Midterm Exam Solutions, AST 205, Fall 2003

Wednesday, October 22, 3:00-4:20 PM

1. True or False (10 points)

a. False (elliptical orbits are also possible.)

b. True.

c. True.

d. False (the luminosity of the Sun is close to constant).

e. False (none of the planets have been seen directly; we see the reflex motion of their parent star via the Doppler shift).

2. Essay (20 points).
Among the methods to be mentioned are:

- Periodic Radial velocities of parent stars. 110 planets have been discovered thus far. Disadvantages include no sensitivity to Earth-like planets, poor sensitivity (or very long observing times) for very distant planets, mass ambiguity of sin i, poor sensitivity (at present) for faint or cool stars.
- Periodic Astrometric wiggles. Has been seen definitively in no planets to date. Disadvantages include decreasing sensitivity with distance from us, poor sensitivity to low-mass or close-in planets; long observing times for very distant planets; technically extremely difficult.
- Occultations/eclipses. Has confirmed one radial-velocity planet, and has discovered one other. Disadvantages include poor sensitivity to distant planets, requires almost perfect alignment, and so only a very small fraction of stars with planets will show eclipses, requires very high precision photometry of a very large number of stars.
- Microlensing. No microlensed planets have yet been discovered. Disadvantages are that the microlens planet event is very brief, the stars in question tend to be very distant, and it is impossible to get a posteriori confirmation of a planet.
- Direct imaging. No such planets have yet been found. Principal difficulty is that it is *extremely* difficult because of the more than 10^8 difference in brightness between the star and the planet.

3. The lifetime of a star is its mass divided by the rate at which it uses up fuel. The rate at which it uses up fuel is proportional to its luminosity. The luminosity of each is given by the blackbody formula. Thus, the lifetime t is given by:

$$t \propto \frac{M}{4\pi R^2 T^4}$$

We don't need to worry about the constant of proportionality, or the specifics of the amount of mass that goes into energy (i.e., that pesky factor of 0.007), as all we need is the ratio. In particular:

$$\frac{t_A}{t_B} = \frac{M_A}{M_B}\left(\frac{T_B}{T_A}\right)^4\left(\frac{R_B}{R_A}\right)^2$$

We are told the values of all the ratios on the right hand side. Plugging them all in gives:

$$\frac{t_A}{t_B} = 2 \times \left(\frac{1}{2}\right)^4 \times \left(\frac{1}{1.1}\right)^2 = 1/8/1.2 = 0.1.$$

The lifetime of star A is 10% that of star B.

4. (50 points)
a. (10 points) The transiting object has the same size as Jupiter, and the parent star has the same size as the Sun, ten times the radius. Thus the shadow of the object has an area $(0.1)^2 = 0.01 = 1\%$ the area of the disk of the star. Thus 1% of the light of the star is blocked.
b. (15 points) Kepler's third law tells us a relationship between the period and radius of the orbit. In particular, for periods P around stars of one solar mass measured in years, and radii r measured in AU, $P^2 = r^3$. Here we are told $P = 3$ days, or 1/120 years. Thus $r = 120^{-2/3}$ AU $= 0.04$ AU $=$ 6,000,000 km $= 6 \times 10^{11}$ cm.
c. (15 points) The transit lasts for the length of time it takes for the planet to traverse the diameter of the star. We know the star is the same size of the Sun, and so has a diameter of 1.4×10^{11} cm. What speed is the planet travelling? We know from the previous problem that it is in an orbit of radius 6,000,000 km $= 6 \times 10^{11}$ cm. Thus its speed is

$$v = \frac{2\pi r}{P} \approx \frac{6 \times 6 \times 10^{11}\,\text{cm}}{3\,\text{days} \times 90,000\,\text{sec/day}} = \frac{3.6 \times 10^{12}}{2.7 \times 10^5}\,\text{cm/sec} = 1.3 \times 10^7\,\text{cm/sec}$$

Thus the transit time is the distance divided by that speed, or:

$$t = \frac{1.4 \times 10^{11}\,\text{cm}}{1.3 \times 10^7\,\text{cm/sec}} = 10^4\,\text{sec},$$

or roughly 3 hours.
d. (10 points). The observation of the fraction of light covered by the object (part a) determines the radius of the object relative to the star. This measurement gives us some information about the mass: since we know that all massive planets and brown dwarfs have roughly the diameter of Jupiter, we know that the mass is somewhere between 0.3 and 80 Jupiter masses. This is a large range, however, and can't distinguish between a planet and a brown dwarf. To really determine the mass, we would need a radial velocity measurement (the measurement of an astrometric wobble, together with the distance to the star, will do the trick as well, but at present we don't have the technical capability). Note that because the orbit is exactly edge-on, there is no ambiguity about sin i.