Astro 205. Problem set 2. Solutions

1. Consider three main sequence stars: (1) the Sun (mass 2×10^{33} gm, luminosity 4×10^{33} ergs/sec). (2) a high mass star, $M = 60\ M_\odot$, $L = 10^6\ L_\odot$, and (3) a low mass star, $M = 0.1\ M_\odot$, $L = 10^{-3}\ L_\odot$. If all are initially composed of pure hydrogen and "convert mass to energy" via $E = 0.007\ m\ c^2$, what is the maximum main sequence lifetime of each star in years? ($c = 3 \times 10^{10}$ cm/sec, 1 year $= 3.2 \times 10^7$ seconds).

Solution

The total amount of energy available for the star is

$$E = 0.007 N m_H c^2$$

Here, N is the initial number of H atoms present in the star, m_H is the mass of a hydrogen nucleus (1.7×10^{-24} gm, although you don't need it) and c is the speed of light (3×10^{10} cm/sec). If you use these units, E is in ergs. Of course, if the star is pure hydrogen to start with, then its mass is $M_\star = N m_H$, and

$$E = 0.007 M_\star c^2$$

The luminosity $L_\star$ is the rate at which it emits energy. If the star shines steadily, then the total time for which it can shine is

$$T = \frac{E}{L} \text{ seconds} = \frac{0.007 M_\star c^2}{3.2 \times 10^7 L_\star} \text{ years}$$

So if we express luminosity $L_\star$ in $L_\odot$ (4×10^{33}ergs/sec and mass $M_\star$ in $L_\odot$ (4×10^{33} erg/sec) then the time is

$$\begin{aligned} T &= 0.007 M_\star \times 2 \times 10^{33} \times (3 \times 10 \frac{1])^2}{L_\star \times 4 \times 10^{33}} \times 3.2 \times 10^7 \\ &= \frac{M_\star}{L_\star} \times 10^{11} \text{ years} \end{aligned}$$

Then star (a) ($M_{star} = 1\ M_\odot$, $L_\star = 1\ L_\odot$) shines for at most 10^{11} years; star (b) ($M_\star = 60\ M_\odot$, $L_\star = 10^6\ L_\odot$) shines for at most 6×10^6 (6 million) years, while star (c) ($M_\star = 0.1\ M_\odot$, $L = 10^{-3} L_\odot$) shines for at most 10^{13} years.

Notice that the estimated lifetime for the Sun is about 10 times its actual main sequence lifetime. This is because the Sun's energy is produced in its core and radiated to the outside, so that only the hydrogen in the core (about 10% of the total mass) is burned on the main sequence. Low mass stars, on the other hand, transfer their energy to the outside mostly by convection, and so mix new hydrogen into the core. These low-mass stars therefore last a very long time indeed.

2. *Black Bodies* radiate at all wavelengths, but the distribution of the radiation with wavelength depends on the temperature. Roughly

$$\lambda_{\rm max} \rm T \;=\; 0.3$$

where $\lambda_{\rm max}$ is the wavelength at which the body is brightest (and is measured in cm) and T is the temperature in K. Calculate $\lambda_{\rm max}$ for T = 6000 K (the Sun); T = 300 K (the Earth): and T = 50 K (a large dust disk around a star). In what part of the electromagnetic spectrum does the peak of the radiation fall? (to help: 1 Angstrom (Å) = 10^{-8} cm; 1 micron = 10^{-4} cm). You'll have to look up these wavelengths in some diagram of the electromagnetic spectrum - one is accessible from the course web page). Which of these wavelengths is observable from the ground?

Solution

$$\lambda_{\rm max} \;=\; 0.3 \text{ cm K}$$

(a) T = 6000 K, λ_{max} = 5000 Å. This is in the middle of the visible light band, and is detectable from the ground.

(b) T = 300 K, λ_{max} = 10 microns, in the infrared, and visible from the ground only with difficulty. Work at these wavelengths is done from space, as in NASA's newly-launched Great Observatory, SIRTF.

(c) T = 50 K, λ_{max} = 60 microns, far-infrared, not detectable from the ground.

3. How many planets have been discovered outside the Solar System as of today's date? (see Geoff Marcy's web page, accessible from the course web page. "today" is the day you do this homework.)

"Solution" 110 planets as of Sept 30 2003

4. (no answer required) Try out the binary star orbit simulator on the web page.