Math 135 – Intermediate Algebra

Homework 8 – Solutions

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Problems from section 6.1 of Akst & Bragg

Problems 3 to 11

Identify the values for which the given rational expression is undefined.

3. Expression: $\dfrac{2n-1}{4}$

The denominator of this expression is never 0, regardless of the value of n.
This expression is therefore $\boxed{\text{defined for all real numbers}}$.

5. Expression: $\dfrac{4x+5}{6x-3}$

The denominator of this expression is 0 when

$$\begin{aligned} 6x-3 &= 0 \\ 6x &= 3 \\ x &= \frac{3}{6} \\ x &= \frac{1}{2} \end{aligned}$$

This expression is therefore $\boxed{\text{undefined for } x = \frac{1}{2}}$.

7. Expression: $\dfrac{n+6}{n^2-8n+12}$

The denominator of this expression is 0 when

$$\begin{aligned} n^2-8n+12 &= 0 \\ (n-6)(n-2) &= 0 \\ n=6 \quad &\text{or} \quad n=2 \end{aligned}$$

This expression is therefore $\boxed{\text{undefined for } n=6 \text{ or } n=2}$.

9. Expression: $\dfrac{x^2+10}{x^2-9}$

The denominator of this expression is 0 when

$$\begin{aligned} x^2-9 &= 0 \\ (x+3)(x-3) &= 0 \\ x=-3 \quad &\text{or} \quad x=3 \end{aligned}$$

This expression is therefore $\boxed{\text{undefined for } x=-3 \text{ or } x=3}$.

11. Expression: $\dfrac{x^2-2x+1}{2x^2+x-3}$

The denominator of this expression is 0 when

$$\begin{aligned} 2x^2+x-3 &= 0 \\ (2x+3)(x-1) &= 0 \\ x=-\frac{3}{2} &\quad \text{or} \quad x=1 \end{aligned}$$

This expression is therefore $\boxed{\text{undefined for } x=-\frac{3}{2} \text{ or } x=1}$.

Problems 21 to 25 and 31 to 33

Simplify, if possible.

21. Expression: $\dfrac{6x^3-4x}{2x^2}$

$$\frac{6x^3-4x}{2x^2} = \frac{3x^3-2x}{x^2} = \boxed{\frac{3x^2-2}{x}}$$

23. Expression: $\dfrac{-12p^2q}{8p+4q}$

$$\frac{-12p^2q}{8p+4q} = \boxed{\frac{-3p^2q}{2p+q}}$$

25. Expression: $\dfrac{-7+a}{a-7}$

$$\frac{-7+a}{a-7} = \frac{a-7}{a-7} = \boxed{1}$$

31. Expression: $\dfrac{x^2-16}{4x+16}$

$$\frac{x^2-16}{4x+16} = \frac{(x+4)(x-4)}{4x+16} = \frac{(x+4)(x-4)}{4\,(x+4)} = \boxed{\frac{x-4}{4}}$$

33. Expression: $\dfrac{n^2-6n+9}{n^2+n-12}$

$$\frac{n^2-6n+9}{n^2+n-12} = \frac{(n-3)^2}{(n-3)(n+4)} = \boxed{\frac{n-3}{n+4}}$$

Problems 45 to 49 and 53

Multiply. Express answers in lowest terms.

45. Operation: $\dfrac{8y-4}{6y^2-12y}\cdot\dfrac{3y}{2y-1}$

$$\frac{8y-4}{6y^2-12y}\cdot\frac{3y}{2y-1} = \frac{4\,(2y-1)}{3y\,(2y-4)}\cdot\frac{3y}{2y-1} = \frac{4}{2y-4} = \boxed{\frac{2}{y-2}}$$

