Optics of the Atmosphere and Seeing

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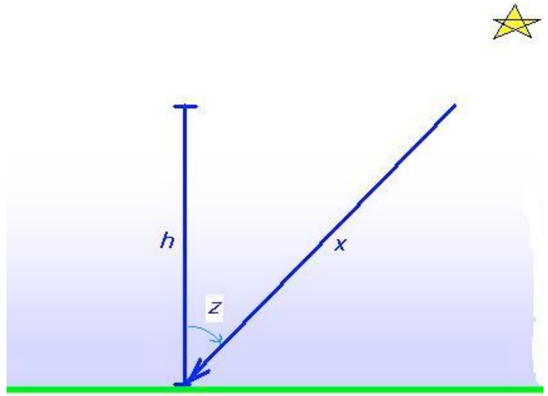
Outline

- Review general concepts:
  - Airmass
  - Atmospheric refraction
  - Atmospheric dispersion
- Seeing
  - General idea
  - Theory
  - how does image look like?
  - How it depends on
    - physical sites
    - time
  - Other sources: dome and mirror seeing
- Summary

Airmass

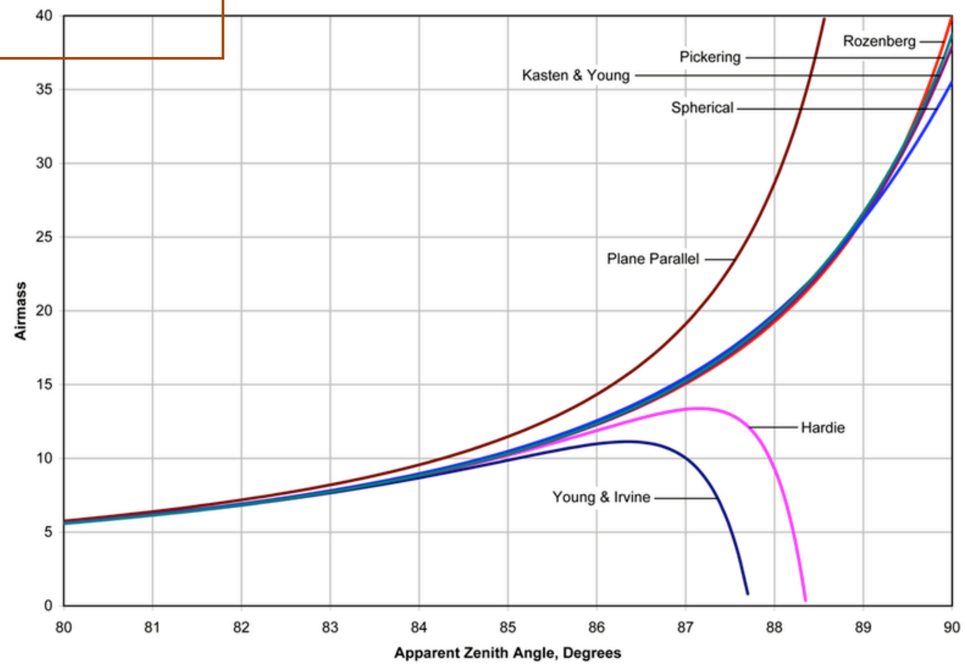
- Zenith distance z
- Airmass (X): mass column density to that at the zenith at sea level, i.e., the sea-level airmass at the zenith is 1.



$$X = \sec(z) \quad \text{for small } z$$

$$X = \sec(z) - 0.001817(\sec(z)-1) - 0.002875(\sec(z)-1)^2 - 0.0008083(\sec(z)-1)^3 \quad \text{for } X > 2$$

For values of z approaching 90° there are several interpolative formulas:
-X is around 40 for z=90°



http://en.wikipedia.org/wiki/Airmass

Atmospheric refraction

- Direction of light changes as it passes through the atmosphere.

- Start from Snell´s law $\boxed{\mu_1 \sin(\theta_1) = \mu_2 \sin(\theta_2)}$
- Define

z_0 : true zenith distance
z : observed zenith distance
z_n : observed zenith distance at layer n

Top of the atmosphere

$$\boxed{\frac{\sin(z_0)}{\sin(z_N)} = \frac{\mu_N}{1}}$$

Induction using infinitesimal layers

$$\frac{\sin(z_n)}{\sin(z_{n-1})} = \frac{\mu_{n-1}}{\mu_n}, \quad \frac{\sin(z_{n-1})}{\sin(z_{n-2})} = \frac{\mu_{n-2}}{\mu_n}$$

$$\Rightarrow \frac{\sin(z_n)}{\sin(z_{n-2})} = \frac{\mu_{n-2}}{\mu_n} \text{ and so on... to get}$$

$$\sin(z_0) = \mu \sin(z)$$

Refraction depends only on refraction index near earth´s surface

Atmospheric refraction

- We define the astronomical refraction r as the angular displacement:

$$\boxed{\sin(z+r) = \mu \sin(z)}$$

- In most cases r is small

$$\boxed{\begin{aligned} &\sin(z) + r\cos(z) = \mu \sin(z) \\ &\Rightarrow r = (\mu - 1)\tan(z) \\ &\qquad \equiv \; R\tan(z) \end{aligned}}$$

