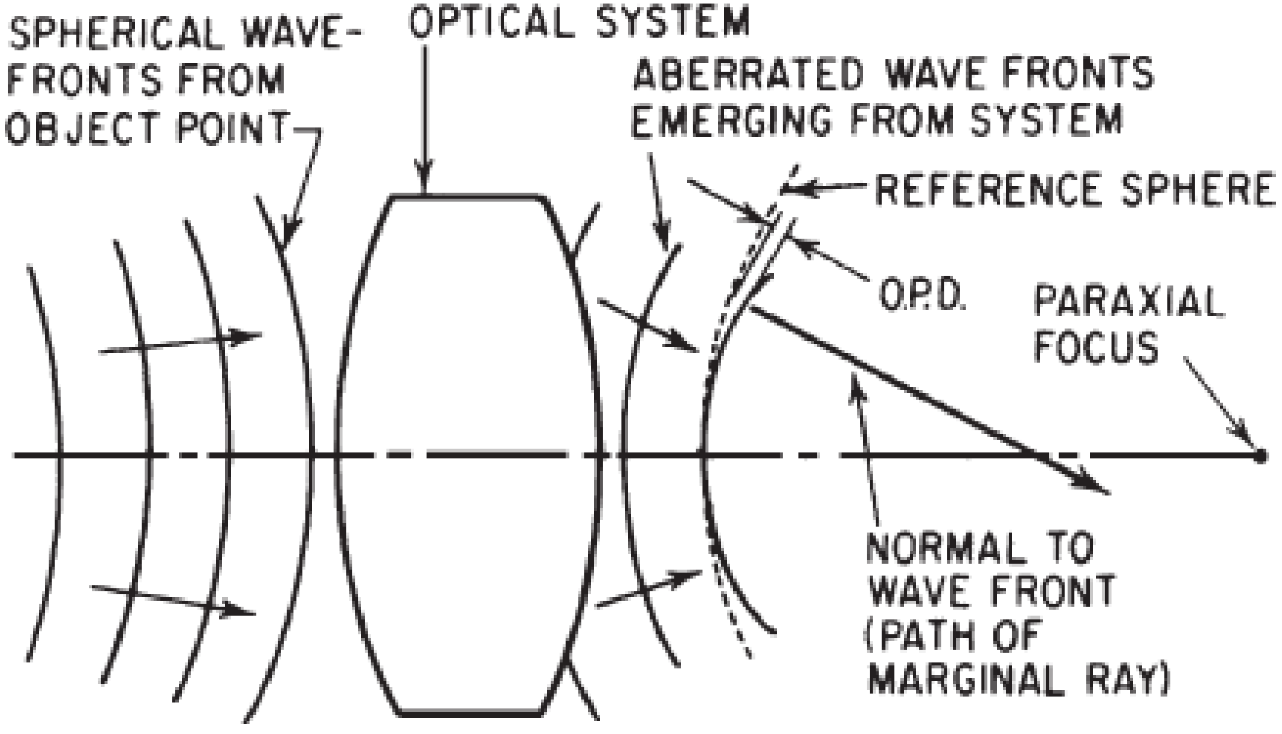
P. Chromatic aberration



http://physics.uoregon.edu/~jimbrau/BrauImNew/Chap05

Wavefront aberration

Reinterpretation of aberrations in terms of the **wave description** of light.