47. Operation: $\dfrac{p^2+p-30}{p^3-2p^2}\cdot\dfrac{2p^2-4p}{p^2+12p+36}$

$$\begin{aligned}
\frac{p^2+p-30}{p^3-2p^2}\cdot\frac{2p^2-4p}{p^2+12p+36} &= \frac{(p-5)(p+6)}{p^2\,(p-2)}\cdot\frac{2p\,(p-2)}{(p+6)^2}\\
&= \frac{(p-5)}{p}\cdot\frac{2}{p+6}\\
&= \boxed{\frac{2\,(p-5)}{p\,(p+6)}}
\end{aligned}$$

49. Operation: $\dfrac{x^2-2xy}{4y^2-x^2}\cdot\dfrac{x^2+xy-2y^2}{x^2+y^2}$

$$\begin{aligned}
\frac{x^2-2xy}{4y^2-x^2}\cdot\frac{x^2+xy-2y^2}{x^2+y^2} &= \frac{x\,(x-2y)}{(2y-x)(2y+x)}\cdot\frac{(x+2y)(x-y)}{x^2+y^2}\\
&= \frac{-x\,(2y-x)}{(2y-x)(2y+x)}\cdot\frac{(2y+x)(x-y)}{x^2+y^2}\\
&= -x\,\frac{x-y}{x^2+y^2}\\
&= \boxed{x\,\frac{y-x}{x^2+y^2}}
\end{aligned}$$

53. Operation: $\dfrac{2n^2+11n+12}{n^2+3n-4}\cdot\dfrac{n^2-9n+8}{6n^2+5n-6}$

$$\begin{aligned}
\frac{2n^2+11n+12}{n^2+3n-4}\cdot\frac{n^2-9n+8}{6n^2+5n-6} &= \frac{(n+4)(2n+3)}{(n-1)(n+4)}\cdot\frac{(n-1)(n-8)}{(2n+3)(3n-2)}\\
&= \boxed{\frac{n-8}{3n-2}}
\end{aligned}$$

Problems 61 and 63

Divide. Express answers in lowest terms.

61. Operation: $\dfrac{4t-10}{12t^2}\div\dfrac{2t-5}{6t^2+9t}$

$$\frac{4t-10}{12t^2}\div\frac{2t-5}{6t^2+9t}=\frac{4t-10}{12t^2}\cdot\frac{6t^2+9t}{2t-5}=\frac{2\,(2t-5)}{2\cdot 2t\cdot 3t}\cdot\frac{3t\,(2t+3)}{2t-5}=\frac{1}{2t}\cdot(2t+3)=\boxed{\frac{2t+3}{2t}}$$

63. Operation: $\dfrac{a^2-2a}{3a^3+9a^2}\div\dfrac{a^2-4a+4}{a^2+a-6}$

$$\frac{a^2-2a}{3a^3+9a^2}\div\frac{a^2-4a+4}{a^2+a-6}=\frac{a^2-2a}{3a^3+9a^2}\cdot\frac{a^2+a-6}{a^2-4a+4}=\frac{a\,(a-2)}{3a\cdot a\,(a+3)}\cdot\frac{(a+3)(a-2)}{(a-2)^2}=\boxed{\frac{1}{3a}}$$

Problem 79

An environmental agency determines that it will cost approximately $125p\div(100-p)$ *million dollars to clean* $p\%$ *of the chemical pollutants in a river. Under what circumstance is this expression undefined?*

The expression $125p\div(100-p)$ is undefined when the denominator is 0, *i.e.*, when $\boxed{p=100}$.
The expression is therefore not defined when it comes to cleaning 100% of the chemical pollutants in a river.

Problem 83

In studying electrical circuits, a physics student can use an expression to find the power needed to maintain an electric current. With V the potential difference in volts and R the resistance in ohms, it reads

$$\left(\frac{V}{R}\right)^2 \cdot R$$

Simplify this expression.

$$\left(\frac{V}{R}\right)^2 \cdot R = \frac{V^2}{R^2} \cdot R = \boxed{\frac{V^2}{R}}$$

Problem 85

Inflation is a sustained increase in prices of goods and services. When the rate of inflation, i, grows, the purchasing power of the dollar decreases. To calculate the amount by which the value of a dollar decreases, economists use the following expression:

$$\frac{1}{100+i} \div \frac{1}{i}$$

Write this quotient as a single rational expression.

$$\frac{1}{100+i} \div \frac{1}{i} = \frac{1}{100+i} \cdot i = \boxed{\frac{i}{100+i}}$$

Problems from section 6.2 of Akst & Bragg

Problems 1 to 7

Perform the indicated operation. Simplify, if possible.

