AST 403 Problem Set #3 Due March 1 2011

1. a. The Earth can be represented roughly as a polytrope of Emden index $n = 0$, *i.e.* constant density. Assuming this, and given that $M_E = 6 \times 10^{27}\,\mathrm{g}$ and $R_E = 6.4 \times 10^8\,\mathrm{cm}$, calculate its central density, central pressure, and gravitational potential energy. Convert the pressure to atmospheres ($1\,\mathrm{atm} = 1.01 \times 10^6\,\mathrm{dyne\,cm^{-2}}$).
b. Calculate the corresponding quantities for Jupiter, which is composed mostly of degenerate molecular hydrogen rather than rock and is better approximated by $n = 1$, $M_J = 2 \times 10^{30}\,\mathrm{g} = 10^{-3} M_\odot$, $R_J = 7 \times 10^9\,\mathrm{cm} = 0.1 R_\odot$.

2. Write a computer program to solve the Lane-Emden equation for arbitrary n in the range $0 < n < 5$. Print out $\xi_{\max}$ and $\mu_{\max}$ (the dimensionless radius and mass). Check the accuracy of your program against the analytic case $n = 1$. Quote your results for a stated value of n chosen randomly from the interval $2 < n < 4$ (*i.e.* not a rational number of small (n) denominator, since those results are widely tabulated).