Math 135: Intermediate Algebra Homework 9 Solutions Dec 11, 2007

Section 6.4

Solve and check the solution of:

1.

$$\frac{x}{10} + \frac{3}{5} = \frac{x}{20}$$

Answer:

$$20\left(\frac{x}{10} + \frac{3}{5}\right) = 20\left(\frac{x}{20}\right)$$
$$2x + 12 = x$$
$$x = -12$$

Check:

$$\frac{\frac{x}{10} + \frac{3}{5}}{\frac{10}{10} + \frac{3}{5}} = \frac{\frac{x}{20}}{\frac{12}{10}} + \frac{3}{5} = \frac{-12}{20} - \frac{6}{5} + \frac{3}{5} = -\frac{3}{5} - \frac{3}{5} - \frac{3}{5} = -\frac{3}{5}$$

Solution checks.

3.

$$\frac{r}{8} + \frac{r-4}{12} = \frac{r}{24}$$

Answer:

$$24\left(\frac{r}{8} + \frac{r-4}{12}\right) = 24\left(\frac{r}{24}\right)$$
$$3r + 2(r-4) = r$$
$$3r + 2r - 8 = r$$
$$4r - 8 = 0$$
$$r = 2$$

Check:

$$\frac{\frac{r}{8} + \frac{r-4}{12}}{\frac{2}{8} + \frac{2-4}{12}} = \frac{\frac{r}{24}}{\frac{2}{24}}$$
$$\frac{\frac{3}{12} + \frac{-2}{12}}{\frac{1}{22}} = \frac{1}{12}$$
$$\frac{\frac{1}{12}}{\frac{1}{12}} = \frac{1}{12}$$

Solution checks.

5.

$$\frac{x+5}{x} - \frac{2}{3} = 0$$

Answer:

$$x\left(\frac{x+5}{x}-\frac{2}{3}\right) = x(0)$$
$$x+5-\frac{2}{3}x = 0$$
$$\frac{1}{3}x+5 = 0$$
$$x = -15$$

Check:

$$\frac{x+5}{x} - \frac{2}{3} = 0$$

$$\frac{-15+5}{-15} - \frac{2}{3} = 0$$

$$\frac{-10}{-15} - \frac{2}{3} = 0$$

$$\frac{2}{3} - \frac{2}{3} = 0$$

$$0 = 0$$

Solution checks.

7.

$$\frac{7}{r+20} = \frac{2}{r}$$

Answer:

$$r(r+20)\left(\frac{7}{r+20}\right) = r(r+20)\left(\frac{2}{r}\right)$$

$$7r = 2(r+20)$$

$$7r = 2r+40$$

$$5r = 40$$

$$r = 8$$

Check:

$$\begin{array}{rcrcr} \frac{7}{r+20} & = & \frac{2}{r} \\ \frac{7}{8+20} & = & \frac{2}{8} \\ \frac{7}{28} & = & \frac{1}{4} \\ \frac{1}{4} & = & \frac{1}{4} \end{array}$$

Solution checks.

9.

$$\frac{2}{p} - \frac{3p+1}{4p} = -1$$

Answer

$$4p\left(\frac{2}{p} - \frac{3p+1}{4p}\right) = 4p(-1)$$

$$8 - (3p+1) = -4p$$

$$-3p+7 = -4p$$

$$p = -7$$

Check:

$$\frac{2}{p} - \frac{3p+1}{4p} = -1$$

$$\frac{2}{-7} - \frac{3(-7)+1}{4(-7)} = -1$$

$$-\frac{2}{7} - \frac{-20}{-28} = -1$$

$$-\frac{2}{7} - \frac{5}{7} = -1$$

$$-\frac{7}{7} = -1$$

$$-1 = -1$$

Solution checks.

11.

$$\frac{8}{a-1}-4=\frac{2a}{a-1}$$

Answer:

$$(a-1)\left(\frac{8}{a-1}-4\right) = (a-1)\left(\frac{2a}{a-1}\right) 8-4(a-1) = 2a 8-4a+4 = 2a -6a+12 = 0 a = 2$$

Check:

$$\frac{\frac{8}{a-1} - 4}{\frac{8}{2-1} - 4} = \frac{2a}{a-1}$$
$$\frac{\frac{8}{2-1} - 4}{\frac{2}{2-1}} = \frac{2(2)}{2-1}$$
$$\frac{\frac{8}{1} - 4}{\frac{8}{1} - 4} = \frac{4}{1}$$
$$\frac{8-4}{4} = 4$$

Solution checks.

