Astro 205. Problem set 4. Solutions

1. Consider a planet in a circular orbit around a star of mass 1 $M_\odot$ (2×10^{33} gm). Suppose we observe the system edge-on, and that the accuracy with which we measure the reflex velocity of the star is 3 meters/second, i.e. we cannot detect motions smaller than this. Assume that you can observe for arbitrarily long times so can measure long periods. If a planet of given mass is close to the star, the reflex velocity is larger. For planets further from the star, the reflex velocity is smaller, until for large distances it's too small to detect. If a planet has a small mass, it produces a smaller reflex velocity than does a high-mass planet. Thus for a planet of a particular mass, there's a range of distances from the star over which it can be detected. Draw a graph of planet mass (in Jupiter masses, 0.001 times that of the Sun) versus distance from the star (in A.U.) and shade in the region on this diagram in which you could detect a planet. You'll need a minimum radius too: use the solar radius, 7×10^{10} cm.

If the planet's mass is m_p, its orbital speed V_p (let's assume circular orbits for clarity) and the star's mass is $M_\star$, then

$$M_\star V_\star = m_p V_p \tag{1}$$

by conservation of momentum, where $V_\star$ is the reflex velocity of the star.

Now the planet's speed around the star is given by

$$\frac{m_p V_p^2}{r_p} = \frac{G m_p M_\star}{r_p^2} \tag{2}$$

where G is the constant of gravity and r_p is the distance between the planet and the star. Equation (2) gives

$$V_p = (GM_\star/r_p)^{\frac{1}{2}}$$

Substituting this into Equation (1) gives

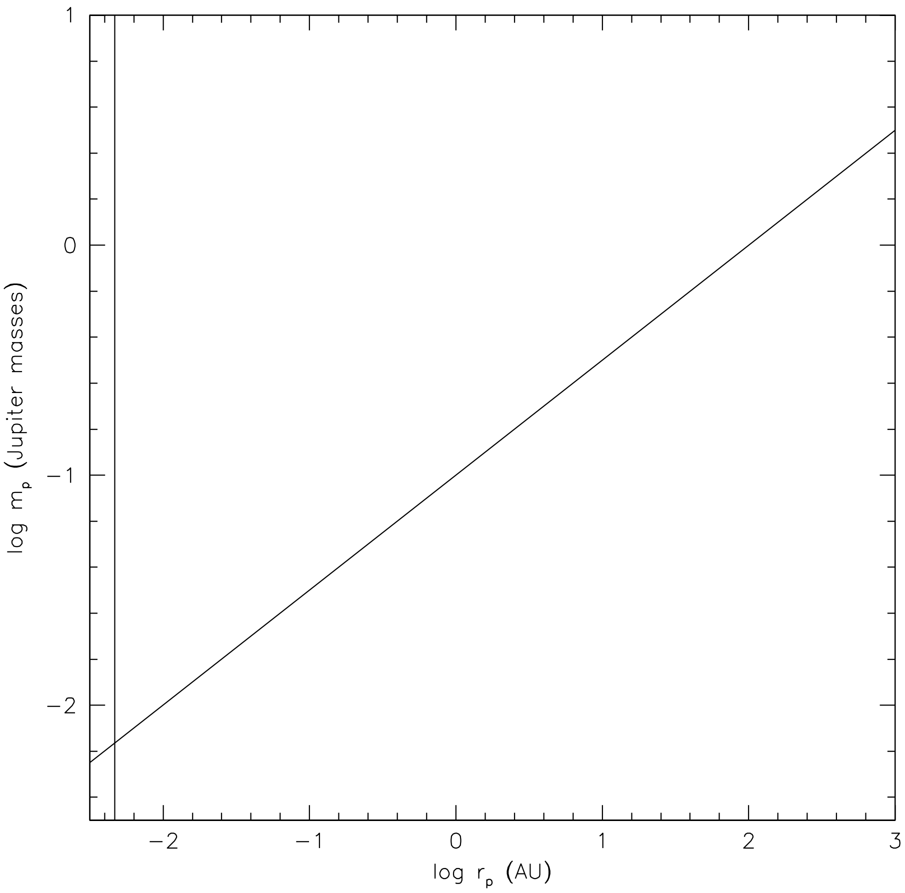
$$M_\star V_\star = m_p \left(\frac{GM_\star}{r_p}\right)^{\frac{1}{2}}$$

Now it's time to put numbers in. Let's let $V_\star = 3$ meters/sec = 300 centimeters/second - this is the smallest velocity we can measure. $M_\star = 2\times10^{33}$ gm, put the planet mass in Jupiter masses (2×10^{30} gm or $0.001M_\odot$, and r_p in AU (1.5×10^{13} cm). Then

$$m_p = 0.1 r_p^{\frac{1}{2}}$$

This is an equation giving you the minimum mass of a planet which can be detected as a function of distance from the star. So if $r_p = 1$ AU, $m_p = 0.1$ M(Jupiter); if $r_p = 10$ AU, $m_p = 1$ M(Jupiter), and so on.

Now the *closest* you can get to the star is its photosphere, otherwise you're inside it. What is the minimum mass a detectable planet could have at this distance? 1.4×10^{28} gm, which is larger than the Earth's mass (6×10^{27} gm. Therefore, the present accuracy is not sufficient to detect Earth-mass planets orbiting solar-mass stars. Here's the graph. The region in which you can detect planets is above and to the left of the diagonal line and to the right of the vertical line.



2. Suppose a planetary system has two planets of the same mass in circular orbits orbiting it in the same plane at the same distance, with the planets on opposite sides of the star (this configuration is a favorite of science fiction stories along the "shadow Earth" lines, in which the Earth has an unseen companion sharing our orbit). Would you be able to detect these planets by radial velocity wobble?

The position of the center of mass of this system is

$$\mathrm{r} \quad = \quad \frac{\mathrm{m_1 r_1 + m_2 r_2 + m_3 r_3}}{\mathrm{m_1 + m_2 + m_3}}$$

Let $\mathrm{r_2}$ be the position of the star. Then $\mathrm{r_1 = r_2 + x}$ and $\mathrm{r_3 = r_2 - x}$ where x is the radius of the planets' orbits. Since $m_1 = m_3$,

$$\begin{aligned} r &= \frac{m_1(r_2+x) + m_2 r_2 + m_3(r_2 - x)}{m_1+m_2+m_3} \\ &= \frac{m_1 r_2 + m_2 r_2 + m_3 r_2}{m_1+m_2+m_3} \\ &= r_2 \end{aligned}$$

Since the center of mass is located at the same place as the center of the star, the star doesn't move as the planets orbit, and so these planets can be detected neither by Doppler (radial velocity) wobble nor by astrometric (position) wobble.