Problem Set 8 Solutions, AST 205, Fall 2003

1. (6 points total).
(a). There are 6 billion people on Earth, with an average weight of 150 pounds, or roughly 7.5×10^4 grams. Thus all of us together have a mass of about 5×10^{14} grams.
(b). A neutron star has a density:

$$\rho = \frac{M}{4/3\pi R^3} = \frac{1.4 \times 2 \times 10^{33}\,\text{gm}}{4 \times (1.5 \times 10^6\,\text{cm})^3} = 2 \times 10^{14}\,\text{g/cm}^3.$$

(c). To figure out how much volume of neutron star stuff all of humanity would squeeze into, we divide the relevant mass by the relevant density:

$$\text{Volume} = \frac{\text{Mass}}{\text{Density}} = \frac{5 \times 10^{14}\,\text{gm}}{2 \times 10^{14}\,\text{g/cm}^3} = 2.5\,\text{cm}^3.$$

That is, all of humanity would be packed down to about 1/2 teaspoon. Youch!

2. Let us start by assuming that the pulsar is in the plane of the Earth's orbit around the Sun. As the Earth orbits the Sun, its distance to the pulsar changes by 2 astronomical units. The pulses from the pulsar travel at the speed of light. We know that one Astronomical Unit is 8 light minutes, so this delay is 16 minutes, absolutely enormous compared with the size of the delay caused by the pulsar planets. Luckily, we know the orbit of Earth around the Sun *very* well, and so can easily correct for this effect.
Now consider that the line between the pulsar and the Sun is perpendicular to the plane of the pulsar's orbit. In this case, there is absolutely no change in the distance from the Earth to the pulsar. In general, if the line to the pulsar makes an angle i with the plane of the orbit, the difference in arrival times will be 16 minutes $\times \cos i$.
3. (3 points)
(a). The first position can be occupied by 20 amino acids. The second one can be occupied by 20 amino acids, independent of the choice of the first. The third can be can be occupied by 20 amino acids, independent of the choice of the first and second. And so on. In this case, the total number of different amino acids of length x is $20 \times 20 \times 20...$ x times, or 20^x.
(b). For $x = 100$, if I type 20^{100} into my calculator, it can't handle it (the number is too big!). So we have to be cleverer:

$$20^{100} = 2^{100} \times 10^{100}.$$

Now, I happen to know that $2^{10} = 1024$, or roughly 10^3. So:

$$2^100 = \left(2^{10}\right)^{10} \approx \left(10^3\right)^{10} = 10^{30}.$$

Therefore:

$$20^{100} \approx 10^{30} \times 10^{100} = 10^{130}.$$

That is a truly enormous number.
(c). Each codon codes for a single amino acid, so if there are x amino acids, plus a start and stop codon, there are $x + 2$ codons. Each codon is made of three base pairs, so there are $3\,(x + 2)$ base pairs.
(d). There are a number of amino acids that can be generated by more than one codon. So a given protein (string of amino acids) can be generated by more than one nucleic acid string. Therefore there are more nucleic acid strings than unique proteins of a given length.