For red light R is around 1 arc minute

Atmospheric dispersion

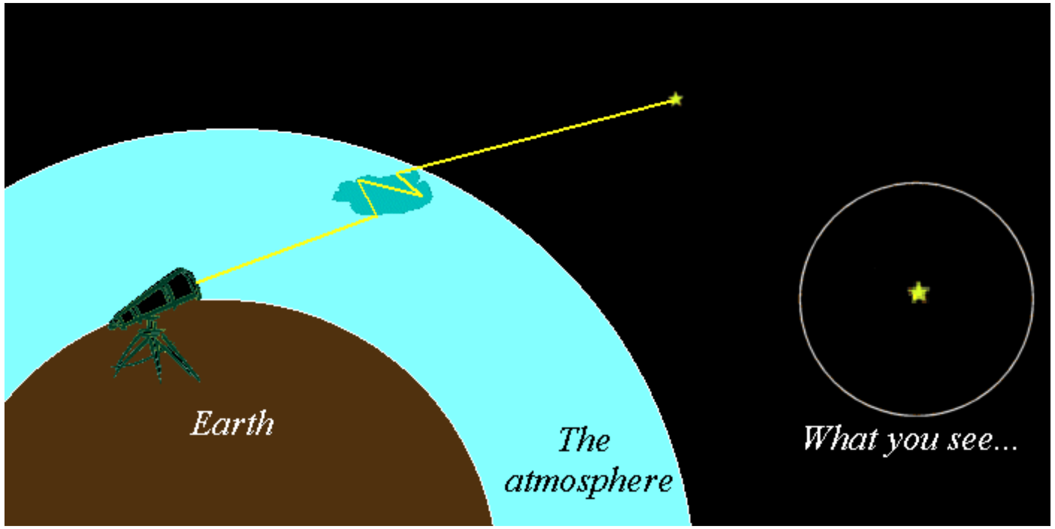
- Since R depends on frequency: atmospheric dispersion
  - Location of an object depends of wavelength:
    - implications for multiobjects spectrographs , where slits are placed on objects to accuracy of an arcsecond

Lambda (A)	R (arcsec)
3000	63.4
4000	61.4
5000	60.6
6000	60.2
7000	59.9
10000	59.6
40000	59.3

An image of Venus, showing chromatic dispersion.

Seeing: main idea

- General idea: "Earth's atmosphere is turbulent and variations in the index of refraction cause the plane wavefront from distant objects to be distorded"



Seeing: general idea

- Scintillation: amplitude variations in time
  - Timecales of several milliseconds and up
  - small for large apertures.
- Positional and image quality:
  - Small apertures: diffraction pattern moving around
  - Large apertures: diffraction pattern moving on scale or arcsec.

Double star Zeta Aquarii (which has a separation of 2 arcseconds) being messed up by atmospheric seeing, which varies from moment to moment.

Seeing: Theory

- "The idea is to describe statistically the observed amplitude and phase of an incoming plane wave front passing through a turbulent media"

i) Assuming the turbulence of the atmosphere is isotropic and scale invariant from Kolmogorov's theory we obtain the structure function for the kinetic energy:

$$\boxed{D_V = \left\langle (V(R+r) - V(R))^2 \right\rangle \propto r^{2/3}}$$

Dimensional analysis implies

$$\boxed{D_T = \left\langle (\delta T(R+r) - \delta T(R))^2 \right\rangle \propto r^{2/3}}$$

ii) Fluctuations in the refractive index n depens linearly on temperature (fluctuations in pressure are assumed to come quickly to mechanical equilibrium)

$$\boxed{\begin{array}{l} D_n = \left\langle (n(R+r) - n(R))^2 \right\rangle = C_n{}^2 r^{2/3}, \\ \text{where } C_n \text{ is refractive - index structure coefficient} \end{array}}$$

Seeing: Theory

iii) The phase structure function at the entrance of the telescope is (see arguments in Tatarski 1971)

$$\boxed{D(r) = 2.91 k^2 r^{5/3} \int C_n^2 ds}$$

Which can be expressed in a more common way as

$$D(r) = 6.88 \left(\frac{r}{r_0} \right)^{5/3} \text{, where}$$

$$\mathrm{r}_0 = 0.185 \cdot \lambda^{6/5} \cdot \cos(z)^{3/5} \left(\int C_n^2 dh \right)^{-3/5} \text{ is known as the Fried parameter}$$

Fried 1965

Seeing: Theory

- **Larger r_0 means better seeing**
- In fact, it can be (roughly) interpreted to be inversely proportional to the image size d from seeing $\boxed{d \propto \lambda / r_0}$
- For diffraction-limited images (aperture D): $\boxed{d \propto \lambda / D}$

and seeing dominates if $\boxed{r_0 < D}$

- A seeing-limited astronomical telescope will improve as

$$\boxed{d \propto \lambda / r_0 \propto \lambda^{-1/5}}$$

- Then, seeing is more important than diffraction at shorter wavelengths (and larger apertures D):
  - Crossover in the IR for most astronomical-sized telescopes (5 microns for D=4m)

Seeing: how does image look like?

- A way of describing the quality of an image is to specify the Point Spread Function.
- A way to do this is starting from its Modulation Transfer Function MFT (Fourier transform pair of the PSF).

$$MFT(\nu) = e^{-1/2D(\nu/\nu_c)} = e^{-(\nu/\nu_c)^n},$$

where D is the structure function and ν is the spatial frequency.

