21 November 2006 by JJG

Stellar Nucleosynthesis

Figure 1 shows the relative abundances of solar system elements versus atomic number Z, the number of protons in the nucleus. A theory of nucleosynthesis should explain this pattern. One believes, of course, that all but the lightest elements ($Z > 4$) are made in stars rather than the early universe, and primarily through nuclear fusion.

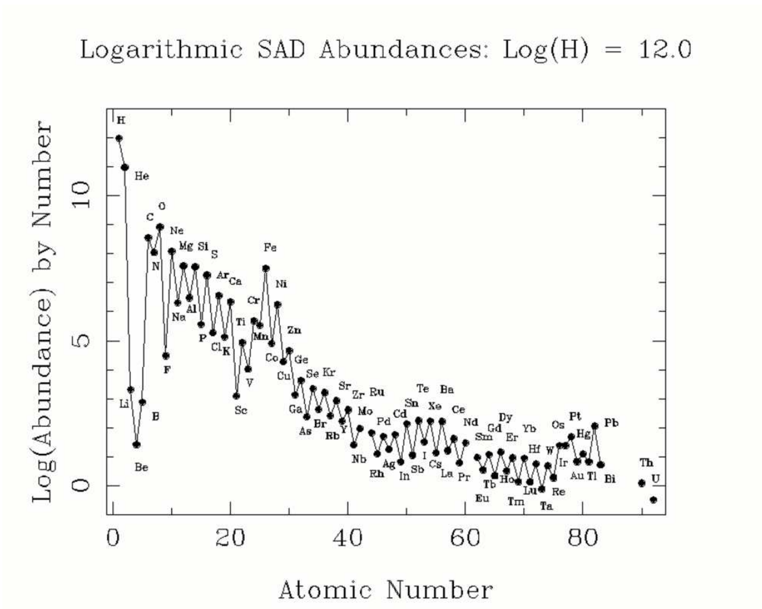

Figure 1: Solar abundances by number, *i.e.* vertical axis is $\log(X_Z/\bar{A}_Z) + \text{constant}$, where X_Z is the abundance of element Z by mass and $\bar{A}_Z$ is the mean atomic weight of that element. (C. R. Crowley, U Mich).

It is clear that the pattern in Fig. 1 is not the result of thermal equilibrium: if it were, then iron (^{56}Fe) would be the most abundant element since it is the most tightly bound. In fact, nuclear masses can be approximated by the *semi-empirical mass formula* [1]

$$M(A,Z) = (A-Z)m_n + Z(m_p + m_e) \\ - a_1 A + a_2 A^{2/3} + a_3 \frac{(A/2 - Z)^2}{A} + a_4 \frac{Z^2}{A^{1/3}} + a_5 \frac{\delta}{A^{3/4}} \,. \qquad (1)$$

The coefficients have the following approximate values measured in $\mathrm{MeV/c^2}$:

$$a_1 - 15.53,\ a_2 = 17.804,\ a_3 = 94.77,\ a_4 = 0.7103,\ a_5 = 33.6.$$

The first two terms on the righthand side represent the "free" rest masses of the neutron, proton, and electron: $m_n \approx 939.57$, $m_p \approx 938.27$, $m_e \approx 0.511\ \mathrm{MeV/c^2}$. The term in a_1 represents an increase in the binding energy (*i.e.*, decrease in nuclear mass) due to nearest-neighbor interactions between nucleons: to lowest order, nuclei are rather like drops of liquid, in which the interactions are very short range, and are attractive at low pressure but are strongly repulsive under compression; consequently, the liquid prefers to maintain a roughly constant density, which for nuclei is $\rho_{\rm nuc} \approx 2 \times 10^{14}\,\mathrm{g\,cm^{-3}}$, correponding to an interparticle spacing $a_0 \equiv (3\rho_{\rm nuc}/4\pi m_p)^{1/3} \approx 1.3\,{\rm fm}$. Thus the volume of the nucleus $\propto A$. Just as in most liquids, there is a positive energy associated with the surface area of the drop because particles near the surface have fewer neighbors to bond with: this is represented by the positive term $a_2 A^{2/3}$. The next term (in a_3) favors comparable numbers of neutrons $(A-Z)$ and protons (Z) due to the exclusion principle between identical particles. However, the term in a_4 is the electrostatic energy of the protons, which causes a shift toward neutron-rich nuclei (*i.e.*, $A - Z > Z$) with increasing atomic number. The last term reflects an attractive pairing between like nucleons: $\delta = -1$ if the number of protons and the number of neutrons are both even, $\delta = 0$ if A or Z but not both is odd, and $\delta = +1$ if both are odd. Nuclei with larger binding energies per nucleon

$$\frac{B}{A} \equiv \frac{M(A,Z) - (A-Z)m_n - Z(m_p + m_e)}{A} \tag{2}$$

tend of course to be stabler. This quantity is plotted in Figure 2.

