Astro 205. Problem set 3. Solutions

1. Compute the Sun's orbital speed about the center of mass of the solar system (just consider the effect of Jupiter). Does Jupiter really produce the largest wobble? Check by calculating the effect for a couple of other planets, assuming a system of one planet only each time.

By conservation of momentum,

$$m_p v_p = M_\odot V_\odot$$

The speed of the planets in their orbits is given by

$$\frac{V_p^2}{r} = \frac{GM_\odot}{r_p^2}$$

where V_p is planet's velocity, r_p is the distance between the planet and the Sun, and G is the constant of gravity. So

$$V_p^2 = \frac{GM_\odot}{r_p}$$

and hence

$$V_\odot = \frac{m_p}{M_\odot} V_p$$

$$= \frac{m_p}{M_\odot} \left(\frac{GM_\odot}{r_p} \right)^{1/2}$$

Put the numbers in. Use Jupiter masses (M_J) and AU. $M_J/M_\odot = 10^{-3}$. Then

$$V_\odot = 10^{-3} m_p \sqrt{\frac{6.7 \times 10^{-8} \times 2 \times 10^{33}}{1.5 \times 10^{13} r_p(\mathrm{AU})}}$$

$$= \frac{30\mathrm{meters/sec}\ m_p(\mathrm{in}\ M_J)}{(r_p(\mathrm{AU}))^{\frac{1}{2}}}$$

For Jupiter, r = 5.2 AU so $V_\odot$ = 13 meters/sec. This method favors high mass planets, close in. Saturn: M = 0.3 M(Jupiter), distance = 9.54 AU, so $V_\odot$ = 2.9 meters/sec, just on the level of detectability, but less that that of Jupiter. What about Earth? (the most massive inner planet): $V_\odot$ = 0.09 m/sec, far below the level of detectability. So yes, Jupiter provides by far the largest radial velocity wobble.

2. Remember that at the atomic level, motion is heat. Suppose there is a star of temperature 6,000 K (this is the photospheric temperature). What is the average speed of the atoms in the photosphere? Remember that E = kT per atom, and that kinetic energy

$= 3/2$ m V^2. k is Boltzmann's constant, 1.4×10^{-16} erg/K, and m is the mass of the hydrogen atom, 1.7×10^{-24} gm. Compare this speed with the Sun's orbital speed from Problem 1. Comments?

I fear I gave slightly wrong formulae here, but let's proceed. The kinetic energy of atoms in a gas is proportional to their temperature:

$$E = kT = 3/2mV^2$$

k = Boltzmann constant, T = 6000 K, m = mass of hydrogen atom. So

$$V = \left(\frac{2 \times 1.4 \times 10^{-16} \times 6000}{3 \times 1.7 \times 10^{-24}}\right)^{\frac{1}{2}} \quad \text{cm/sec}$$

or 5750 meters/second. This is the characteristic velocity width of the spectral line. Jupiter's motion moves this line back and forth by 13 meters/sec, or 0.002 of its width! A hard measurement to make.

3. The police are measuring your speed on the NJ Turnpike by radar. If the radar wavelength is 10.000000 cm, what's the largest wavelength shift that they'll observe if you're not speeding? (I think the speed limit on the turnpike is 65 m.p.h.).

OK, the Doppler shift gives

$$\frac{\lambda - \lambda_o}{\lambda_o} = \pm\frac{2V(car)}{c}$$

where λ is the wavelength of the radar bounced back from your car, V(car) is the speed of your car, λ_o is the wavelength at which the wave is emitted (10 cm), and c is the speed of light. The wavelength is longer if the car is receding from the radar, shorter if the car is approaching. Why the factor of 2? Well, the car is receding from the radar, so sees a longer wavelength. This wave is reflected back to the radar detector, which sees a longer wavelength because the object reflecting the wave is moving (sam argument if the car is approaching). This is the largest possible wavelength shift because remember that the Doppler effect measures the line of sight velocity only. Now you have to convert miles per hour to km/hour (darn those Imperial units anyway) - there are 1.6 km in a mile, so at the speed limit your car is travelling 104 km/hour or 3000 cm/sec. 2V/c is then 1.9×10^{-7}, so the largest wavelength shift is 1.9×10^{-6} cm, plenty large enough to detect.

4. Consider two objects, both at the same temperature, one a brown dwarf of mass 40 M(Jupiter) and the other a planet of mass 2 M(Jupiter). Assume each radiates like a perfect black body and that there are no sources of outside heat. Which will cool the fastest?

The common-sense answer to this one is correct (although the cooling is much more complicated for real objects than we're saying here). The luminosity of a black body is

$$L = 4\pi R^2 \sigma T^4$$

The luminosity, remember, is the rate of energy loss (in ergs/sec in cgs units). Since both objects have the same radius, they will radiate the same amount of energy per unit time if they are at the same temperature. However, the more massive body has more energy to lose, so will take longer to cool. The cooling time is basically proportional to mass.