- Then, the Point Spread Function PSF is

Racine 1996

$$I(\theta) = \int_0^\infty J_0(\nu\theta) e^{-(\nu/\nu_c)^n} \nu d\nu$$

Seeing: how does image look like?

- Analytical solutions
  - n=1 → Lorentzian
  - n=2 → Gaussian
- No analytical solution for n=5/3

  → Numerical integration:

$$\boxed{FWHM = 0.976\lambda / r_0}$$

- → Fits

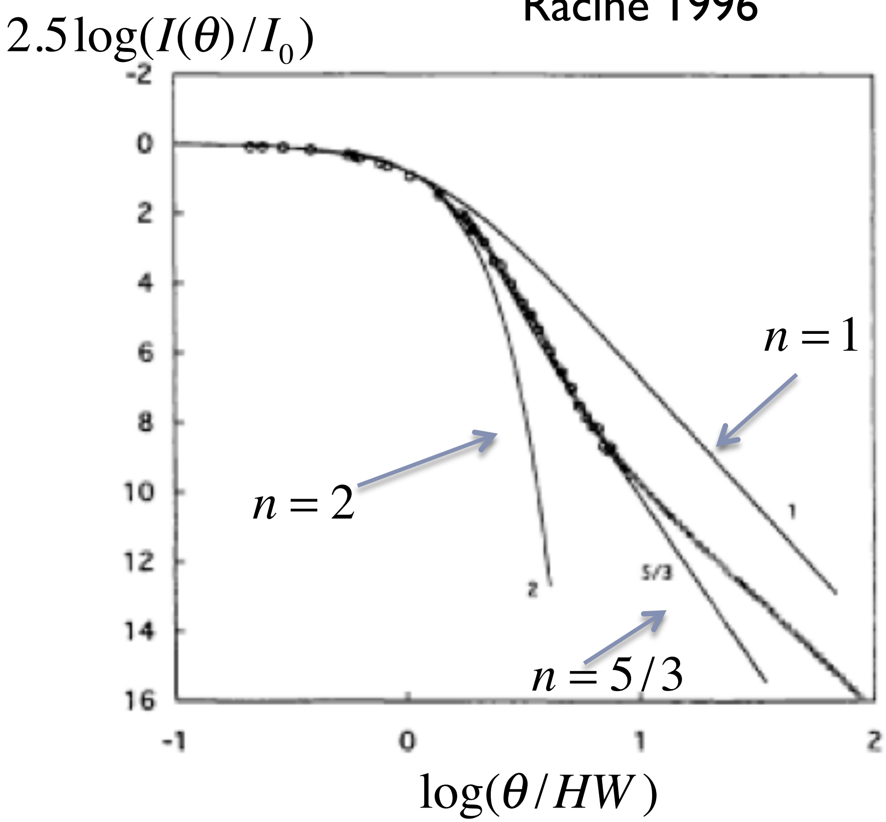
  Moffat function (Moffat 1969)

$$\boxed{I(\theta) = \frac{I_0}{(1+(\theta/R)^2)^\beta}}$$

Double Gaussian:

$$\boxed{\begin{array}{l} I(\theta) = I_0(\alpha e^{-(\theta/\sigma_1)^2} + (1-\alpha)e^{-(\theta/\sigma_2)^2}) \\ \sigma_2 = 2\sigma_1, \quad \alpha = 0.9 \end{array}}$$

Racine 1996



Roughly Gaussian in core, but more extended wings

Seeing: how does image look like?

Typical ways to characterize seeing (or r_0)

i) The most common way of describing the seeing is by specifying the FWHM:

- Usually fitting a Gaussian

ii) Encircled energy as a function of radius

iii) Strehl ratio: ratio of PSF peak to that of a perfect PSF limited by diffraction only (e.g. Racine 1996)

- As r_0 decreases (D/r_0 increases)
  - Strehl ratio decreases
  - seeing disk collect more flux

$$strehl = S = \frac{I_{\max}^{obs} / F^{obs}}{I_{\max}^{PSF,diff} / F^{PSF,diff}}$$

D/r_0	S_{long}
0.0	1.000
0.1	0.978
0.5	0.741
1.0	0.445
2.0	0.175
3.0	0.089
3.5	0.067
4.0	0.053
5.0	0.035
7.0	0.019
10.0	0.009

Racine 1996

Seeing: how does image look like?

- Scintillation: amplitude variation in turbulence timescales … several milliseconds
  - Similar effect to refraction patterns in a pool
  - Affects short exposure images