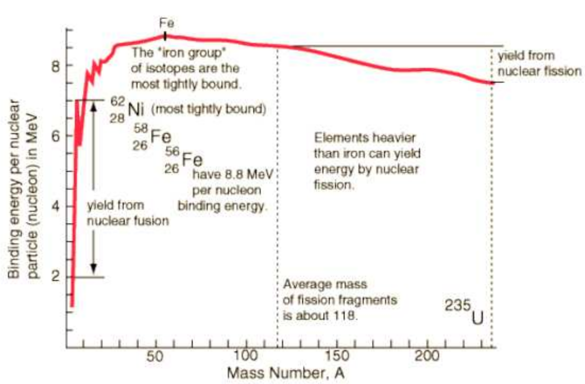

Figure 2: Binding energy per nucleon versus atomic number (`http://hyperphysics.phy-astr.gsu.edu`).

While iron is less abundant than it would be in thermal equilibrium, Figure 1 shows that it is nevertheless very abundant compared to nearby elements. Furthermore, there is a pronounced tendency for elements with odd Z to be less abundant than those with even Z, reflecting perhaps the influence of the last term in eq. (1). Although it is not accounted for in that formula, nuclei whose Z or $A-Z$ is one of the *magic numbers* $2, 8, 20, 28, 50, 82, 126$ are more strongly bound than their neighbors, and doubly magic nuclei such as ^{16}O ($Z=8$), ^{40}Ca ($Z=20$), and ^{208}Pb ($Z=82$) are particularly strongly bound, because they represent closed shells of nucleons orbiting within the mean-field nuclear potential. These "magic" isotopes are also more abundant than their neighbors. So the binding energy per nucleon clearly has some explanatory power.

The reason that the heavier nuclei are less abundant is of course that the coulomb barrier makes them difficult to form by fusion except at extreme temperatures. In the cores of massive stars at the end of their evolution, the temperature approaches $10^{10}\,\mathrm{K} \approx 1\,\mathrm{MeV}/k_{\mathrm{B}}$, so that elements up to the iron group (meaning $24 \le Z \le 28$, chromium through nickel) do form. Since these cores are strongly gravitationally bound, much of the heavier elements remain locked up in neutron stars or black holes, but some is returned to the interstellar medium in supernovae.

The elements are often categorized according to the nuclear reactions in which they participate or those which produce them.

$$^{1}\mathrm{H},\ \ ^{2}\mathrm{H},\ \ ^{3}\mathrm{He},\ \ ^{4}\mathrm{He},\ \ ^{7}\mathrm{Li}$$

are primordial or mainly so—outcomes of the first three minutes of the Big-Bang. The other light nuclei ^{6}Li, ^{9}Be, ^{10}B, and ^{11}B are probably produced mainly by spallation—that is, by fragmentation of heavier nuclei in the ISM under bombardment by cosmic rays. The lack of a stable isotope with $A=8$ blocked the build-up of heavier nuclei at the low densities of cosmological nucleosynthesis. In stars, this obstacle is bridged by the triple-alpha reaction, $3\,^{4}\mathrm{He} \to {}^{12}\mathrm{C}$; since this is a three-body process, its rate is negligible except at high density (and high temperature, $T \sim 10^{8}\,\mathrm{K}$). Actually, the triple-alpha reaction can usefully be thought of as a quick succession of two-body reactions:

$$2\,^{4}\mathrm{He} \to {}^{8}\mathrm{Be}^{*}, \quad {}^{8}\mathrm{Be}^{*} + {}^{4}\mathrm{He} \to {}^{12}\mathrm{C},$$

in which the beryllium isotope is a metastable state or resonance with a lifetime $\approx 2.6 \times 10^{-16}\,\mathrm{s}$. Although short, this is very long compared to the time required for an alpha particle to cross its own diameter at its thermal velocity, $\lesssim 10^{-20}\,\mathrm{s}$ at $10^{8}\,\mathrm{K}$, so it contributes substantially to the triple-alpha reaction rate. The subsequent step is also enhanced by a resonance: that is, the sum of the energies (including rest mass) of the $^{8}\mathrm{Be}^{*}$ and the alpha particle ^{4}He after barrier is very close to that of an excited state of the ^{12}C nucleus. In fact, Hoyle *predicted* the existence of this excited state from the requirement that the reaction be fast enough to satisfy astrophysical constraints [2].