Modern Optical Engineering, Smith

Aberration correction

Lens shape and stop position

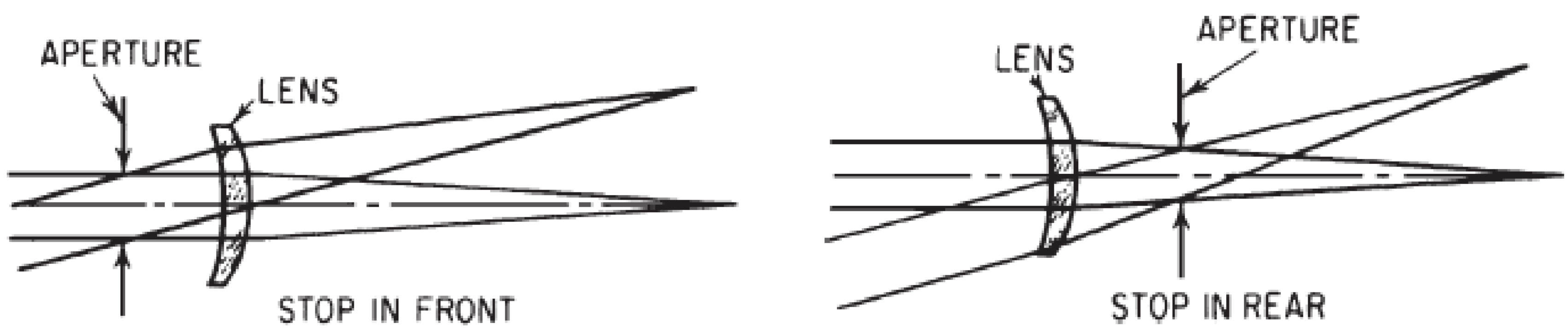
Are used to control aberrations in simple lens systems.

Thin lens equation: → A number of ways we can choose the radii to obtain the *same* focal length.

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Aberrations change with shape

Oblique rays pass through a different part of the lens depending on where the stop is.