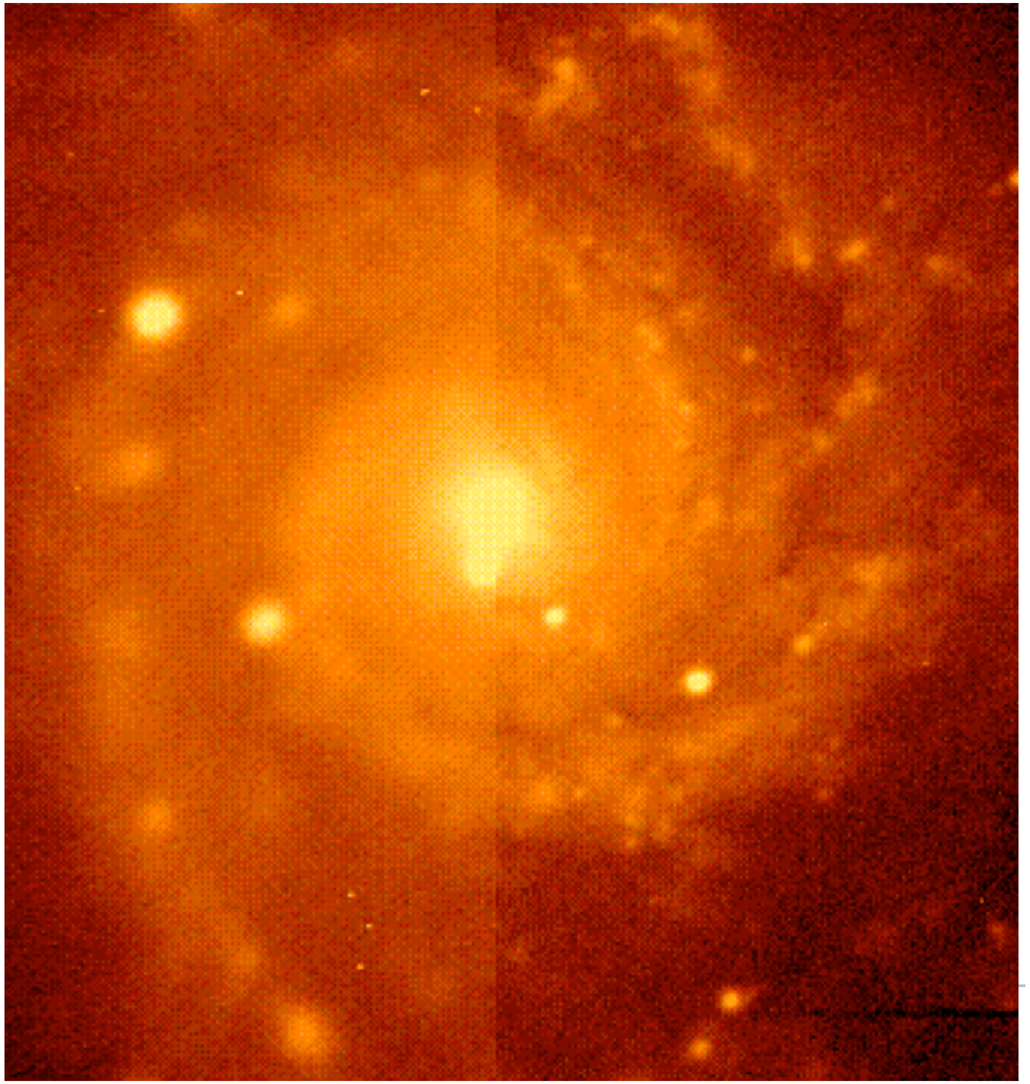
- Positional and image quality:
  - Diffraction patern (Speckles) moving around

    → Seeing disrupts the single spot of the Airy disk into a pattern of similarly-sized spots covering a much larger area
  - For large apertures diffraction patern moving in scales of 1 arcsec

Typical short-exposure image of a binary star (Zeta Bootis)

http://en.wikipedia.org/wiki/Speckle_imaging

Seeing: how does image look like?



This frame compares M74 (NGC 628) as observed on night with different seeing blurs.

Seeing: how it depends on physical sites?

- In the analysis, the only parameter that depends on the physical site is the structure coefficient integrated through the atmosphere:

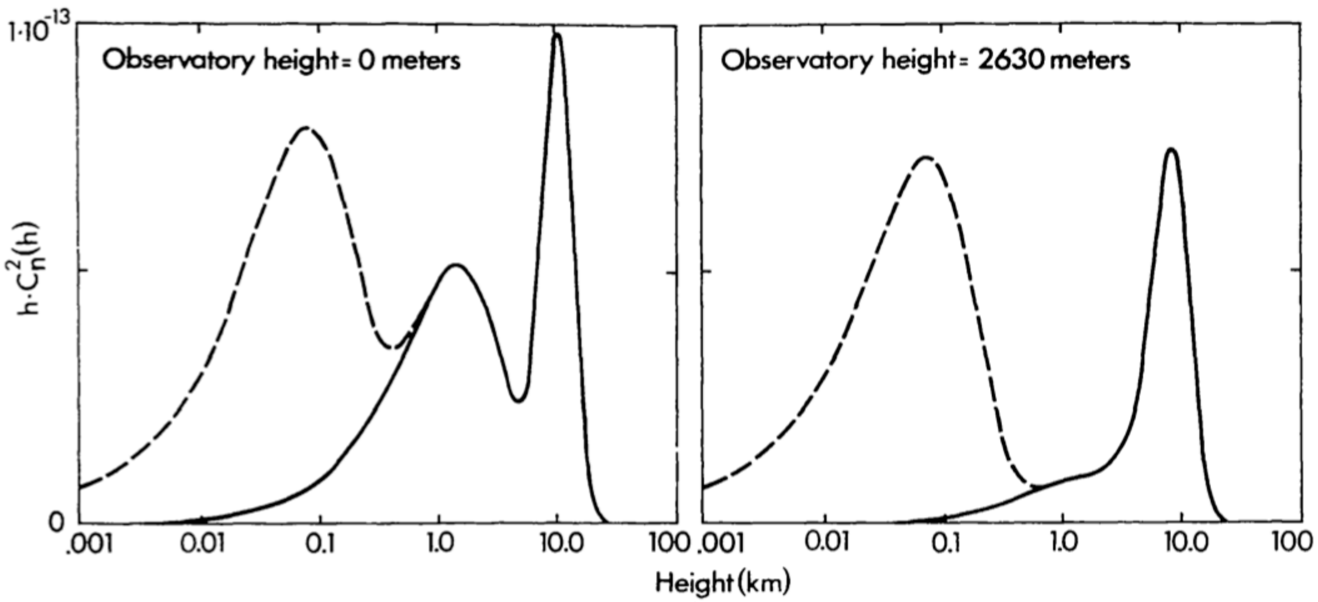
$$\boxed{\int C_n^2 dh} \qquad \left(r_0 \propto \left(\int C_n^2 dh \right)^{-3/5} \right)$$

