Astro 205. Lecture 5, September 29, 2003
The Temperatures of Planets

We want to examine three questions today: the temperatures of planets due to the radiation from their central star (which will also include a discussion of planet orbits and of the lifetimes of stars): the heating of planets by gravitational contraction: and the mass of the lowest mass star possible (which will also include a physical definition of "planet").

Insolation

As an example, let's discuss the temperatures of planets in the inner solar system. The Sun *steadily* produces 4×10^{33} erg s^{-1} of electromagnetic radiation, and the planetary orbits are close to circular, so the mean amount of radiation falling on the planets does not change very much.

We'll now estimate the mean temperature of a planet. The *flux* F at distance D from the Sun is

$$\mathrm{F} \;=\; \frac{\mathrm{L}_\odot}{4\pi \mathrm{D}^2} \quad \mathrm{erg\ cm^{-2}s^{-1}}$$

where $\mathrm{L}_\odot$ is the solar luminosity. The *area* of a planet which faces the Sun is its projected area $\pi \mathrm{R}_\mathrm{p}^2$, where R_p is the radius of the planet. The total energy absorbed by the planet per second is then

$$E \;=\; \pi R_p^2 F_\odot (1-a) \;=\; \frac{\pi R_p^2 L_\odot (1-a)}{4\pi D^2}$$

where a is the *albedo*, the fraction of light falling on a body which is reflected. The Earth's albedo is ~ 0.3, that of the Moon ~ 0.08.

This energy falls on the planet and warms it up. All warm bodies radiate: the radiation per unit time is $4\pi \mathrm{R}_\mathrm{p}^2 \sigma \mathrm{T}_\mathrm{p}^4 \epsilon$, where ϵ is the efficiency ans σ is the Stefan-Boltzmann constant, 5.7×10^{-5} erg/cm^2/s/K^4. The planet is not a perfect black body, so $\epsilon < 1$. In equilibrium

$$4\pi \mathrm{R}_\mathrm{p}^2 \sigma \mathrm{T}_\mathrm{p}^4 \epsilon \;=\; \frac{\pi \mathrm{R}_\mathrm{p}^2 (1-\mathrm{a}) \mathrm{L}_\odot}{4\pi \mathrm{D}^2}$$

(note that R_p cancels out. In other words, this equation describes the temperature euilibrium of *any* planet at the Earth's distance from the Sun if solar radiation is the only source of heat). So

$$\mathrm{T}_\mathrm{p} \;=\; \left(\frac{(1-\mathrm{a})\mathrm{L}_\odot}{16\pi \mathrm{D}^2 \sigma \epsilon} \right)^{0.25}$$

This is an approximate derivation, and is more or less OK for the subsolar region of the planet. This equation gives the planet's *equilibrium temperature*. Let's estimate this for Earth. First, assume a = 0 and $\epsilon = 1$; we get $\mathrm{T}_\mathrm{p} = 280$ K, just above the freezing point of water (273 K). The Earth thus lies at a distance from the Sun at which water is in liquid form, and as we know water is essential to life.

What is the range of distances from the Sun at which water is in liquid form? The temperature at which water freezes is 273 K, and the temperature at which it boils is 373 K. Let's rewrite the above equation to find distance D corresponding to these temperatures (we'll take a = 0 and $\epsilon = 1$):

$$\mathrm{D} \;=\; \left(\frac{\mathrm{L}_\odot}{16\pi \sigma \mathrm{T}_\mathrm{p}^4} \right)^{\frac{1}{2}}$$

setting $\mathrm{T}_\mathrm{p} = 273$ K gives D = 1.6×10^{13} cm (about 1.1 A.U.) and setting $\mathrm{T}_\mathrm{p} = 373$ K gives 8.5 $\times 10^{12}$ cm (about 0.6 A.U.) You could call this the *comfort zone* for the Sun. Two planets, Earth and Venus, lie within this zone.

The distance from the Sun is not the only quantity affecting the planet's temperature. The Earth's *seasons* are due to the tip of 23° of its axis of rotation with respect to its path around the Sun - in the northern hemisphere summer the northern hemisphere receives much more radiation than does the southern hemisphere, and vice versa in the northern hemisphere winter. Note that the seasons don't affect the Earth's mean temperature. The slight eccentricity of the Earth's orbit is too small to have much effect on the Earth's temperature. A most important component of a planet which affects its temperature is its *atmosphere*.

