Astro 205. Lecture 6, October 1, 2003
Spectroscopy: theory, telescopes, spectrographs

Review, and some considerations for the detectability of planets

In the last five lectures, we rushed through two important pieces of physics:

1. Stellar Radiation

Stars shine by producing energy in their interiors by the fusion of lighter elements to heavier elements. For most of their life, stars shine by fusing hydrogen to helium. The rate at which the stars produce energy in their interiors is balanced by the rate at which they radiate it from their "surface" - the photosphere. The rate at which a star produces energy is regulated by gravity: the star must produce enough energy to keep its interior hot enough to produce enough pressure to withstand the inward force of gravity. In this way, the star's energy output can remain essentially constant for billions of years. Stars fusing hydrogen to helium at a steady rate are called main sequence stars. Most of the mass in a new-formed star is hydrogen, so it has large reservoirs of "fuel".

The lowest possible mass star has a mass of about $0.08\ \mathrm{M}_\odot$ or 80 M(Jupiter). Objects of lower mass than this are sufficiently supported by electron degeneracy pressure that they do not become hot enough in their interior to undergo nuclear fusion. These objects are called brown dwarfs.

There's a very interesting point about the radii of bodies of mass between the highest-mass brown dwarf (about 80 Jupiter masses) and about 0.3 times the mass of Jupiter. Gravitationally bound objects across this entire mass range are degenerate in the interior. Now if a self-gravitating, non-degenerate body has constant density ρ,

$$\mathrm{M} = \frac{4}{3}\pi \mathrm{R}^3\rho$$

(remember that self-gravitating bodies take up a spherical configuration), then

$$\mathrm{R} \propto \mathrm{M}^{\frac{1}{3}}$$

and as the mass grows, so does the radius.

But for degenerate matter, as we saw last time, the larger the mass the *smaller* the radius:

$$\mathrm{R} \propto \mathrm{M}^{-\frac{1}{3}}$$

Thus for objects in the range about 80 M(Jupiter) to 0.3 M(Jupiter) (the limit for normal matter), $\mathrm{R} \propto \mathrm{M}^{-\frac{1}{3}}$ competes with $\mathrm{R} \propto \mathrm{M}^{\frac{1}{3}}$ and thus over this whole mass *the radius is essentially constant and we know its value* - about the same as Jupiter's radius, 0.1 of the radius of the Sun, or about 7×10^9 cm.

Planets orbit a star and shine by reflected light. Why can't we just see them, at least for nearby stars? Let's estimate what fraction of the starlight is reflected by the planet.

Let's imagine a system in which a planet of 2 Jupiter masses orbits a star just like the Sun, mass 1 $M_\odot$ at a distance of 1 A.U. (1.5×10^{13} cm). We know that such a planet will have about the same radius as does Jupiter (see above). Let's say it also has the same albedo as Jupiter (a = 0.7 - remember that the albedo is the fraction of incident light which is reflected). Now let's imagine that we view the system from a distance like that to the nearest stars, a few light years, at a time when the planet is at its maximum elongation from the star (i.e. its projected distance on the sky is a maximum). In this case, we see half of its illuminated face. The total amount of reflected light we see is then

$$B = \frac{0.5\pi R^2 a}{4\pi D^2} L_\star$$

where $L_\star$ is the luminosity of the star. Thus the ratio of the planet's brightness to that of the star is

$$f = \frac{0.5\pi R^2 a}{4\pi D^2}$$

$$f = 2 \times 10^{-8}$$

a huge contrast. The planet and the star will be very close together in the sky, and for nearby stars you could perhaps distinguish a brightness ratio of 100:1 under very good observing conditions. Thus the planet is lost in the glare of the star and it is impossible to see them directly at present. Later in the semester, we will discuss techniques which have some promise for actually seeing planets around other stars, but that technology is not in operation at present.

The other vital thing we have learned about is:

2. Gravity

The solar system consists of a star (the Sun), nine planets, and a lot of debris (comets, meteoroids, asteroids, the Kuiper belt objects, interplanetary dust). The Sun contains most of the mass of the Solar System, and the planets orbit the Sun. Bound orbits are ellipses with the Sun at one focus; a circular orbit is a special case, in which both axes of the ellipse are equal, the Sun is in the center, and the orbital eccentricity is e = 0. The orbits of the Solar System planets are close to circular, and we can approximate them by concentric circles. The planets are bound to the Sun by gravity, and orbit the Sun at speed V such that

$$\frac{mV^2}{r} = \frac{GmM}{r^2} \tag{1}$$

where m is the mass of the planet, M the mass of the Sun, and r is the distance between the Sun and the planet (the radius of the circular orbit).

