

MASS – LUMINOSITY RELATION FOR MASSIVE STARS

Within the Eddington model $\beta \equiv P_g/P = \text{const}$, and a star is an $n = 3$ polytrope. Large mass stars have small β , and hence are dominated by radiation pressure, and the opacity in them is dominated by electron scattering. Let us consider the outer part of such a star assuming it is in a radiative equilibrium. We have the equation of hydrostatic equilibrium:

$$\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho, \quad (\text{s2.1})$$

and the equation of radiative equilibrium

$$\frac{dP_r}{dr} = -\frac{\kappa\rho L_r}{4\pi cr^2}. \quad (\text{s2.2})$$

Dividing these equations side by side we obtain

$$\frac{dP_r}{dP} = \frac{\kappa L_r}{4\pi cGM_r}, \quad (\text{s2.3})$$

Near the stellar surface we have $M_r \approx M$ and $L_r \approx L$, and adopting $\kappa \approx \kappa_e = \text{const}$, we may integrate equation (s2.3) to obtain

$$P_r - P_{r,0} = \frac{\kappa_e L_r}{4\pi cGM_r} (P - P_0), \quad (\text{s2.4})$$

where $P_{r,0} = P_0 = aT_{eff}^4/6$ is the radiation pressure at the surface, i.e. at $\tau = 0$, where the gas pressure $P_g = 0$. At a modest depth below stellar surface pressure is much larger than P_0 , we may neglect the integration constants in (s2.4) to obtain

$$1 - \beta = \frac{P_r}{P} = \frac{\kappa_e L}{4\pi cGM} = \frac{L}{L_{Edd}} = \frac{1 + X}{65300} \frac{M_\odot}{L_\odot} \frac{L}{M}, \quad (\text{s2.5})$$

where L_{Edd} is the **Eddington luminosity**.

Equation (s2.5) gives a relation between stellar mass, luminosity and β . The Eddington model gives a relation between stellar mass and β :

$$\frac{M}{M_\odot} = \frac{18.1}{\mu^2} \frac{(1 - \beta)^{1/2}}{\beta^2}, \quad (\text{s2.6})$$

where μ is a mean molecular weight in units of mass of a hydrogen atom, $\mu^{-1} = 2X + 0.75Y + 0.5Z$, and X, Y, Z , are the hydrogen, helium, and heavy element abundance by mass fraction. Combining equations (s2.5) and (s2.6) we obtain a mass luminosity relation

$$\frac{M}{M_\odot} = \frac{18.1}{\mu^2} \frac{(L/L_{Edd})^{1/2}}{(1 - L/L_{Edd})^2}, \quad L_{Edd} \equiv \frac{4\pi cGM}{\kappa_e}. \quad (\text{s2.7})$$

For very massive stars this gives approximate relation

$$L \approx L_{Edd} = \frac{65300L_\odot}{(1 + X)M_\odot} M, \quad M \gg 100M_\odot. \quad (\text{s2.8})$$

For lower mass stars $L \ll L_{Edd}$, and (s2.7) gives

$$L \approx L_{Edd} \mu^4 \left(\frac{M}{18.1M_\odot} \right)^2 \approx 30 L_\odot \left(\frac{M}{M_\odot} \right)^3, \quad X = 0.7, \quad Z = 0.02. \quad (\text{s2.9})$$

It is remarkable that we obtained the mass luminosity relation without any reference to the stellar energy sources. This can be understood in the following terms. Within our approximation the opacity of matter is constant (per unit mass), the photons diffuse out at the rate they can, which is independent on stellar temperature or density. So the heat losses are fixed by the constant opacity. If there are no nuclear energy sources then the star will be losing energy, and it will have to contract, to pump some gravitational energy into thermal energy. The temperature of a contracting star rises, and at some point heat released in nuclear reactions may balance the radiative heat losses. If this happens then stellar contraction stops, and the star radiates away the energy generated in nuclear burning. Whenever nuclear fuel is exhausted the ever present heat losses will force farther contraction of the star. Therefore, it is the stellar radius that depends on the presence or absence of nuclear burning, while the luminosity is controlled by the opacity.

Notice, that luminosity depends on chemical composition not only through the opacity, but also through the equation of hydrostatic equilibrium. This shows up in the equation (s2.9) through the term μ^4 . The larger is the mean molecular weight, the higher is the luminosity. This has paradoxical consequence. Imagine a main sequence star, i.e. a star that burns hydrogen at the rate required to balance radiative heat losses. According to equation (s2.9) the luminosity depends on chemical composition according to

$$L \sim \frac{\mu^4}{\kappa_e} = (1 + X)^{-1} (2X + 0.75Y + 0.5Z)^{-4} \sim X^{-2.6} \quad \text{for } X \approx 0.7, \quad (\text{s2.10})$$

i.e. the less hydrogen there is the brighter is a hydrogen burning star!

In stars that are not dominated by radiation pressure there are other opacity sources that exceed electron scattering. If the opacity is increased then the rate of photon diffusion is reduced, and stellar luminosity is reduced as well. For this reason the mass luminosity relation (equations s2.7, s2.8, and s2.9) gives an upper limit to the luminosity of stars approximated with the Eddington model.