The Earth has a fairly high albedo, so should be colder than the above estimate, because it reflects a significant fraction of the solar radiation; but in fact it's quite a bit warmer - why? The answer is the *greenhouse effect*, so called after the glass houses, or greenhouses, in which fruit, vegetables and flowers can be grown in northern climates. If you build an enclosure from glass, sunlight can pass through the glass and heat the ground inside. The warm ground radiates, but since its temperature is about 300 K, its peak radiation is at wavelengths around 10^{-3} cm, about 10 microns, i.e. in the infrared. Glass is opaque at infrared wavelengths, so the radiation is trapped and can't escape (there's a strong vibrational resonance in the SiO bond at 9.7μm). Various atmospheric gases, in particular water vapor, carbon monoxide, and methane, have this same effect (it is rarely bitter cold on a cloudy day or night, and deserts, where there are no clouds, can be scorching hot in the daytime and very cold at night). This *greenhouse effect* raises the Earth's temperature well above the freezing point of water.

Venus is closer to the Sun than is the Earth, so should be somewhat warmer. However, it is a lot warmer - its surface temperature is 740 K! The atmospheric composition is also quite different: 96.4% CO_2 and 3.4% N_2. A hypothesis for what happened to Venus is that it started out with water in liquid form, but as the Sun's luminosity slowly increased (see below), Venus' oceans began to boil, filling the atmosphere with water vapor, a greenhouse gas. This made the planet hotter yet, causing a *runaway greenhouse effect*. Most of the CO_2 which would have been in the Earth's atmosphere is bound up in rocks (carbonates such as limestone) but at high temperatures the gas would be released from the rocks. Venus has thus come to equilibrium at a temperature far too high to support life.

The presence of an atmosphere is essential to maintaining a relatively constant temperature on a planet, but the atmosphere should be neither too opaque nor too transparent, but just right.

This should serve for us as a warning. The Earth is in balance, but if that balance were tipped, we could settle into a new balance which is hostile to life. If the temperature were to decrease, water would freeze, giving the Earth simultaneously a high albedo and a low greenhouse effect, which would keep the planet frozen. If the temperature rises, more H_2O and CO_2 are released into the atmosphere, and these greenhouse gases increase the temperature further, giving a runaway greenhouse effect. The actions of the human species (burning fossil fuel, clearcutting forests) could lead to global warming and a runaway greenhouse effect.

Mars is outside the comfort zone, at least as estimated above: but if it were a more massive planet, with a denser atmosphere, it might have been able to maintain a strong greenhouse effect and have liquid water.

Has the Sun's radiation been constant throughout its life? We know that it must have been closely constant, because sedimentary rocks, produced by liquid water, have been present basically throughout the Earth's 3.5 billion year history. The Sun's radiation was not constant at the beginning of its life, when it formed, but over its 4.5 billion year lifetime, the Sun's luminosity has not quite remained constant but has slowly increased due to the conversion of hydrogen to helium in the Sun's core and a corresponding increase in the mean mass per particle. The gas pressure is

$$\mathrm{P_g} = \frac{\rho \mathrm{kT}}{\mu \mathrm{m_H}}$$

where μ is the mean molecular weight. For ionized hydrogen this is 0.5, while for ionized helium it is 4/3. Thus the nuclear reaction rate has to go up slowly to produce enough gas pressure to balance the gravitational pressure, and the star's luminosity slowly grows. The Sun's luminosity has increased by about 30% during its lifetime because of this effect. There is a corresponding slow growth in the radius. This happens for all main sequence stars.

The Main Sequence and Stellar Lifetimes

The energy source for stars is the nuclear fusion of hydrogen to helium in their interiors, known as the stellar *core*. Fusion of lighter to heavier elements releases energy, but the first fusion, hydrogen to helium, produces the bulk of the energy released when hydrogen is successively fused to heavier and heavier elements. Fortunately, stars are mostly composed of hydrogen. The reaction in the cores of main sequence stars is

$$4\mathrm{H} \quad \rightarrow \quad \mathrm{He}$$

Since the pressures at the centers of massive stars are so high, their nuclear reaction rates are very high and although they have more mass to "burn", they run out of hydrogen much more rapidly than do low mass stars.

The main sequence lifetimes of stars is given in the following table. Note that hot young blue stars don't last very long and the fact that you see them in our own and other galaxies means that *star formation continues to the present time.*

Data for Main Sequence Stars
age of Universe = 13.9 billion years

$\mathrm{M(M_\odot)}$	τ_{ms} (10^9 yrs)	$\mathrm{L(L_\odot)}$	$\mathrm{T_{eff}(K)}$	Sp.T.
0.08	1000?	0.0008?	2000	L4
0.15	300	0.003	3000	M7
0.8	25	0.4	5000	K1
1.0	10	1	5800	G2
2.0	0.75	20	10,000	A0
10	0.02	8,000	25,000	B0
60	0.003	500,000	40,000	O5