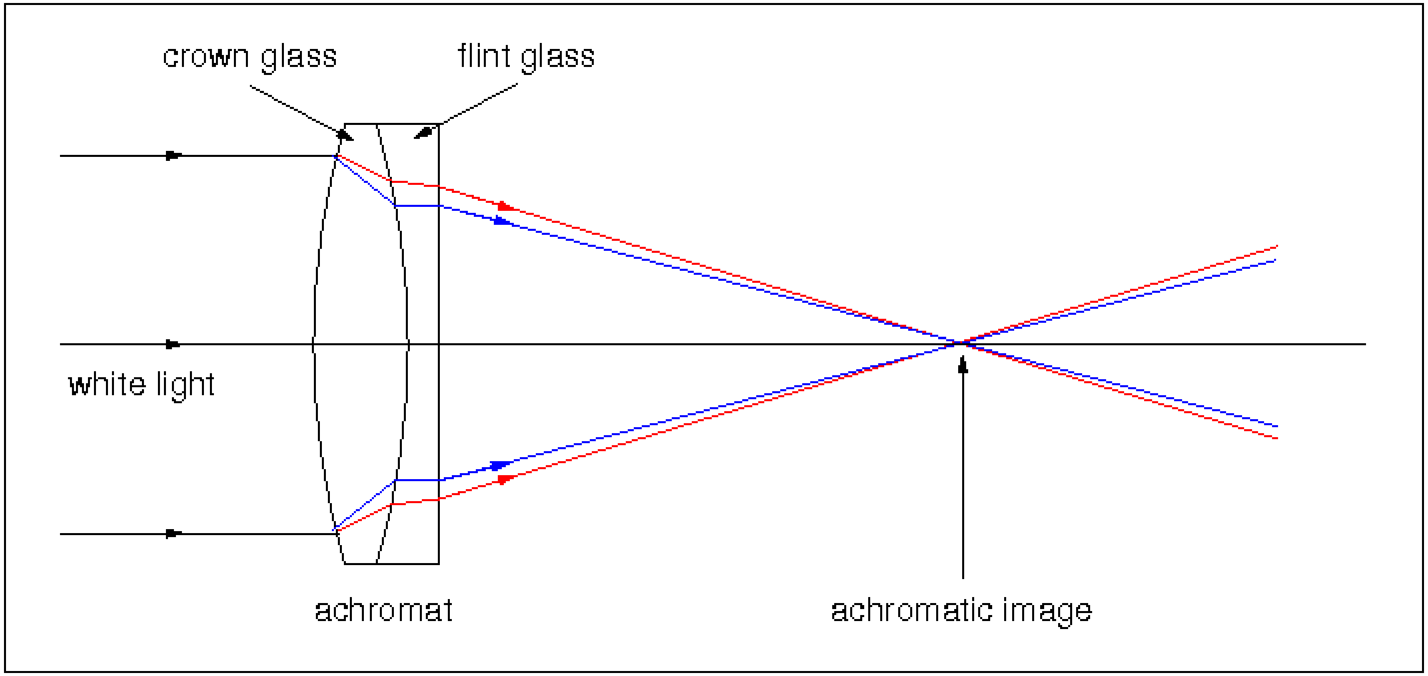
Modern Optical Engineering, Smith

Aberration correction

Usually a higher precision is needed and we need to correct aberrations by other means. The way to do this is to...

... combine optical elements with aberrations of opposite sign such that they cancel each other.

RESIDUALS: Corrections usually work for a given **zone** or incidence **angle**.



Correction to chromatic aberration

http://www.vikdhillon.staff.shef.ac.uk/teaching/phy217/telescopes/phy217_tel_refractors.html

Aberration correction

Ray tracing → Computational procedures for determining the effects of each aberration.

↓

Because of symmetry, we do not need to trace a very large number of rays

COMA: To determine the effect of coma we trace three tangential rays from an off-axis object point:

- A ray passing through the **center** of the entrance pupil.
- A ray passing through the **lower rim** of the entrance pupil.
- A ray passing through the **upper rim** of the entrance pupil.

→ Determine height of intersection with focal plane.

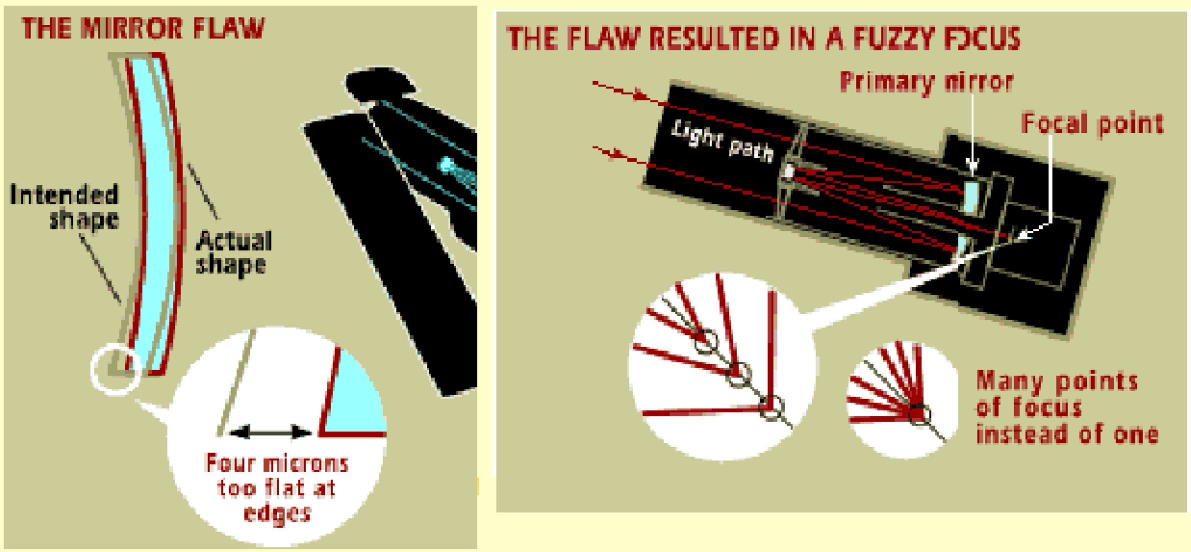
ZEMAX

Software for lens design, illumination, laser beam propagation, and many other applications. Supports multi-spectral source file input.