- Of course, the smaller the better
- It varies from site to site and also in time
- People measure this…

  Examples:
  - *SCIDAR: imaging the shadow patterns in the scintillation of starlight*
  - *RADAR mapping of turbulence*
  - *Balloon-borne thermometers: measure how quickly the air temperature is fluctuating with time due to turbulence*

Seeing: how it depends on physical sites?

- At most sites, there seems to be three regimes:
  - *surface layers*: wind-surface interactions and manmade seeing
  - *planetary boundary layer*: influenced by diurnal heating
  - *free atmosphere*: at 10 km high wind shears (tropopause)

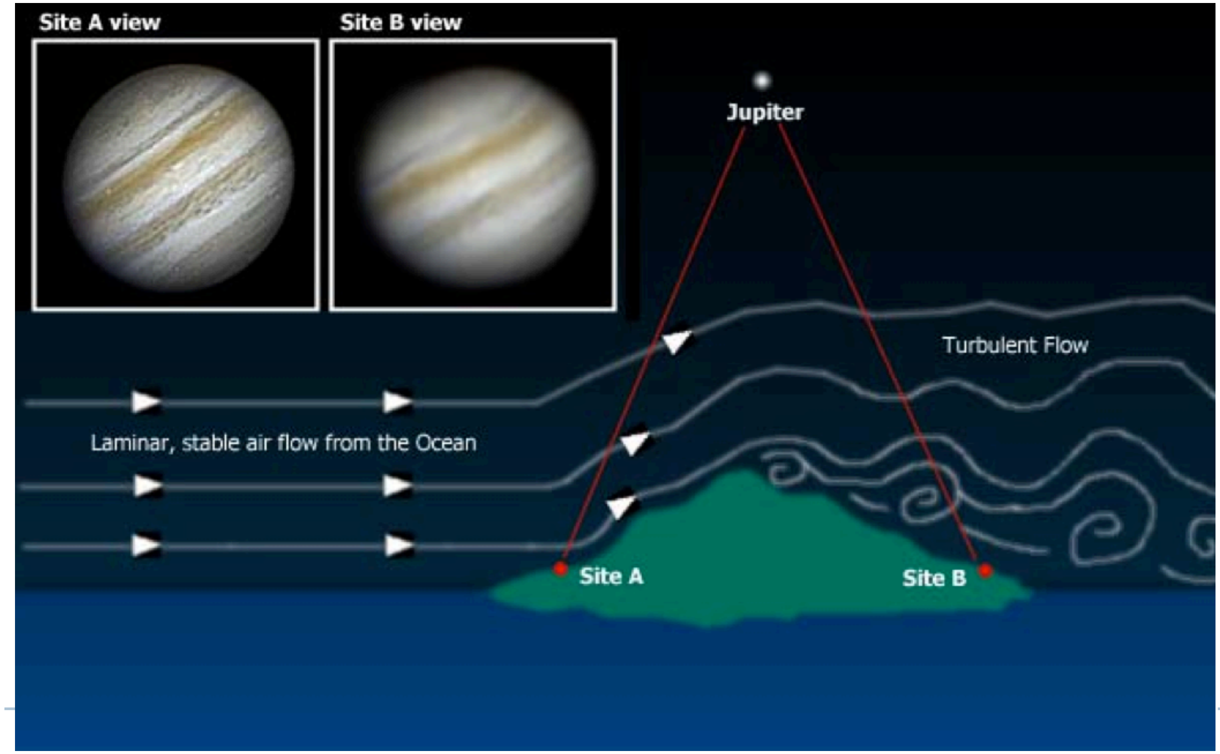


Remember… $r_0 = 0.185 \cdot \lambda^{6/5} \cdot \cos(z)^{3/5} \left(\int C_n^2 dh \right)^{-3/5}$

Beckers, J. 1993

Seeing: how it depends on physical sites?

- The world's finest locations for a stable atmosphere are:
  - mountain top observatories
  - located above frequently occurring temperature inversion layers
  - prevailing winds have crossed many miles of ocean.