Gravitational Energy and Internal Heat: Planets in the Solar System

Both Jupiter and Saturn radiate energy at about twice the rate they receive it from the Sun. In Jupiter's case, this is probably due to the energy released in gravitational collapse and the fact that Jupiter, being massive, cannot radiate as efficiently as the other planets. This is because the rate at which energy can be radiated is proportional to the surface area, i.e. to R^2, where R is the radius, while the heat content is proportional to the volume, i.e. to R^3. Thus energy radiated per unit mass is proportional to 1/R, all other things being equal. The energy released by gravitational collapse can be simply estimated as

$$\mathrm{E} \sim \frac{\mathrm{GM}^2}{\mathrm{R}} \quad \mathrm{ergs}$$

(you drop mass M from infinity to radius R onto mass M), a considerable amount of energy - for example, were the Sun to shrink till it was about half the diameter of the Earth, the total gravitational energy released would be about the same as the Sun radiates throughout its life. This release of gravitational energy is probably also the reason why the Earth's interior is molten; radioactivity in the Earth's crust melted some iron, which then sank to the middle, releasing energy and melting more iron, till almost all the iron is at the center of the Earth.

The Moon is solid throughout. As a much smaller body, it has mostly cooled down (it is still losing a little heat). The Apollo missions left seismographs on the Moon, which can pick up the waves propagated by small moonquakes (mostly due to the shifting of the crust due to libration) and meteor impacts. These show that the Moon has no liquid core.

The Bottom End of the Main Sequence and Brown Dwarfs

Large, self gravitating bodies, like stars and planets, naturally take up the minimum energy configuration, a sphere. Why are all such bodies not stars? To see why this is an interesting question, go back to the hydrostatic equation (the approximate equation will do):

$$\mathrm{P_c} = \frac{\mathrm{GM^2}}{\mathrm{R^4}}$$

The gas pressure is

$$\mathrm{P_{gas}} = \mathrm{nkT}$$

so that if the object is supported by gas pressure, equilibrium gives:

$$\frac{\mathrm{GM^2}}{\mathrm{R^4}} = \frac{\rho \mathrm{kT}}{\mu \mathrm{m_H}}$$

Since

$$\rho = \frac{\mathrm{M}}{\frac{4}{3}\pi \mathrm{R^3}}$$

We have, neglecting numbers:

$$\frac{\mathrm{GM^2}}{\mathrm{R^4}} \sim \frac{\mathrm{MkT}}{\mathrm{R^3}\mu \mathrm{m_H}}$$

solving for T:

$$\mathrm{T} \sim \frac{\mathrm{GM}\mu \mathrm{m_H}}{\mathrm{kR}}$$

i.e. as $\mathrm{R} \to 0$, $\mathrm{T} \to \infty$, which says that *any* gravitating body could become a star simply by shrinking to a small enough radius that the temperature rises sufficiently for nuclear reactions to start. We know that this doesn't happen, but what stops it?

For bodies like the Earth, made of normal matter, the answer is normal van der Waals forces between the atoms/molecules - the "fluid" out of which the Earth is made is incompressible. But as the mass of a self-gravitating body grows, so does the pressure at its center. At sufficiently high pressures, as we saw when discussing the interior of the Sun, normal matter is "crushed", but not to nuclear densities, because of *electron degeneracy pressure.* This is, basically, the pressure exerted by electrons occupying different states and indeed electrons must do so in order to "exist" - no two electrons can occupy the same state.

Each electron has momentum $\mathrm{p_e}$. The *Heisenberg Uncertainty Principle* says

$$\Delta \mathrm{p} \Delta \mathrm{x} \geq \mathrm{h}$$

Electrons of the same momentum must be at least Δx apart - no two electrons can be the same. The momentum difference of two electrons must then be of order

$$\Delta \mathrm{p}(\mathrm{e_1}, \mathrm{e_2}) \sim \frac{\mathrm{h}}{\Delta \mathrm{x}}$$

Let n_e be the number density of electrons, i.e. the number per cubic centimeter. The pressure exerted by the electrons is

$$P_e = n_e m_e V_e V_e = n_e m_e V_e^2$$

If n_e is the number of electrons per cubic centimeter, the mean distance between electrons must be

$$\Delta x \sim \frac{1}{n_e^{\frac{1}{3}}}$$

Thus

$$p \sim \frac{h}{\Delta x} \sim h n_e^{\frac{1}{3}}$$

and

$$V_e = \frac{p}{m_e} \sim \frac{h}{m_e} n_e^{\frac{1}{3}}$$

So

$$P = n_e m_e V_e^2 = n_e m_e \frac{h^2 n_e^{\frac{2}{3}}}{m_e^2}$$

$$= \frac{n_e^{\frac{5}{3}} h^2}{m_e}$$

Since the total density ρ is proportional to the electron density, we can just replace n_e by ρ (forgetting about all sorts of constants). The mean density ρ is given by

$$\rho = M / \frac{4\pi R^3}{3}$$

In balance, the gravitational pressure = the electron degeneracy pressure:

$$\frac{GM^2}{R^4} = \frac{h^2}{m_e}\left(\frac{3M}{4\pi R^3}\right)^{\frac{5}{3}}$$

i.e.