1. Operation: $\dfrac{n-10}{4n^2} + \dfrac{3n+2}{4n^2}$

$$\frac{n-10}{4n^2} + \frac{3n+2}{4n^2} = \frac{n-10+3n+2}{4n^2} = \frac{4n-8}{4n^2} = \frac{4\,(n-2)}{4n^2} = \boxed{\frac{n-2}{n^2}}$$

3. Operation: $\dfrac{4n-15}{n-6} - \dfrac{2n-3}{n-6}$

$$\frac{4n-15}{n-6} - \frac{2n-3}{n-6} = \frac{4n-15-(2n-3)}{n-6} = \frac{4n-15-2n+3}{n-6} = \frac{2n-12}{n-6} = \frac{2\,(n-6)}{n-6} = \boxed{2}$$

5. Operation: $\dfrac{3y-2x}{x^2-y^2} - \dfrac{4y-3x}{x^2-y^2}$

$$\frac{3y-2x}{x^2-y^2} - \frac{4y-3x}{x^2-y^2} = \frac{3y-2x-(4y-3x)}{x^2-y^2} = \frac{3y-2x-4y+3x}{x^2-y^2} = \frac{x-y}{(x-y)(x+y)} = \boxed{\frac{1}{x+y}}$$

7. Operation: $\dfrac{y^2-3}{y^2-y-12}+\dfrac{7+y-y^2}{y^2-y-12}$

$$\frac{y^2-3}{y^2-y-12}+\frac{7+y-y^2}{y^2-y-12}=\frac{y^2-3+7+y-y^2}{y^2-y-12}=\boxed{\frac{y+4}{(y-4)(y+3)}}$$

Problems 9 to 15

Find the LCD of each group of rational expressions. Then write each expression in terms of the LCD.

9. Expressions: $\dfrac{3}{8x^2y^3}$ and $\dfrac{1}{6x^3y^2}$

The first thing to check is that each of these expressions is in lowest terms. As this is the case, we can proceed to looking for the LCD. To do so, a technique that always works consists in (1) calculating the ratio (call it r) of the denominator of the first expression to the denominator of the second one in lowest terms, and (2) multiplying the denominator of r by the denominator of the first expression. The result of step (2) is the LCD. Here, we get

$$r=\frac{8x^2y^3}{6x^3y^2}=\frac{4y\cdot 2x^2y^2}{3x\cdot 2x^2y^2}=\frac{4y}{3x}$$

The denominator of r, with r written in lowest terms, is $3x$ and the denominator of the first expression is $8x^2y^3$, so the LCD we are looking for is

$$\boxed{\text{LCD}}=3x\cdot 8x^2y^3\boxed{=24x^3y^3}$$

To find the first (respectively, second) expression in terms of the LCD, one has to multiply its numerator and denominator by the denominator (resp., numerator) of r in lowest terms, which leads to

$$\boxed{\frac{3}{8x^2y^3}}=\frac{3\cdot 3x}{8x^2y^3\cdot 3x}\boxed{=\frac{9x}{24x^3y^3}}$$

$$\boxed{\frac{1}{6x^3y^2}}=\frac{1\cdot 4y}{6x^3y^2\cdot 4y}\boxed{=\frac{4y}{24x^3y^3}}$$