13.

$$\frac{1}{x+5} = \frac{5}{x+5}$$

Answer:

$$(x+5)\left(\frac{1}{x+5}\right) = (x+5)\left(\frac{5}{x+5}\right)$$
$$1 = 5$$

This is a false statement, so there is no solution.

15.

$$\frac{2}{n} + n = 3$$

Answer:

$$n\left(\frac{2}{n}+n\right) = n(3)$$

$$2+n^2 = 3n$$

$$n^2 - 3n + 2 = 0$$

$$(n-2)(n-1) = 0$$

$$n = 1 \text{ or } 2$$

Check:

$$\frac{2}{n} + n = 3 \qquad \frac{2}{n} + n = 3$$
$$\frac{2}{1} + 1 = 3 \qquad \frac{2}{2} + 2 = 3$$
$$2 + 1 = 3 \qquad 1 + 2 = 3$$
$$3 = 3 \qquad 3 = 3$$

Both solutions check.

17.

Answer:

$$4a\left(\frac{a}{4}\right) = 4a\left(\frac{9}{a}\right)$$
$$a^2 = 36$$
$$a = -6 \text{ or } 6$$

 $\frac{a}{4}=\frac{9}{a}$

Check:

$$\frac{a}{4} = \frac{9}{a} \qquad \frac{a}{4} = \frac{9}{a}$$
$$\frac{-6}{4} = \frac{9}{-6} \qquad \frac{6}{4} = \frac{9}{6}$$
$$-\frac{3}{2} = -\frac{3}{2} \qquad \frac{3}{2} = \frac{3}{2}$$

Both solutions check.

19.

$$\frac{t}{t-1}-\frac{t^2}{t-1}=5$$

Answer

$$(t-1)\left(\frac{t}{t-1} - \frac{t^2}{t-1}\right) = (t-1)(5)$$
$$t-t^2 = 5t-5$$
$$0 = t^2 + 4t - 5$$
$$0 = (t+5)(t-1)$$
$$t = -5 \text{ or } 1$$

Check:

$$\frac{t}{t-1} - \frac{t^2}{t-1} = 5 \qquad \frac{t}{t-1} - \frac{t^2}{t-1} = 5$$
$$\frac{1}{-1-1} - \frac{1^2}{1-1} = 5 \qquad \frac{-5}{-5-1} - \frac{(-5)^2}{-5-1} = 5$$
$$\frac{1}{0} - \frac{1}{0} = 5 \qquad \frac{-5}{-6} - \frac{25}{-6} = 5$$
$$\frac{5}{6} + \frac{25}{6} = 5$$
$$\frac{30}{6} = 5$$
$$5 = 5$$

The solution t = -5 checks, but the solution t = 1 does not, so the only solution is t = -5.

21.

$$4 + \frac{7}{p} = \frac{15}{p^2}$$

Answer:

$$p^{2}\left(4+\frac{7}{p}\right) = p^{2}\left(\frac{15}{p^{2}}\right)$$

$$4p^{2}+7p = 15$$

$$4p^{2}+7p-15 = 0$$

$$(4p-5)(p+3) = 0$$

$$p = -3 \text{ or } \frac{5}{4}$$

Check:

$$4 + \frac{7}{p} = \frac{15}{p^2} \qquad 4 + \frac{7}{p} = \frac{15}{p^2}$$
$$4 + \frac{7}{-3} = \frac{15}{(-3)^2} \qquad 4 + \frac{7}{\frac{5}{4}} = \frac{15}{\left(\frac{5}{4}\right)^2}$$
$$4 - \frac{7}{3} = \frac{15}{9} \qquad 4 + \frac{7 \cdot 4}{5} = \frac{15}{\frac{25}{16}}$$
$$\frac{12}{3} - \frac{7}{3} = \frac{5}{3} \qquad 4 + \frac{28}{5} = \frac{15 \cdot 16}{25}$$
$$\frac{5}{3} = \frac{5}{3} \qquad \frac{20}{5} + \frac{28}{5} = \frac{240}{25}$$
$$\frac{48}{5} = \frac{48}{5}$$

Both solutions check.