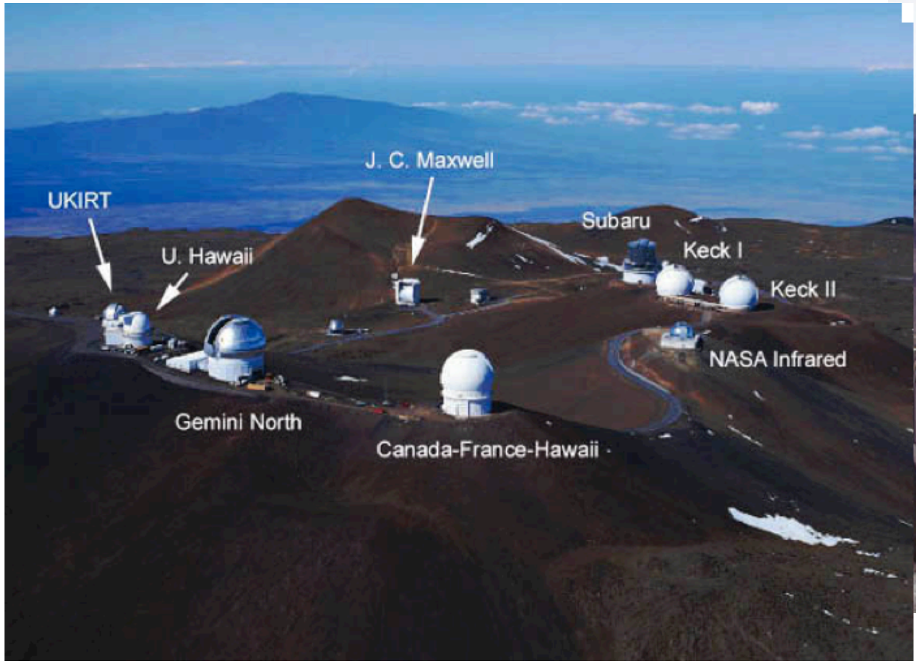


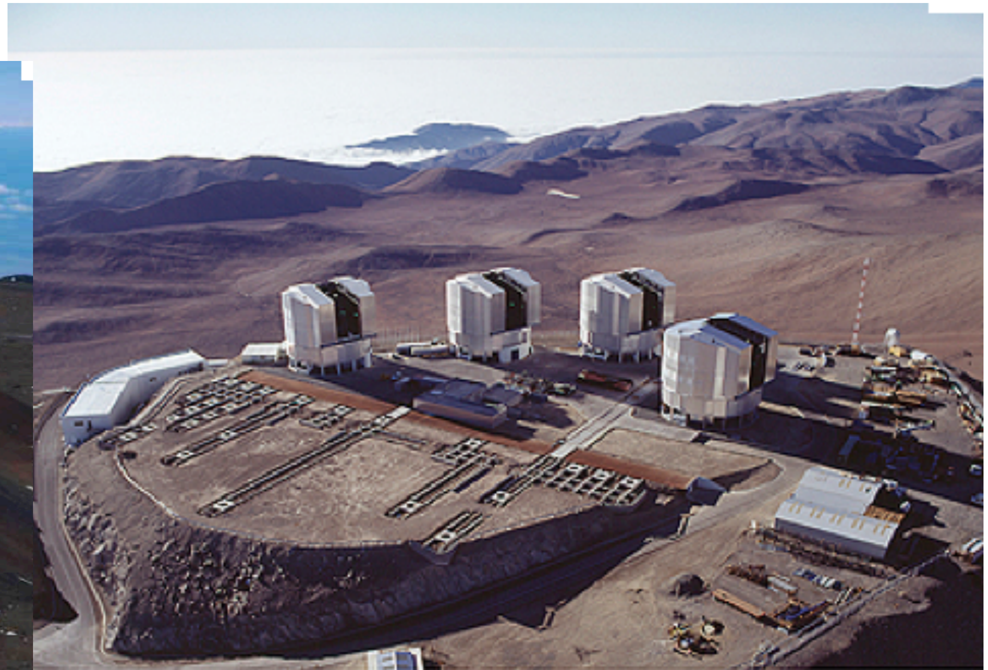
Seeing: how it depends on physical sites?

- Sites such as La Palma, Tenerife, Hawaii, Paranal frequently enjoy superb seeing (with measured as low as 0.11" arc seconds) much of the year laminar flow off the ocean