11. Expressions: $\dfrac{n}{n-2}$ and $\dfrac{7}{n-1}$

Let's proceed as in the previous problem. The two expressions are in lowest terms, so all we need to find is r, also in lowest terms:

$$r=\frac{n-2}{n-1}$$

As this result is already in lowest terms, the LCD reads

$$\boxed{\text{LCD}=(n-1)(n-2)}$$

Therefore, the first and second expressions can be written in terms of the LCD as follows:

$$\boxed{\frac{n}{n-2}=\frac{n\,(n-1)}{(n-1)(n-2)}}\quad\text{and}\quad\boxed{\frac{7}{n-1}=\frac{7\,(n-2)}{(n-1)(n-2)}}$$

13. Expressions: $\dfrac{3}{2t^2+12t}$ and $\dfrac{4}{3t^3+18t^2}$

The two expressions are in lowest terms, so all we need to find is r, also in lowest terms:

$$r = \frac{2t^2+12t}{3t^3+18t^2} = \frac{2t\,(t+6)}{3t^2\,(t+6)} = \frac{2t}{3t^2} = \frac{2}{3t}$$

The LCD therefore reads

$$\boxed{\text{LCD}} = 3t\,(2t^2+12t) = 3t \cdot 2t\,(t+6) \boxed{= 6t^2\,(t+6)}$$

The first and second expressions can now be written in terms of the LCD:

$$\boxed{\frac{3}{2t^2+12t}} = \frac{3 \cdot 3t}{3t\,(2t^2+12t)} \boxed{= \frac{9t}{6t^2\,(t+6)}}$$

$$\boxed{\frac{4}{3t^3+18t^2}} = \frac{2 \cdot 4}{2\,(3t^3+18t^2)} \boxed{= \frac{8}{6t^2\,(t+6)}}$$

15. Expressions: $\dfrac{2p-q}{p^2-9q^2}$ and $\dfrac{3}{p-3q}$

The two expressions are in lowest terms, so all we need to find is r, also in lowest terms:

$$r = \frac{p^2-9q^2}{p-3q} = \frac{(p-3q)(p+3q)}{p-3q} = p+3q$$

Note that the denominator of r is 1 in this problem. The LCD therefore reads

$$\boxed{\text{LCD}} = 1\,(p^2-9q^2) \boxed{= (p-3q)(p+3q)}$$

The first and second expressions can thus be written in terms of the LCD as follows:

$$\boxed{\frac{2p-q}{p^2-9q^2} = \frac{2p-q}{(p-3q)(p+3q)}}$$

$$\boxed{\frac{3}{p-3q} = \frac{3\,(p+3q)}{(p-3q)(p+3q)}}$$

Problems 21 to 37

Perform the indicated operations.

21. Operation: $\dfrac{1}{8x^2y} + \dfrac{1}{12xy^2}$

$$\frac{1}{8x^2y} + \frac{1}{12xy^2} = \frac{3y}{24x^2y^2} + \frac{2x}{24x^2y^2} \boxed{= \frac{3y+2x}{24x^2y^2}}$$

23. Operation: $\dfrac{3}{p} - 4p$

$$\frac{3}{p} - 4p = \frac{3}{p} - \frac{4p^2}{p} = \boxed{\frac{3 - 4p^2}{p}}$$

25. Operation: $\dfrac{n}{n+4} + \dfrac{2}{n+1}$

$$\frac{n}{n+4} + \frac{2}{n+1} = \frac{n\,(n+1)}{(n+1)(n+4)} + \frac{2\,(n+4)}{(n+1)(n+4)} = \frac{n\,(n+1) + 2\,(n+4)}{(n+1)(n+4)} = \boxed{\frac{n^2 + 3n + 8}{(n+1)(n+4)}}$$

27. Operation: $\dfrac{5}{x-2y} + \dfrac{2}{2x-y}$

$$\begin{aligned}\frac{5}{x-2y} + \frac{2}{2x-y} &= \frac{5\,(2x-y)}{(x-2y)(2x-y)} + \frac{2\,(x-2y)}{(x-2y)(2x-y)} = \frac{5\,(2x-y) + 2\,(x-2y)}{(x-2y)(2x-y)} \\ &= \frac{10x - 5y + 2x - 4y}{(x-2y)(2x-y)} = \frac{12x - 9y}{(x-2y)(2x-y)} = \boxed{\frac{3\,(4x-3y)}{(x-2y)(2x-y)}}\end{aligned}$$

