## Math 135: Intermediate Algebra

## Worksheet 3

Oct 11, 2007

1. Company Z makes widgets.
(a) The company sells widgets for $\$ 50.00$ each. Write down a function $R(w)$ and draw a graph showing Company Z's revenue as a function of $w$, the number of widgets it sells.
(b) It costs the company $\$ 1000.00$ per month to run its widget factory, plus $\$ 10.00$ for every widget it makes. Write a function $C(w)$ and draw a graph showing Company Z's costs as a function of $w$.
(c) What are the $y$-intercepts of the two graphs you have just drawn. What are the slopes?
(d) Suppose the company makes and sells 100 widgets. How much profit (revenue minus cost) does it make?
(e) Looking at your graphs, find the number of widgets $w$ at which the lines for $R(w)$ and $C(w)$ cross? In words, what is the significance of this point?
(f) Algebraically, find the number of widgets $w$ where the revenue $R(w)$ is equal to the $\operatorname{cost} C(w)$.
2. Fun with graphs - each question below refers to these six graphs. Identify the graph(s), if any, that have:






(a) Positive slope.
(b) $y$-intercept greater than or equal to zero.
(c) The most negative slope.
(d) Zero slope.
(e) Zero $y$-intercept.
(f) Positive $x$-intercept.
3. Surveyors make extensive use of the fomulae for distance and midpoint. Suppose a surveyor makes a map of an area and uses $x$ to represent distance East of the Central Tower and $y$ to represent distance

North of the Central Tower. The map is shown below.

(a) What are the coordinates of the corner of Appel Hall closest to the Central Tower?
(b) What is the distance from the nearest corner of Chester Hall to the Central Tower?
(c) Baker Hall has a door in the center of its East Wall. Find the coordinates of the door.
(d) A path leads form the door of Baker Hall to the Central tower. Find the midpoint of the path.
(e) How long is a path that runs from the Eastern tip of the fountain to the Southwestern corner of Chester Hall? What is the midpoint of the path?
4. Car 1 leaves Los Angeles at the same time car 2 leaves San Francisco. Suppose car 1 is traveling at 50 miles per hour, and car 2 is traveling at 70 miles per hour. Los Angeles and San Francisco are about 500 miles apart.
(a) Write a function $d_{1}(t)$ and draw a graph showing the distance that car 1 has traveled as a function of time $t$.
(b) Write a function $d_{2}(t)$ and draw a graph showing the distance that car 2 has traveled as a function of time.
(c) What are the slopes of the graphs you just drew?
(d) Write a function $d(t)$ and draw a graph showing the distance between the two cars as a function of time.
(e) What is the slope of the graph of $d(t)$ ? How does it relate to the slopes of $d_{1}(t)$ and $d_{2}(t)$ ?
(f) How long after they have left do the two cars pass each other? You can figure this out using your graphs or algebraically.
(g) When the cars pass, how far are they from San Francisco?
5. Income taxes work in "brackets." A bracket is just a way of specifying the slope of a line. This problem will illustrate that.
(a) In a certain state, the lowest income tax bracket is $3 \%$ on incomes up to $\$ 10,000$ per year. Write a funcion $T_{1}(I)$ and draw a graph showing the amount of tax as a function of income.
(b) What is the domain of $T_{1}(I)$ ? What is the range?
(c) What is the slope of the graph you just drew?
(d) The next tax backet is $5 \%$ for income from $\$ 20,001$ to $\$ 40,000$ per year. Thus, someone would pay $3 \%$ on their first $\$ 20,000$ of income and $5 \%$ on the next $\$ 20,000$. For example, a person making $\$ 25,000$ per year would pay $3 \%$ of $\$ 20,000$ and $5 \%$ of $\$ 5,000$. Write a function $T_{2}(I)$ showing the amount of tax for incomes from $\$ 20,001$ to $\$ 40,000$ per year. Graph $T_{2}(I)$ on the same graph as $T_{1}(I)$.
(e) What is the slope of the graph you just drew?
(f) Where do $T_{1}$ and $T_{2}$ meet? Explain in words why they meet there.
(g) The average tax someone pays is just defined as their total tax divided by their income. What is the average tax for someone making $\$ 20,000$ per year? How about $\$ 30,000$ or $\$ 40,000$ ?