VLT at Atacama desert

Mauna Kea



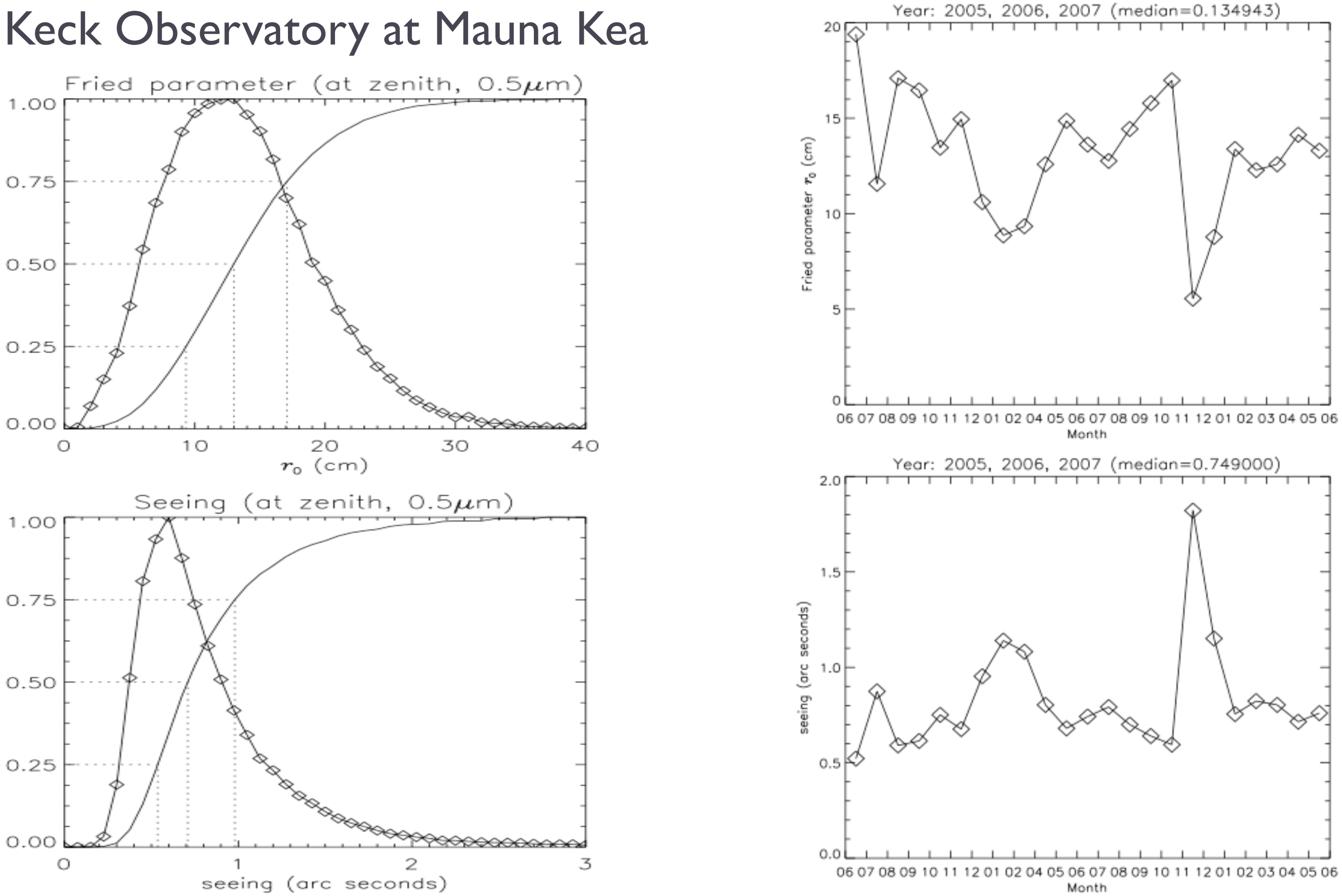


At 2600 mt

- At 2700 mt

Seeing: how it depends on time?

Keck Observatory at Mauna Kea

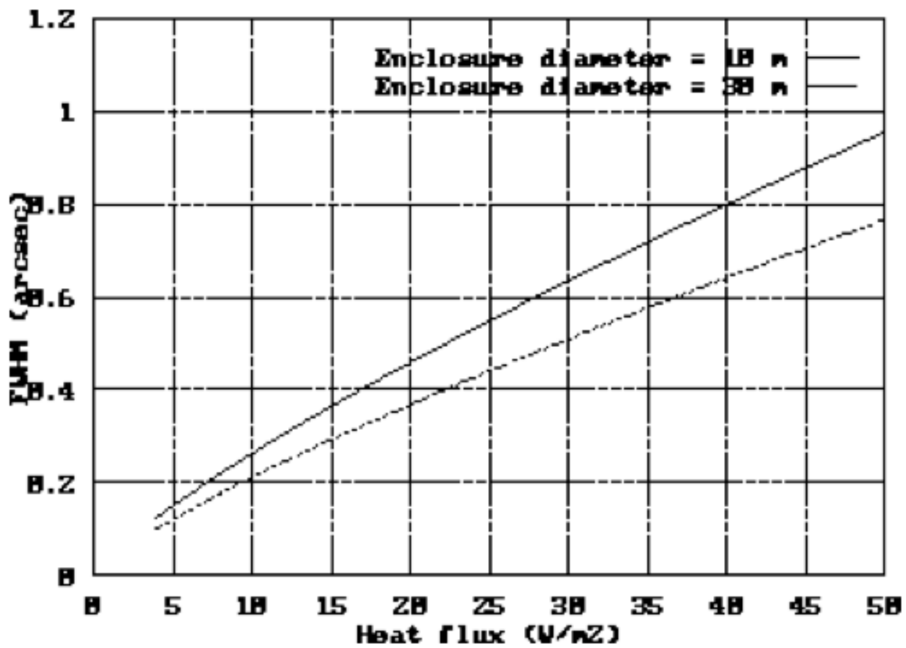


www.oir.caltech.edu/twiki_oir/pub/Keck/NGAO/.../KAON496.pdf

Seeing: other sources

- Dome seeing: turbulence pattern around the dome and interface between inside the dome and outside the dome