Equation (1) gives you a method of determining the mass of the Sun. You can measure the orbital speed V:

$$V = \frac{2\pi r}{P} \tag{2}$$

where P is the period of revolution of the planet, i.e. how long it takes the planet to complete one orbit. So:

$$M = \frac{rV^2}{G} = \frac{4\pi^2 r^3}{P^2 G} \tag{3}$$

gives you the mass of the Sun.

There's a second approximation assumed in the above discussion: that the planets orbit the Sun. Since the planets have mass, what's really going on is that all the objects in the Solar System, including the Sun, orbit the *center of mass* of the Solar System. (The question came up in precept as to whether the center of mass moves. The answer is that in an isolated system, the center of mass is not being acted on by any forces so it continues in a state of rest or of uniform motion in a straight line. The planets move with respect to the center of mass, but it does not move).

For a system consisting of two masses, e.g. one star and one planet, the center of mass lies at

$$r = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2} \tag{4}$$

If we take $r_1 = 0$, then

$$r = \frac{m_2 r_2}{m_1 + m_2}$$

and if $r_2 = 1$ A.U. $= 1.5 \times 10^{13}$ cm, $m_2 = 2 \times 10^{-3} M_\odot$ and $m_1 = M_\odot$ (our model star planetary system defined above), then $r = 3 \times 10^{10}$ cm. and the star's position wobbles back and forth by $\pm 3 \times 10^{10}$ cm over a period of 1 year. But what about the velocity, i.e. the speed with which the star wobbles back and forth? We can calculate this by considering conservation of momentum: the planet and the star are moving in opposite directions with the same momentum, i.e.

$$M_\star V_\star = M_p V_p$$

The orbital speed round a 1 solar mass star at a distance of 1 A.U. is about 30 km/sec (the Earth's orbital speed), so

$$V_\star = \frac{M_p}{M_\star} V_p$$

$= 2 \times 10^{-3} \times 30 = 0.06$ km/sec, 60 meters/sec (about 140 miles per hour). Thus if you see the star moving back and forth, you can find out:

that there's a planet in the system

its period

its mass (since we have some idea of the star's mass from the relationship between brightness and luminosity on the main sequence)

its distance from the star (by applying Kepler's laws)

the orbital eccentricity

A lot of information! There is a hitch, as we shall see shortly, but first let's consider how you might measure velocity $V_\star$.

The velocity of any object towards or away from you can be measured by the *Doppler shift*. This is a shift in the wavelength (and frequency) of light (or more generally electromagnetic radiation) due to the relative motion of the source of light and the observer.

Suppose you observe a star which is at rest with respect to you and is emitting light at a single wavelength λ. Then the wavelength at which you observe this light is

$$\lambda_{\mathrm{obs}} = \lambda$$

But now imagine that the light is moving away from you at velocity V. What happens to the speed of the light and the wavelength of the light as seen by the observer?

First, the speed. By analogy with material objects, you'd expect the velocity of the light to be changed by the velocity of the emitting object. For example, if I stand at the back of a train going away from you at 80 mph and throw a ball towards you at 20 mph, the ball's speed with respect to you, or to the ground, will be 60 mph. *Light doesn't do this.* Its velocity stays at $c = 3 \times 10^5$ km/sec, regardless of the relative motion of the emitter and the observer. Why? Because light is a wave phenomenon whose speed is determined by the properties of the medium through which it propagates. Think of water waves, for example. The speed at which a ripple spreads over the surface of a pond *is determined by the elasticity, density etc.* of the water through which the ripple is moving.

So what happens? Does the relative velocity of the emitter and the observer do anything? Yes, it does. Think of the water ripples again. Drop a stone in the pond, and the concentric ripples spread out across the water. Move through the water (like a boat), though, and the ripples get closer together ahead of the boat and further apart behind it. This happens because the boat is moving and because the speed at which the ripples spread across the water is unchanged - so the wave crests get "pushed" closer together ahead of the boat and spread out further behind. Thus the *wavelength of the ripples is changed by the motion of the boat.*

So suppose you emit a ripple of light at time $t = 0$, and are moving away from me at velocity V. By the time you emit the next ripple, at $t=\lambda/c$, you have moved a distance Vt away from me. So the distance between one ripple and the next is

$$\lambda_{obs} = \lambda + V\frac{\lambda}{c}$$

i.e.

$$\lambda_{\rm obs} \;=\; \lambda\left(1 \;+\; \frac{\rm V}{\rm c}\right)$$

If V is positive, i.e. the object is moving away from you, the wavelength is increased, while iv V is negative, i.e. the object is moving towards you, the wavelength is decreased.