C, N, and O, serve as catalysts in the CNO cycle, but these elements are not significantly produced by hydrogen burning. Nitrogen is an exception: whatever the initial abundances of CNO, when the cycle reaches a steady state, ^{14}N predominates because the reaction that destroys it, $^{14}\mathrm{N}(p,\gamma)\,^{15}\mathrm{O}$, is a bottleneck. Apart from this, the CNO elements are mainly produced in helium burning. Of course ^{12}C is the direct product of the triple-alpha reaction, but in helium-burning regions where alpha particles are plentiful, this can be followed by $^{12}\mathrm{C}(\alpha,\gamma)\,^{16}\mathrm{O}$. In fact, the relative abundance of ^{12}C and ^{16}O resulting from helium abundance is a delicate function of conditions in the burning region and of imperfectly known cross sections; at solar abundance, $^{16}\mathrm{O}/\,^{12}\mathrm{C} \approx 2$. These and heavier nuclei with $Z = A/2$ are referred to as "α nuclei." The heavier of these seem to be produced mainly by hydrostatic carbon or oxygen burning, although explosive burning—in novae and supernovae—also contributes. Insofar as the fuel for these burning stages consists of two-body collisions between nuclei of the same isotope, one might naively expect that atomic weights would simply double at each stage, *i.e.* $2\,^{12}\mathrm{C} \to \,^{24}\mathrm{Mg}$, $2\,^{24}\mathrm{Mg} \to \,^{48}\mathrm{Ca}(Z = 20)$ or $\to \,^{48}\mathrm{Ti}(Z = 22)$ (the obvious alpha nucleus, $^{48}\mathrm{Cr}$, is not stable). However, after barrier penetration, the collision of two identical heavy nuclei usually results in a highly excited state, which decays promptly *via* emission of a nucleon or α particle, which involves strong interactions, rather than wait around for a relatively slow electromagnetic (*i.e.*, emission of a γ) or weak interaction. Thus for example, the dominant exothermic carbon-burning reactions are $2\,^{12}\mathrm{C} \to \,^{23}\mathrm{Na} + p$ and $2\,^{12}\mathrm{C} \to \,^{20}\mathrm{Ne} + \alpha$. In fact, $^{20}\mathrm{Ne}$ is the next most abundant nucleus after $^{16}\mathrm{O}$ and $^{12}\mathrm{C}$ in solar composition. Similarly, $^{31}\mathrm{P} + p$ and $^{30}\mathrm{Si} + 2p$ are the most common "exit channels" for $^{16}\mathrm{O} + \,^{16}\mathrm{O}$ (oxygen burning), rather than the alpha element $^{32}\mathrm{S} + \gamma$. In this way, the earlier parts of the periodic table are filled in.

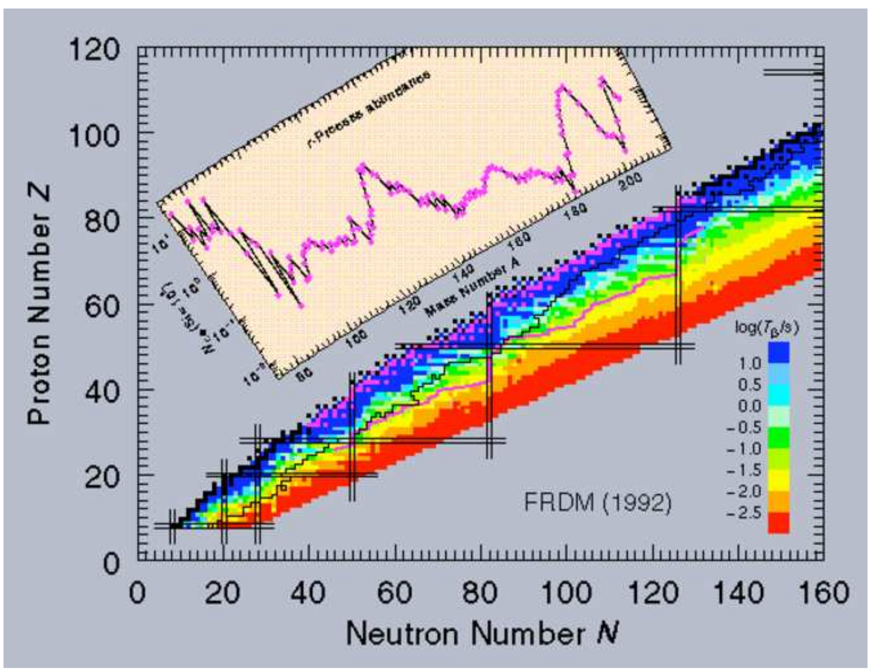

Figure 3: r process. Black squares are non-r-process stable elements. Nearby magenta squares are stable nuclei resulting from beta relaxation of r-process trajectories such as the one shown as the jagged magenta curve. Other colors show beta-decay times (in seconds, with logarithmic steps), decreasing with distance from the locus of the black squares. (`http://dkcmzc.chemie.uni-mainz.de/p̃feiffer/ages.html`)

The elements beyond the iron peak cannot be made in exothermic fusion reactions. While endothermic reactions during burning of lighter nuclei contribute, these elements are produced mainly by neutron capture: the s ("slow") and r ("rapid") processes. There is of course no coulomb barrier to neutron capture, but a supply of free neutrons is required.