23.

$$\frac{r+7}{2} + \frac{4}{r} = \frac{1}{2}$$

Answer:

$$2r\left(\frac{r+7}{2} + \frac{4}{r}\right) = 2r\left(\frac{1}{2}\right)$$

$$r(r+7) + 8 = r$$

$$r^{2} + 7r + 8 = r$$

$$r^{2} + 6r + 8 = 0$$

$$(r+2)(r+4) = 0$$

$$r = -2 \text{ or } -4$$

Check:

$$\frac{\frac{r+7}{2} + \frac{4}{r} = \frac{1}{2}}{\frac{-2+7}{2} + \frac{4}{-2} = \frac{1}{2}} = \frac{\frac{r+7}{2} + \frac{4}{r} = \frac{1}{2}}{\frac{-4+7}{2} + \frac{4}{-4} = \frac{1}{2}}$$
$$\frac{\frac{5}{2} - \frac{4}{2} = \frac{1}{2}}{\frac{5}{2} - \frac{4}{2} = \frac{1}{2}} = \frac{3}{2} - \frac{2}{2} = \frac{1}{2}$$
$$\frac{1}{2} = \frac{1}{2} = \frac{1}{2}$$

Both solutions check.

$$\frac{1}{2} + \frac{2}{2} =$$

$$\frac{1}{x} + \frac{2}{x+10} = \frac{x}{x+10}$$

Answer:

25.

$$x(x+10)\left(\frac{1}{x} + \frac{2}{x+10}\right) = x(x+10)\left(\frac{x}{x+10}\right)$$
$$x+10+2x = x^{2}$$
$$0 = x^{2} - 3x - 10$$
$$(x-5)(x+2) = 0$$
$$x = -2 \text{ or } 5$$

Check:

$$\frac{1}{x} + \frac{2}{x+10} = \frac{x}{x+10} \qquad \frac{1}{x} + \frac{2}{x+10} = \frac{x}{x+10}$$
$$\frac{1}{x-2} + \frac{2}{-2+10} = \frac{-2}{-2+10} \qquad \frac{1}{5} + \frac{2}{5+10} = \frac{5}{5+10}$$
$$-\frac{1}{2} + \frac{2}{8} = \frac{-2}{8} \qquad \frac{1}{5} + \frac{2}{15} = \frac{5}{15}$$
$$-\frac{2}{4} + \frac{1}{4} = -\frac{1}{4} \qquad \frac{3}{15} + \frac{5}{15} = \frac{5}{15}$$
$$-\frac{1}{4} = -\frac{1}{4} \qquad \frac{5}{15} = \frac{5}{15}$$

Both solutions check.

27.

$$\frac{6}{n+2} = \frac{10}{n-2}$$

Answer:

$$(n+2)(n-2)\left(\frac{6}{n+2}\right) = (n+2)(n-2)\left(\frac{10}{n-2}\right)$$

$$6(n-2) = 10(n+2)$$

$$6n-12 = 10n+20$$

$$-4n = 32$$

$$n = -8$$

Check:

$$\frac{\frac{6}{n+2}}{\frac{6}{-8+2}} = \frac{10}{n-2}$$
$$\frac{\frac{6}{-8+2}}{\frac{6}{-6}} = \frac{10}{-10}$$
$$\frac{-1}{-1} = -1$$

Solution checks.

29.

$$\frac{1}{a+1} = \frac{4}{a+8}$$

Answer:

$$(a+1)(a+8)\left(\frac{1}{a+1}\right) = (a+1)(a+8)\left(\frac{4}{a+8}\right)^{2}$$
$$a+8 = 4(a+1)$$
$$a+8 = 4a+4$$
$$-3a = -4$$
$$a = \frac{4}{3}$$

Check:

$$\frac{\frac{1}{a+1}}{\frac{1}{4}+1} = \frac{4}{\frac{4}{3}+8}$$
$$\frac{\frac{1}{\frac{4}{3}}}{\frac{\frac{1}{7}}{\frac{3}{3}}} = \frac{4}{\frac{28}{3}}$$
$$\frac{\frac{3}{7}}{\frac{7}{7}} = \frac{4\cdot 3}{\frac{28}{28}}$$
$$\frac{3}{\frac{7}{7}} = \frac{12}{\frac{28}{28}}$$
$$\frac{3}{\frac{7}{7}} = \frac{3}{\frac{7}{7}}$$

Solution checks.

31.

$$\frac{3-7r}{r^2-9} - \frac{r}{3-r} = \frac{10}{r+3}$$

Answer:

$$\frac{3-7r}{r^2-9} - \frac{r}{3-r} = \frac{10}{r+3}$$
$$\frac{3-7r}{(r+3)(r-3)} + \frac{r}{r-3} = \frac{10}{r+3}$$