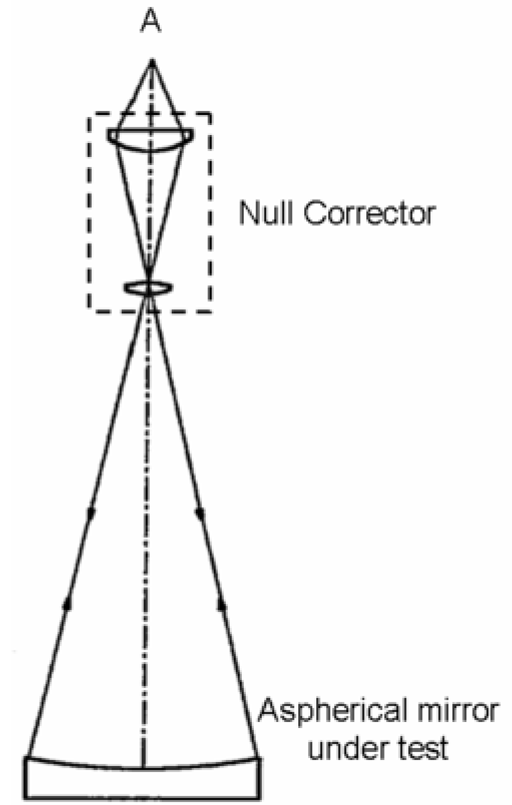
http://www.zemax.com/

What happened to HST?

Hubble Space Telescope **Cassegrain** reflector.



http://www.northerneye.co.uk/



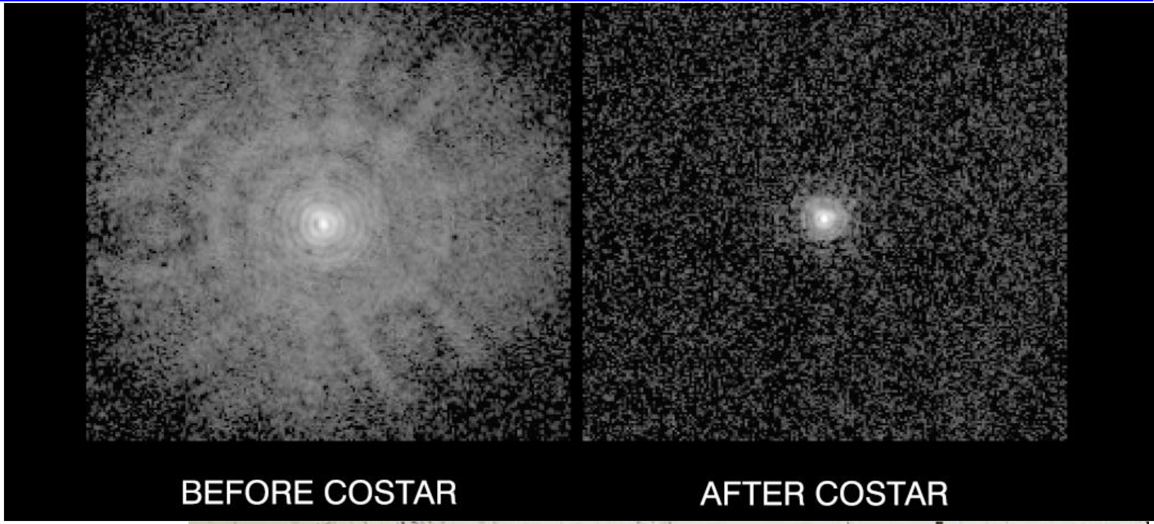
http://en.wikipedia.org/wiki/Null_corrector

The curve to which the primary mirror was ground was incorrect, causing "spherical aberration".

If Hubble's primary mirror were scaled up to the diameter of the Earth, the **biggest bump** would be only *six inches* tall.