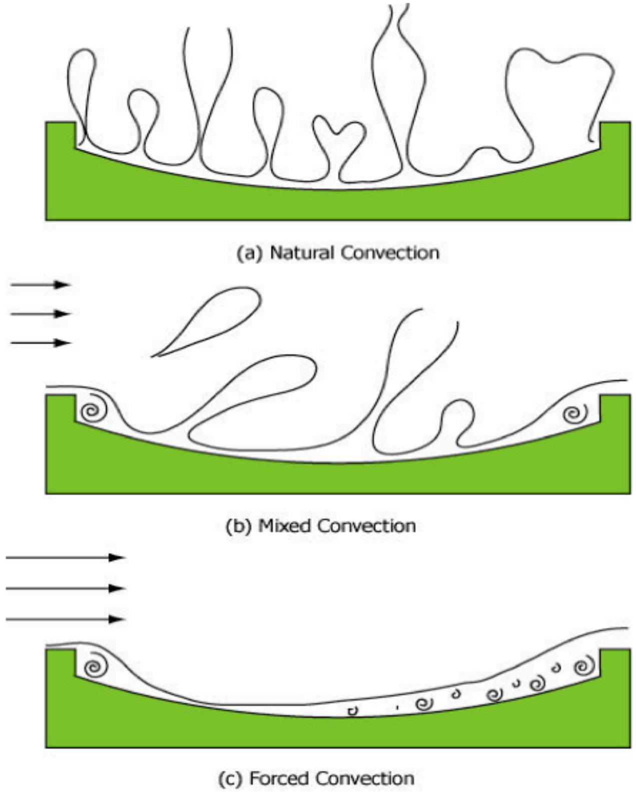
•Small temperature differences can lead to significant image degradation.
•People spend a lot of money in controlling the climate.



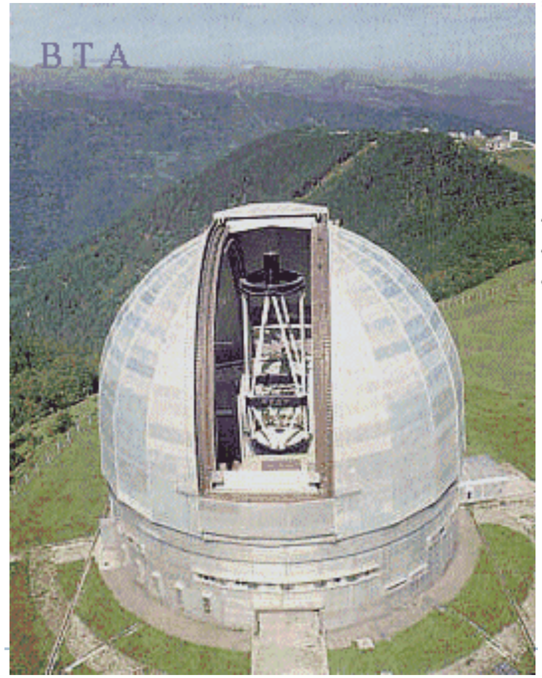
http://www.eso.org/gen-fac/pubs/astclim/papers/lz-thesis/node55.html

Seeing: other sources

- Mirror seeing: turbulence right above the surface of the mirror.
  - Can arise from a difference of temperature of the mirror and the air above it

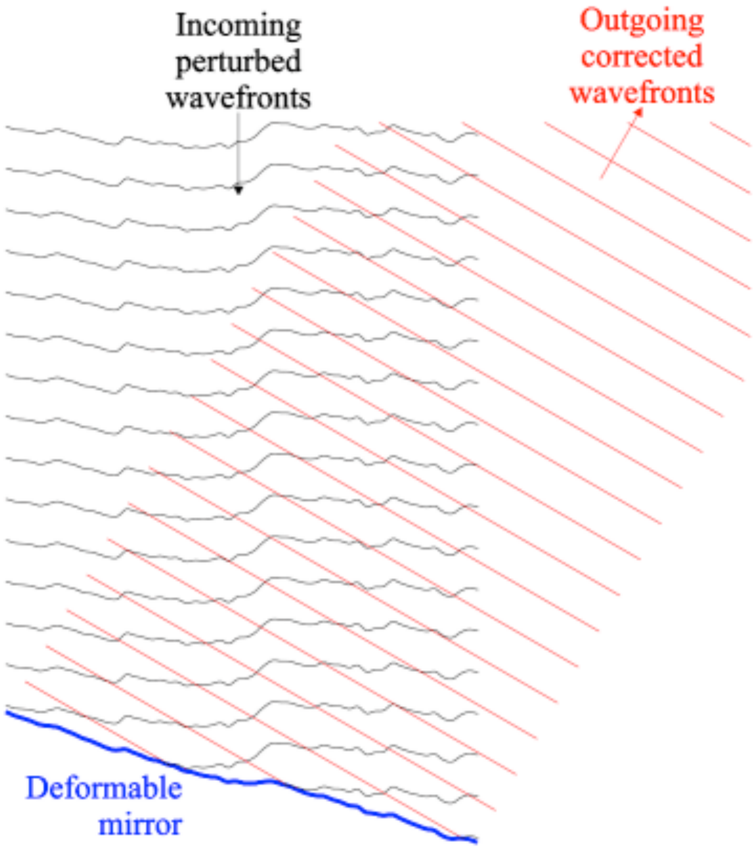


Example:
The 6 meter Rusian telescope Large Altazimuth Telescope (BTA) case
-downwind of peaks in the Caucasus
-Huge thermal mass of the primary



Seeing: solutions

- Adaptive optics: Next talk by Ruobing



Summary

- Refraction depends only on refraction index near earth´s surface
- Differential refraction changes objects position
  - account for wavelength (e.g. multiobject spectrographs)
- Seeing
  - Kolmogorov's theory gives a description
  - Resolution increases with wavelength to the one fifth
  - PSF is close to a Gaussian, but with more extended wings
  - Varies with time and physical site
  - Dome and mirror seeing are also important

References

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