A nucleus of fixed Z cannot absorb arbitrarily many neutrons, however, without becoming unstable, since the neutron has a higher mass than the proton, and more importantly because of the symmetry term ($\propto a_3$) in the semi-empirical mass formula (1). At some point, a beta decay $(A, Z) \to (A, Z+1) + e^- + \bar{\nu}_e$ must occur to restore the balance between protons and neutrons. The distinction between r and s processes has to do with the rate of neutron capture relative to the rate of beta decay. If the neutron flux is low, each capture that results in a β-unstable nucleus is followed by a beta decay before the next capture. This is the s process. The resulting pattern of abundances is in principle well defined by the stability of the nuclei concerned, at least insofar as the initial abundances of the "seed" nuclei resulting from fusion reactions is known. The s process terminates at $209Bi$ (bismuth), the last stable isotope (actually this nucleus is radioactive, but with a half-life longer than a Hubble time). In particular, it cannot produce uranium and thorium.

If the neutron flux is very high, then multiple neutron captures can occur between beta decays. This is the r process. While it is going on, nuclei can be produced that are well away from the "valley of stability" defined by (1) or more accurate formulae: that is, from the value(s) of Z that are most tightly bound for given A. As a rule however, the farther a nucleus lies from the valley, the more rapid beta decay becomes (Fig. 3). Thus the nuclei cannot stray arbitrarily far from the valley, and the resulting abundance pattern can be expected to depend somewhat on the neutron flux.

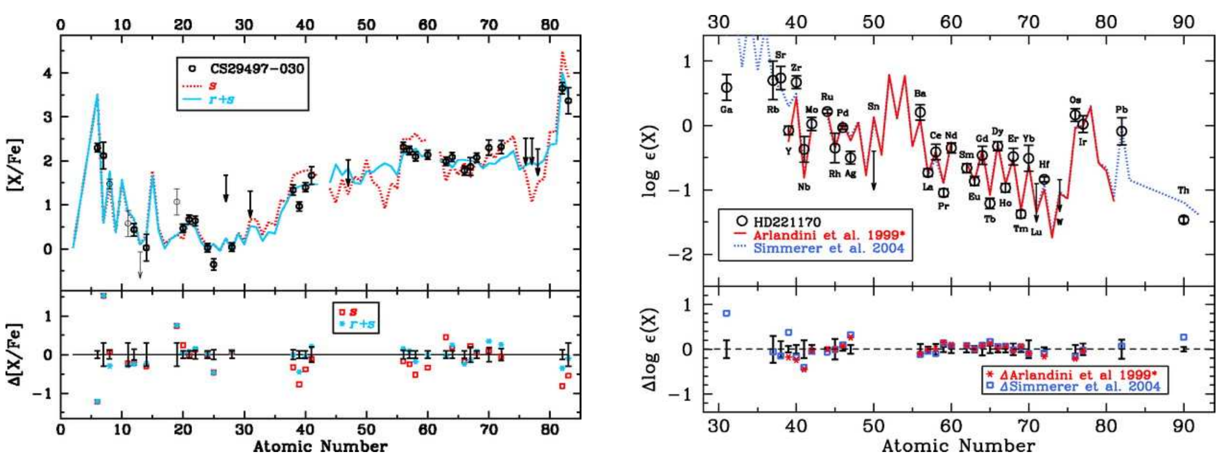

Figure 4: r and s processes in two low-metallicity halo stars compared to theoretical calculations. *Left:* a star dominated by s process but apparently "seeded" by prior r process [4]. *Right:* A pure r-process star [3].

The r and s patterns were first recognized in solar abundances but can also be seen in high-resolution spectroscopy of stars, as shown in Figure 4.

The sites of these processes are still somewhat uncertain, particularly that of the r process. The s process is believed to occur mainly on the asymptotic giant branch in intermediate-mass stars ($1 \lesssim M/M_\odot \lesssim 9$). Helium shell burning is unstable and occurs in "pulses." It is hypothesized that as a result of these pulses, some ^{13}C associated with the CNO cycle in the overlying hydrogen-burning shell is mixed into the helium-burning region. This enables the reaction $^{13}\mathrm{C} + \alpha \to {}^{16}\mathrm{O} + n$, which provides the required free neutrons in the intervals between pulses. The reaction $^{22}\mathrm{Ne} + \alpha \rightarrow {}^{25}\mathrm{Mg} + n$ may also be important to the production of neutrons during the pulses themselves [5]. The r process probably occurs at some point during the violent process of core collapse and supernova explosion of massive stars, but a consensus has not been reached concerning the details.

References

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