Then, if we multiply both sides of the equation by (r+3)(r-3), we get

$$3 - 7r + r(r+3) = 10(r-3)$$

$$3 - 7r + r^{2} + 3r = 10r - 30$$

$$r^{3} - 14r + 33 = 0$$

$$(r - 3)(r - 11) = 0$$

$$r = 3 \text{ or } 11$$

Check r = 3:

$$\begin{array}{rcl} \frac{3-7r}{r^2-9}-\frac{r}{3-r} &=& \frac{10}{r+3}\\ \frac{3-7(3)}{3^2-9}-\frac{3}{3-3} &=& \frac{10}{3+3}\\ \frac{3-21}{9-9}-\frac{3}{0} &=& \frac{10}{6} \end{array}$$

We have something over 0, so r = 3 does not check. Check r = 11:

$$\frac{3-7r}{r^2-9} - \frac{r}{3-r} = \frac{10}{r+3}$$
$$\frac{3-7(11)}{11^2-9} - \frac{11}{3-11} = \frac{10}{11+3}$$
$$\frac{3-77}{121-9} - \frac{11}{-8} = \frac{10}{14}$$
$$\frac{-74}{112} + \frac{11}{8} = \frac{5}{7}$$
$$-\frac{37}{56} + \frac{77}{56} = \frac{5}{7}$$
$$\frac{40}{56} = \frac{5}{7}$$
$$\frac{5}{7} = \frac{5}{7}$$

Solution checks. So the the solution is r = 11.

61. The human resources department of a company surveyed employees to determine employee satisfaction with the company's benefits packages. Of the 150 employees surveyed, 124 said they were satisfied. At this rate, approximately how many of the company's 1128 employees are satisfied with their benefits package? Answer:

Let n be the number of employees satisfied with the benefits package. Then we have

$$\frac{n}{1128} = \frac{124}{150}$$
$$\frac{n}{1128} = \frac{62}{75}$$
$$n = 1128 \left(\frac{62}{75}\right)$$
$$= \frac{23312}{25}$$
$$= 932.48$$

Since people only come in whole numbers, we round to the nearest whole. Approximately 932 of the 1128 employees will be satisfied.

63. The speed of an airplane in still air is 420 mph. Find the speed of the wind if the plane traveled 1125 mi with the wind in the same time it took to travel 975 mi against the wind. Answer:

Let w be the speed of the wind. Then the speed with the wind is 420 + w, and the speed against the wind is 420 - w. Making a table:

Direction	Speed	Distance	Time
With wind	420 + w	1125	$\frac{1125}{420+w}$
Against wind	420 - w	975	$\frac{975}{420-w}$

Since we are told that the two times are equal, we can write an equation and solve:

$$\frac{1125}{420+w} = \frac{975}{420-w}$$

Then, if we multiply both sides of the equation by (420 + w)(420 - w), we get

$$1125(420 - w) = 975(420 + w)$$

$$472500 - 1125w = 409500 + 975w$$

$$-2100w = -63000$$

$$w = 30$$

So the wind speed is 30 mph.

69. One grounds crew can prepare a college's football field in 9 hr. Another grounds crew can prepare the same field in 6 hr. Working together, how long will it take them to prepare the field?

Answer:

Let t be the time it takes them both together. The first crew can prepare the field by itself in 9 hours, so it does $\frac{1}{9}$ of the field per hour. The second can prepare the field by itself in 6 hours, so it completes $\frac{1}{6}$ of the field per hour. Combining this information into a table:

Crew	Rate	Time	Amount done
1	$\frac{1}{9}$	t	$\frac{t}{9}$
2	$\frac{1}{6}$	t	$\frac{t}{6}$

Since they complete one field working together, we have

$$\frac{t}{9} + \frac{t}{6} = 1$$

18 $\left(\frac{t}{9} + \frac{t}{6}\right) = 18(1)$

$$\begin{array}{rcl} 2t + 3t &=& 18 \\ 5t &=& 18 \\ t &=& \frac{18}{5} = 3.6 \end{array}$$

So working together the two crews take 3.6 hours to complete the field.

 $\sqrt{-4} = 2i$

 $\sqrt{-\frac{1}{16}} = \frac{1}{4}i$

 $\sqrt{-3} = i\sqrt{3}$

Section 7.6

Rewrite in terms of i:

1.

3.

7.

9.

5.

$$\sqrt{-18} = i\sqrt{18} = 3i\sqrt{2}$$

 $\sqrt{-500} = i\sqrt{500} = 10i\sqrt{5}$

$$-\sqrt{-9} = -3i$$

13.

11.

$$6\sqrt{-\frac{5}{16}} = 6i\sqrt{\frac{5}{16}} = \frac{3}{2}i\sqrt{5}$$

15.

$$-\frac{1}{4}\sqrt{-12} = -\frac{1}{4}i\sqrt{12} = -\frac{1}{2}i\sqrt{3}$$