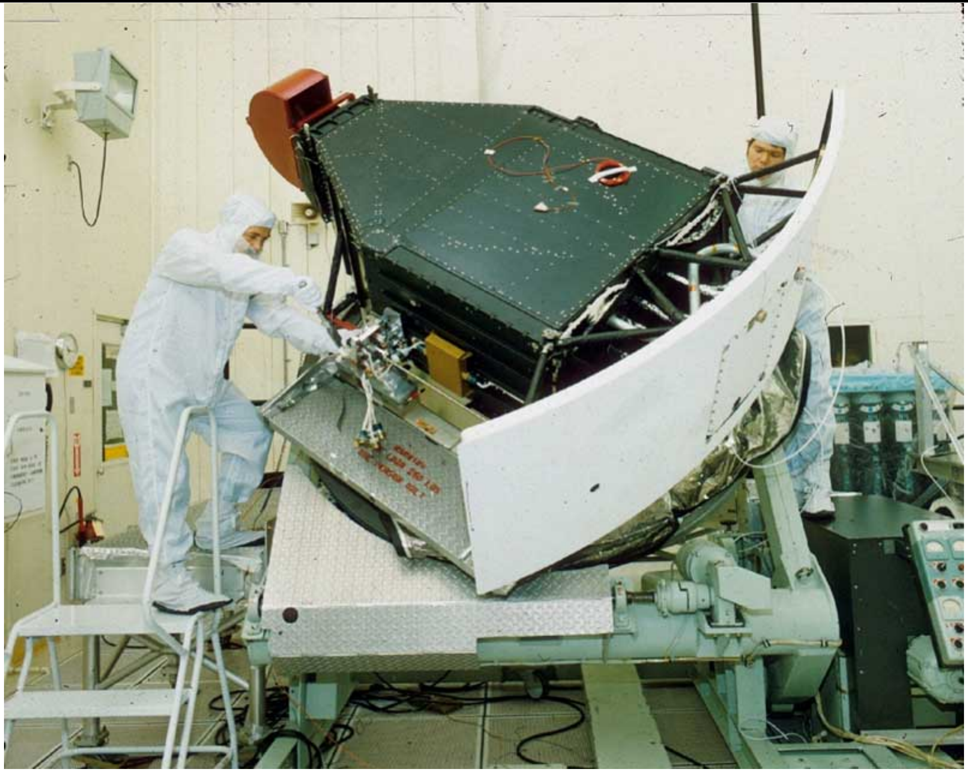
The **null corrector** used to test the primary mirror was designed correctly, but built *incorrectly*.

What happened to HST?



A series of **small mirrors** correct for the flaw: Corrective Optics Space Telescope Axial Replacement (COSTAR).

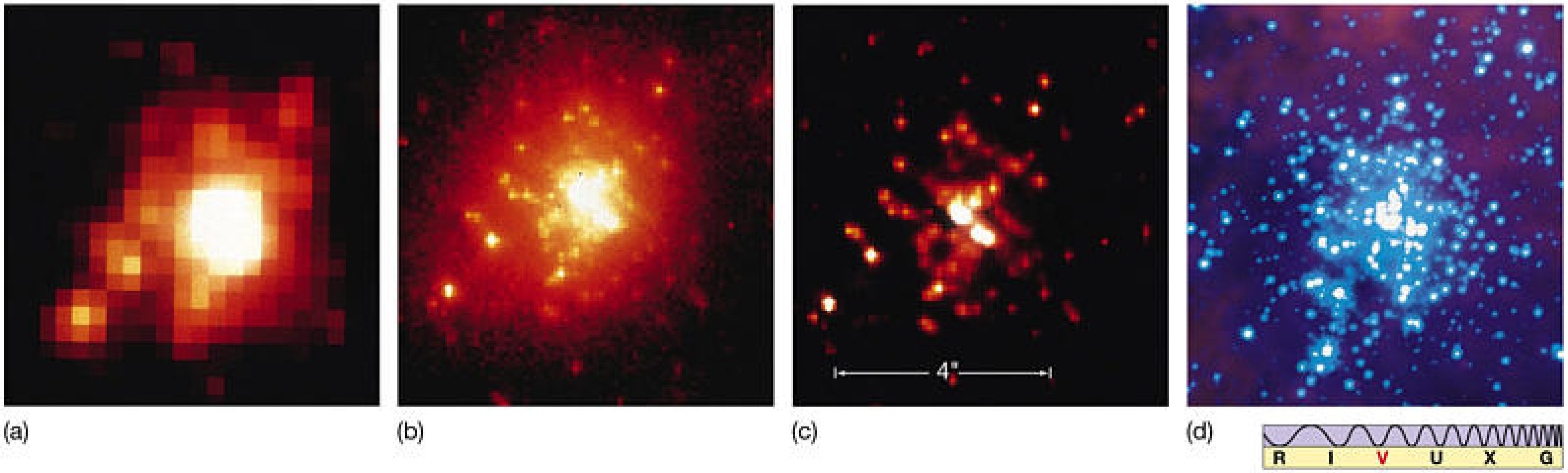
The Wide Field and Planetary Camera had to be entirely replaced by **WFPC2**, with a new optical design to compensate for the aberrations caused by the primary.