And the larger is V, the greater is the shift.

Wonderful! You can measure the speed of any object in the Universe towards or away from you by measuring the wavelengths of its light. Note the limitation, though: you can only measure motion towards or away from you, i.e. the component along your line of sight. If the motion of the object is at an angle i to your line of sight, you measure $\rm V_z$ = V cosine (i).

So all we need is light (or more generally electromagnetic radiation) whose intrinsic wavelength we know. This is provided for us by absorption and emission by atoms and molecules.

Atomic and Molecular Spectroscopy

The "classical" picture of atomic structure, which will do just fine for present purposes, is a positivel-charged nucleus surrounded by a cloud of negatively charged electrons. The atoms can be *neutral*, i.e. number of electrons = number of protons, or ionized, i.e. one or more electrons is removed. The electrons orbit the nucleus, but their energies are *quantized*. Approximately what this means is that the electrons can only occupy certain orbits, those in which the number of electron wavelengths in an orbit is a whole number. Here, "electron wavelength" is the wavelength describing the electron, and depends on the electron's momentum. Thus the electrons occupy a series of *energy levels* which is unique to each atom or ion.

Electrons can lose or gain energy by moving between energy levels. Since the energy levels are not continuous, the energy loss or gain is at certain fixed energies. The way electrons gain or lose energy is to absorb or emit *electromagnetic radiation*. This happens for each atom or ion at a fixed set of energies/wavelengths/frequencies. Thus you can tell that a given atom is present because of the emission or absorption of particular wavelengths of light. And you can tell how much there is of a given atom by how strong the emission or absorption is. And you can tell how fast the emitting object is moving towards or away from you by the observed wavelengths.

The emission and absorption features from an atom are called *spectral lines*. Each atom, ion or molecule has a unique set of spectral lines.

Emission line spectra are produced by hot gas. In the atoms of this gas, the electrons are raised to high energy levels because the gas is hot, and spontaneously drop to lower energy levels, emitting a photon at the exact wavelength corresponding to that energy difference:

$$h\nu \;=\; E_1 - E_2$$

where ν, the frequency, $= c/\lambda$.

Absorption line spectra are produced when a cold gas is in front of a hot object. If this object is a black body, or something like a black body, it emits a continuum spectrum, i.e. radiation at all wavelengths (by contrast with the spectrum of an atom, which is a line spectrum). If this radiation passes through cold gas, light at the particular wavelengths corresponding to that cold gas is absorbed.

Stars have absorption line spectra. Since they are producing energy in their interiors, the star gets colder towards its outside, giving the situation of cold gas in front of a hot body. Hence the absorption line spectrum.

The stellar spectrum tells you a whole lot. What lines are present? How strong are they? This tells you what the star is made of, and gives much information about the temperature. This is because at high temperatures, some atoms are ionized, so their spectral lines don't appear. Take the simplest case, hydrogen. At temperatures over about 20,000 K, all the hydrogen is ionized, so you see no absorption lines from hdrogen in the spectrum. At lower temperatures, the hydrogen is atomic, so you see hydrogen absorption lines. At still lower temperatures (less than about 3500 K) all the hydrogen is molecular, so you don't see hydrogen lines any more.

The spectra of stars were classified long ago, by a letter (gross type) and number (fine type) system. The letters are, in decreasing order of temperature, OBAFGKMLT, and within each spectral type there's a further subdivision of 0-9. The Sun's spectral type is G2.

Spectra are measured by spectrographs. These instruments disperse the light passing through them, ordering the light by wavelength. Light passing through a prism, for example, may enter as a parallel beam but is spread out by wavelength as it leaves the prism, because the speed of light in glass or whatever the prism is made of is wavelength dependent. Spectra of stars tell us what spectral type (mass, lifetime, temperature) a star is: what its composition is: and how fast it is moving towards or away from us. We need a lot of light to analyze: hence the use of large telescopes, which collect light from astronomical objects in proportion to their area. So stellar spectra are measured by spectrographs fed by a beam of light from a large telescope.

Next time, we'll look at the challenges of detecting the motions of stars due to their orbiting planets, and at the properties of those planetary systems.