OPACITY

A good description of opacity is provided by Schwarzschild in "Structure and Evolution of Stars" (Chapter II) . In practical applications opacities calculated and tabulated by the Los Alamos group are used (e.g. Cox, A. N., and Tabor, J. E. 1976, *Ap. J. Suppl.,* **31** , 271) . Here some approximate analytical formulae are given. These are fairly accurate for electron scattering (Paczynski, B., 1983, *Ap. J.,* **267** , 315) and for electron conductivity (Yakovlev, D. G., and Urpin, V. A. 1980, *Soviet Astron.,* **24** , 303) . All other are very crude approximations to the tabulated opacities.

A low density, relatively low temperature fully ionized gas has opacity dominated by Thompson electron scattering. This is is given as

$$\kappa_{Th}\rho = n_e \sigma_{Th}, \tag{on.1}$$

where $n_e = \rho(1+X)/2H$ is the number of free electrons per cubic centemeter, and $\sigma_{Th} = 8\pi r_e^2/3$ $= 0.665 \times 10^{-24}\,\mathrm{cm}^2$ is Thompson scattering cross-section per electron. Putting numerical values for all constants we obtain

$$\kappa_{Th} = 0.2\,(1+X)\,, \qquad \left[\mathrm{cm}^2\,\mathrm{g}^{-1}\right]. \tag{on.2}$$

When temperature is high, so that energy of photons becomes an appreciable fraction of the electron rest mass, the cross-section for scattering is reduced according to the Klein-Nishina formula, and the electron scattering opacity is reduced. When electron gas becomes degenerate and most energy levels below E_{Fermi} are filled in, photons may be scattered only on some electrons, and the electron scattering opacity is reduced too. The numerical results presented by Buchler, J. R., and Yueh, W. R., 1976, *Ap. J.,* **210** , 440) can be fitted with the following fairly accurate formula

$$\kappa_e = 0.2\,(1+X)\left[1 + 2.7\times10^{11}\frac{\rho}{T^2}\right]^{-1}\left[1+\left(\frac{T}{4.5\times10^8}\right)^{0.86}\right]^{-1}. \tag{on.3}$$

The opacity due to free-free, bound-free, and bound-bound electronic transitions can be approximated with the so called "Kramers formula":

$$\kappa_K \approx 4\times10^{25}\,(1+X)\,(Z+0.001)\,\frac{\rho}{T^{3.5}}. \tag{on.4}$$

This formula is important when hydrogen and helium and other elements are partly ionized, roughly speaking for $T \geq 2\times10^4\ K$.

The opacity due to the negative hydrogen ion, H^-, is the dominant opacity source in the solar atmosphere, in general for $4\times10^3 \leq T \leq 8\times10^3\ K$:

$$\kappa_{H^-} \approx 1.1\times10^{-25}\ Z^{0.5}\rho^{0.5}T^{7.7}. \tag{on.5}$$

Notice, the extremely steep temperature dependence of the H^- opacity.

At very low temperature molecules dominate the opacity, with H_2O and CO being the most important for $1.5\times10^3 \leq T \leq 3\times10^3\ K$:

$$\kappa_m \approx 0.1\ Z. \tag{on.6}$$

The total radiative opacity may be crudely approximated with the following formula, which may be used over a huge range of temperatures, $1.5\times10^3 \leq T \leq 10^9\ K$:

$$\kappa_{rad} \approx \kappa_m + \left(\kappa_{H^-}^{-1} + (\kappa_e+\kappa_K)^{-1}\right)^{-1}. \tag{on.7}$$

At very high density and low temperature the radiative opacity becomes very high, as long as the gas remains substantially ionized and non-degenerate [eq. (on.4)], and the electron thermal conductivity becomes a much more efficient heat carrier. A reasonable approximation to the electron conductivity expressed in a form of "opacity" is:

$$\kappa_{cond} \approx 2.6 \times 10^{-7} \ \langle Z^* \rangle \ \frac{T^2}{\rho^2} \ \left(1 + \left(\frac{\rho}{2 \times 10^6} \right)^{2/3} \right) , \tag{on.8}$$

where $\langle Z^* \rangle$ is the average electric charge per ion.

Finally, the total effective opacity, which includes both heat carriers: photons and electrons, can be calculated as:

$$\kappa = \left(\kappa_{rad}^{-1} + \kappa_{cond}^{-1} \right)^{-1} . \tag{on.9}$$

This last formula is exact, as the two heat carriers offer two independent modes of energy transport, i.e. the corresponding coefficients of heat conductivity are additive, which is equivalent to the inverse of the corresponding "opacities" to be additive.

Throughout this chapter all opacities are given in c.g.s. units, i.e. in $[cm^2 g^{-1}]$. Also, in all formulae X and Z indicate the hydrogen and heavy element fraction by mass.