29. Operation: $\dfrac{7a}{a-3} + \dfrac{5a+6}{3-a}$

$$\begin{aligned}\frac{7a}{a-3} + \frac{5a+6}{3-a} &= \frac{7a}{a-3} - \frac{5a+6}{a-3} = \frac{7a - (5a+6)}{a-3} \\ &= \frac{7a - 5a - 6}{a-3} = \frac{2a-6}{a-3} = \frac{2\,(a-3)}{a-3} = \boxed{2}\end{aligned}$$

31. Operation: $\dfrac{1}{4-2r} + \dfrac{7}{3r^2 - 6r}$

$$\begin{aligned}\frac{1}{4-2r} + \frac{7}{3r^2-6r} &= \frac{1}{2\,(2-r)} + \frac{7}{3r\,(r-2)} = \frac{1}{2\,(2-r)} - \frac{7}{3r\,(2-r)} \\ &= \frac{3r}{6r\,(2-r)} - \frac{14}{6r\,(2-r)} \\ &= \boxed{\frac{3r-14}{6r\,(2-r)}}\end{aligned}$$

33. Operation: $\dfrac{4t-1}{t^2-16} + \dfrac{2}{4-t}$

$$\begin{aligned}\frac{4t-1}{t^2-16} + \frac{2}{4-t} &= \frac{4t-1}{(t-4)(t+4)} - \frac{2}{t-4} = \frac{4t-1}{(t-4)(t+4)} - \frac{2\,(t+4)}{(t-4)(t+4)} \\ &= \frac{4t-1-2\,(t+4)}{(t-4)(t+4)} = \frac{4t-1-2t-8}{(t-4)(t+4)} \\ &= \boxed{\frac{2t-9}{(t-4)(t+4)}}\end{aligned}$$

35. Operation: $\dfrac{n^2-4}{n^2-10n+21}-\dfrac{4}{n-7}$

$$\begin{aligned}\frac{n^2-4}{n^2-10n+21}-\frac{4}{n-7} &= \frac{n^2-4}{(n-7)(n-3)}-\frac{4\,(n-3)}{(n-7)(n-3)}=\frac{n^2-4-4\,(n-3)}{(n-7)(n-3)}\\ &= \frac{n^2-4-4n+12}{(n-7)(n-3)}=\boxed{\frac{n^2-4n+8}{(n-7)(n-3)}}\end{aligned}$$

37. Operation: $\dfrac{6}{u^2+11uv+30v^2}+\dfrac{3}{u^2+2uv-24v^2}$

$$\begin{aligned}\frac{6}{u^2+11uv+30v^2}+\frac{3}{u^2+2uv-24v^2} &= \frac{6}{(u+6v)(u+5v)}+\frac{3}{(u+6v)(u-4v)}\\ &= \frac{6\,(u-4v)}{(u+6v)(u+5v)(u-4v)}+\frac{3\,(u+5v)}{(u+6v)(u+5v)(u-4v)}\\ &= \frac{6\,(u-4v)+3\,(u+5v)}{(u+6v)(u+5v)(u-4v)}=\frac{6u-24v+3u+15v}{(u+6v)(u+5v)(u-4v)}\\ &= \frac{9u-9v}{(u+6v)(u+5v)(u-4v)}=\boxed{\frac{9\,(u-v)}{(u+6v)(u+5v)(u-4v)}}\end{aligned}$$

Problems 51 and 53

Given $f(x)$ and $g(x)$, find $f(x)+g(x)$ and $f(x)-g(x)$.