http://blogs.smithsonianmag.com/

What happened to HST?

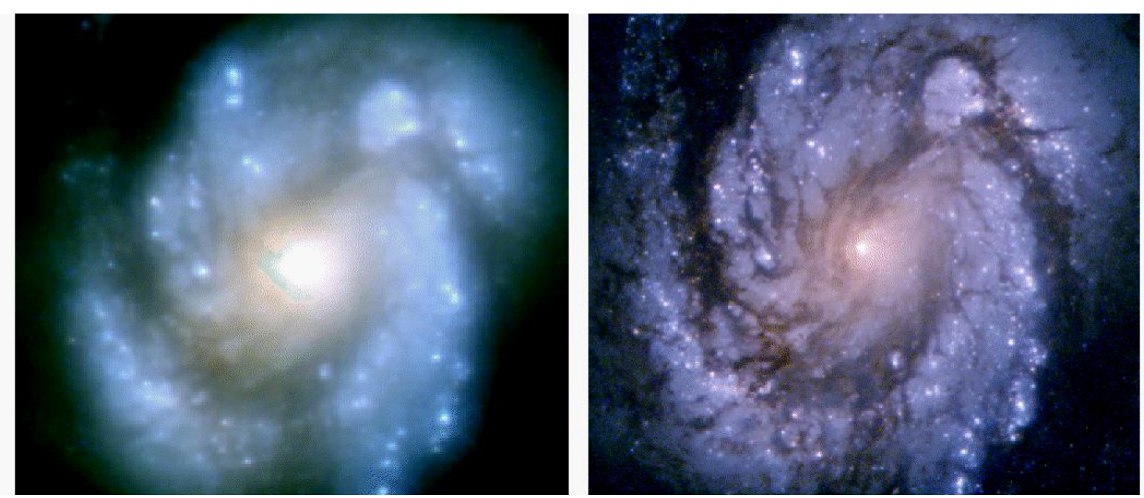
Star cluster R136



http://physics.uoregon.edu/~jimbrau/BrauImNew/Chap05/

(a) from Earth (b) HST original image (c) HST original imaged processed (d) HST repaired

M100



http://hubblesite.org

References

Astronomical optics, D. Schroeder, Academic Press, 2000.

Modern Optical Engineering, W. Smith, McGraw-Hill, 2000.

ganymede.nmsu.edu/holtz/a535/ay535notes/

Lectures on Physics, Vol. 1, R. Feynman, Addison-Wesley, 1966.

Springer handbook of lasers and optics, F. Trager, Springer, 2007.

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http://blogs.smithsonianmag.com/

http://en.wikipedia.org/wiki/Null_corrector

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