$$\frac{GM^2}{R^4} = \frac{h^2}{m_e}\left(\frac{3}{4\pi}\right)^{\frac{5}{3}} \frac{M^{\frac{5}{3}}}{R^5}$$

i.e.

$$R \sim \frac{h^2}{G m_e}\left(\frac{3}{4\pi}\right)^{\frac{5}{3}} M^{-\frac{1}{3}}$$

i.e. as M gets larger R gets smaller. This happens because the matter is more tightly compressed as the gravitational pressure increases.

The pressure is then

$$P_{degen} \sim f\rho^{\frac{5}{3}}$$

where f contains all those constants. Then the equation of hydrostatic equilibrium is, when both gas pressure and degeneracy pressure are included

$$\frac{GM^2}{R^4} \sim \frac{MkT}{R^3 \mu m_H} + f\frac{M^{\frac{5}{3}}}{R^5}$$

Solve for T:

$$T \sim \frac{\mu m_H}{k}\left(\frac{GM}{R} - \frac{fM^{\frac{2}{3}}}{R^2}\right)$$

which shows that $T = 0$ at $R = \infty$ and $R = R_o = f/GM^{\frac{1}{3}}$. So we can find the maximum value of T by differentiating:

$$\frac{dT}{dR} = 0 = -\frac{\mu m_H}{k}\left(\frac{GM}{R^2} - \frac{2fM^{\frac{2}{3}}}{R^3}\right)$$

which gives a maximum temperature at $R_1 = 2R_o$. The maximum temperature can then be found by substituting $R = R_1$:

$$T_{max} = \frac{\mu m_H}{k}\left(GM\frac{GM^{\frac{1}{3}}}{2f} - fM^{\frac{2}{3}}\left(\frac{GM^{\frac{1}{3}}}{2f}\right)^2\right)$$

$$T_{max} = \frac{\mu m_H G^2}{4kf}M^{\frac{4}{3}}$$

If we need $T = 10^7$ K for nuclear reactions to start, then for pure hydrogen, this corresponds to a mass of 0.08 $M_\odot$ or 80 M_J, 80 Jupiter masses (Jupiter is about 1/1000 the mass of the Sun). This is the *lowest mass star possible.*

However, we can get a very low level of nuclear burning at a lower mass than this from *deuterium* burning. Deuterium, or 'heavy hydrogen', has one proton and one neutron in its nucleus, and since the mass-to-charge ratio is twice as high as that of hydrogen, it is easier for deuterium to overcome the electrostatic barrier and hence deuterium burning can take place at a lower temperature (and hence in a lower mass star) than can hydrogen burning. These objects are very cool and faint, and are known as *brown dwarfs.* They are very hard to find, but several have been found recently - the first was the companion to a nearby star called Gliese 229. The companion is at a distance from the brighter star of 44 A.U. and is only 10^{-5} as bright. A spectrum of this star shows that its atmosphere, like Jupiter's, contains methane!

This finally leads us to a physical definition of "planet". The term planet means "wanderer" because the planets as seen from Earth change their positions with respect to the "fixed stars". With the discovery of brown dwarfs and massive planets, a physical definition has been proposed (may not last long). Stars, brown dwarfs and planets are bodies massive enough to be spherical under the pull of gravity. Stars are massive enough to burn hydrogen. Brown dwarfs are massive enough to burn deuterium (but not for long because there is so little of it, so they then gradually cool down). Planets are too low mass for deuterium burning, and orbit stars. Brown dwarfs probably tell us quite a lot about the properties of massive planets, which we can't see directly at present because of the glare of the stars they're orbiting.

Key Concepts for Today

1. Calculation of equilibrium temperature: energy absorbed per second = energy radiated per second.

2. Because of the inverse square law, the larger the distance between a planet and a star the smaller the amount of energy received by the planet and the lower its equilibrium temperature.

3. All stars have a comfort zone - a range of distances between which water is in liquid form. Earth and Venus lie in the Sun's comfort zone. However, the Sun's luminosity is very slowly growing, and the comfort zone is slowly moving outwards.

4. Massive bodies cool more slowly than low-mass bodies because they have a smaller surface area to volume ratio.

5. The lowest mass star possible has about 80 Jupiter masses. Bodies of lower mass than this are supported by electron degeneracy pressure or by electrostatic pressure which don't allow the centers to become hot and dense enough for nuclear fusion.

6. Low mass stars burn on the main sequence for far longer than do high mass stars.