51. $f(x)=\dfrac{8}{x^2+10x+24}$ and $g(x)=\dfrac{2}{x^2-5x-36}$

Let's first rewrite $f(x)$ and $g(x)$ in lowest terms:

$$f(x)=\frac{8}{x^2+10x+24}=\frac{8}{(x+4)(x+6)}\quad\text{and}\quad g(x)=\frac{2}{x^2-5x-36}=\frac{2}{(x+4)(x-9)}$$

We can now find $f(x)+g(x)$ and $f(x)-g(x)$:

$$\begin{aligned}f(x)+g(x) &= \frac{8}{(x+4)(x+6)}+\frac{2}{(x+4)(x-9)}=\frac{8\,(x-9)+2\,(x+6)}{(x+4)(x+6)(x-9)}\\ &= \frac{8x-72+2x+12}{(x+4)(x+6)(x-9)}=\frac{10x-60}{(x+4)(x+6)(x-9)}\\ &= \boxed{\frac{10\,(x-6)}{(x+4)(x+6)(x-9)}}\end{aligned}$$

$$\begin{aligned}f(x)-g(x) &= \frac{8}{(x+4)(x+6)}-\frac{2}{(x+4)(x-9)}=\frac{8\,(x-9)-2\,(x+6)}{(x+4)(x+6)(x-9)}\\ &= \frac{8x-72-2x-12}{(x+4)(x+6)(x-9)}=\frac{6x-84}{(x+4)(x+6)(x-9)}\\ &= \boxed{\frac{6\,(x-14)}{(x+4)(x+6)(x-9)}}\end{aligned}$$

53. $f(x) = \dfrac{x+4}{3x^2-12x}$ and $g(x) = -\dfrac{2}{x^2}$

$g(x)$ is already given in lowest terms. As for $f(x)$, we find:

$$f(x) = \frac{x+4}{3x^2-12x} = \frac{x+4}{3x\,(x-4)}$$

Therefore,

$$\begin{aligned}
f(x)+g(x) &= \frac{x+4}{3x\,(x-4)} - \frac{2}{x^2} = \frac{x\,(x+4)}{3x^2\,(x-4)} - \frac{2\cdot 3\,(x-4)}{3x^2\,(x-4)} \\
&= \frac{x\,(x+4) - 6\,(x-4)}{3x^2\,(x-4)} = \frac{x^2+4x-6x+24}{3x^2\,(x-4)} \\
&= \boxed{\frac{x^2-2x+24}{3x^2\,(x-4)}}
\end{aligned}$$

$$\begin{aligned}
f(x)-g(x) &= \frac{x+4}{3x\,(x-4)} + \frac{2}{x^2} = \frac{x\,(x+4)}{3x^2\,(x-4)} + \frac{2\cdot 3\,(x-4)}{3x^2\,(x-4)} \\
&= \frac{x\,(x+4) + 6\,(x-4)}{3x^2\,(x-4)} = \frac{x^2+4x+6x-24}{3x^2\,(x-4)} \\
&= \frac{x^2+10x-24}{3x^2\,(x-4)} = \boxed{\frac{(x-2)(x+12)}{3x^2\,(x-4)}}
\end{aligned}$$

Problem 55

A cylindrical water tank is drained for a short period of time. The change in the water level can be found using the expression $V_1 \div (\pi r^2) - V_2 \div (\pi r^2)$*, where* V_1 *and* V_2 *represent the original and new volume, respectively, and* r *is the radius of the tank. Write the change in water level as a single rational expression.*

$$\frac{V_1}{\pi r^2} - \frac{V_2}{\pi r^2} = \boxed{\frac{V_1 - V_2}{\pi r^2}}$$

Problem 57

To determine the percent growth in sales from the previous year, the owner of a company uses the expression $100\,(S_1 \div S_0 - 1)$*, where* S_1 *represents the current year's sales and* S_0 *represents last year's sales. Write this expression as a single rational expression.*

$$100\left(\frac{S_1}{S_0} - 1\right) = 100\left(\frac{S_1}{S_0} - \frac{S_0}{S_0}\right) = 100\left(\frac{S_1 - S_0}{S_0}\right) = \boxed{\frac{100\,(S_1 - S_0)}{S_0}}$$