Solutions to Problem Set #6

Problem 1.

(1a,b,c) The peak wavelength (λ) and temperature (T) of blackbody radiation obey Wien's law, λT =constant, and we may estimate the constant from the information given. Hence

$$\lambda \approx \frac{6000\,\text{K} \times 0.6\,\mu\text{m}}{T} \; .$$

Plugging $T = 300\,\text{K}$, $20\,\text{K}$, and $2.7\,\text{K}$ into this relation, one obtains $\lambda = 12\,\mu\text{m}$, $180\,\mu\text{m}$ ($0.18\,\text{mm}$), and $1300\,\mu\text{m}$ ($1.3\,\text{mm}$), respectively.

(2b-d) The angular resolution of a telescope is related to the diameter of its aperture (D) and the wavelength of observation (λ) by

$$\theta = 1.22\frac{\lambda}{D}\,\text{radians} \;\approx 0.126 \left(\frac{\lambda}{1\,\mu\text{m}}\right)\left(\frac{2\,\text{m}}{D}\right)\,\text{arcsec.}$$

Therefore

T [K]	λ_{peak} [μm]	θ [arcsec]
6000	0.6	0.075
300	12.	1.5
20	180.	23.
2.7	1300.	160.

(3) HST: optical to near infrared: $\lambda \sim 1\,\mu\text{m}$, suitable for stars. SIRTF: $\lambda \approx 3 - 180\,\mu\text{m}$, suitable for $T \approx 20 - 700\,\text{K}$—interstellar molecules and dust, at cold end, to planets and brown dwarfs, at hot end. OVRO: $\lambda \approx 1 - 3\,\text{mm}$: $T \sim 1 - 4\,\text{K}$: suitable for Cosmic Microwave Background (2.7 K) and long-wavelength tail of molecular emission. WMAP: $\lambda \approx 3 - 14\,\text{mm}$: $T \sim 0.3 - 1\,\text{K}$: long-wavelength tail of CMB.

Problem 2

Observational constraints: Radial velocity searches are most sensitive to planets with orbits of small semimajor axis (a), both because the radial velocity scales as $a^{-1/2}$, and because the orbital period scales as $a^{3/2}$. With regard to the latter, good orbital determinations require data spanning at least one full orbit; Jupiter itself ($a = 5.2\text{AU}$, $P = 12\,\text{yr}$) would probably not yet have been detected around stars surveyed for $< 10\,\text{yr}$, and these are the majority. In short, the incidence of "Jupiters" could be substantially more than 10% if most of these planets have $a > 2\text{AU}$.

Most of the stars surveyed are F and G stars; cooler and fainter but intrinsically more numerous K and M stars are underrepresented in the sample and might have a substantially higher or lower abundance of planets than the sample mean. At present, the preliminary indication is that planets are less frequent among the fainter stars. Binary- and higher-multiplicity stars are also underrepresented.

Furthermore, radial-velocity surveys are limited to stars very near the sun (because these are bright enough for exquisite spectroscopy). Most of these stars have metallicity similar to that of the sun. Yet the incidence of planets appears to increase with stellar metallicity. The sun has a fairly typical metallicity for the Galaxy as a whole, but stars in the outer disk and in the Galactic halo are relatively metal-poor, while those near the Galactic center are more metal rich. Thus the average planetary abundance for the Galaxy as a whole could be rather different from what is found in the solar neighborhood.

Planet formation, migration, etc.*:* Migration is not relevant for the present-day incidence of planets, since migration probably occurs only during pre-main-sequence phases, except insofar as *failure* to migrate may leave planets in distant orbits where they are difficult to detect in radial-velocity surveys. However, it is possible (given the current state of understanding) that some inward-migrating planets plunged into their parent stars, so that the incidence of planets *formed* could be higher than the incidence of planets surviving to be observed. It is also possible that some were ejected from their orbits by interactions with other planets or binary companion stars.