Astro 403 Problem Set #2
Feb 10 2011. Due Feb 24 2011

1. Derive the free-fall time of a sphere of initial density ρ_o, initial radius r_o and mass M. Dimensional analysis gives

$$t_{ff} = \frac{1}{\sqrt{G\rho_o}}$$

but the more accurate derivation gives

$$t_{ff} = \sqrt{\frac{3\pi}{32G\rho_o}}$$

Prove this.

2. (a) Calculate the free-fall time of a molecular cloud of mass 10^5 $M_\odot$ and radius 5 pc. (b) If half of the Galaxy's interstellar medium content (total mass 5×10^9 $M_\odot$) is in such molecular clouds, and free-fall collapse causes the molecular gas to form completely into stars, what is the expected mean star formation rate of the Galaxy at the present time in $M_\odot/yr$? (star formation rates are usually measured in $M_\odot/yr$, i.e. the mass of the stars measured in solar masses $M_\odot$ formed in a year). (c) The current mass of stars in the Galaxy is about $2 \times 10^{11} M_\odot$, and the Universe is 13.7 billion years old. What has been the mean star formation rate in the Galaxy over the lifetime of the Universe? Compare this with the estimate of the current star formation rate which you made above. Comments?

3. Using the scaling relations between the luminosity, mass and radius for a star on the main sequence, and the approximate dependence of the nuclear burning power on mass and radius, derive R(M). Assume that the nuclear burning is due to the CNO cycle, the ideal gas equation of state holds, and that Kramers opacity dominates.