

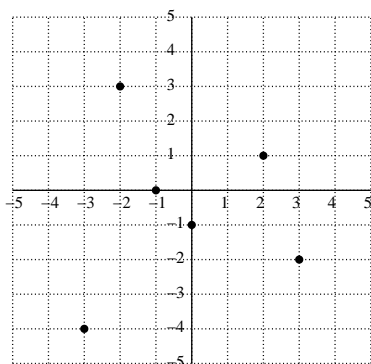
# Math 135 – Intermediate Algebra

## Homework 3 – Solutions

October 16, 2007

### 3.1: Problems 1, 3, 17-23

1. On the coordinate plane, plot the following coordinates.



3. Next to each point, write its coordinates

Clock-wise from upper left:  $(-2, 4)$ ,  $(3, 4)$ ,  $(4.5, 0)$ ,  $(4, -3)$ ,  $(0, -1)$ ,  $(-3, -3)$ .

17. The SAT verbal and math scores for college applicants are displayed on the following graph (not shown).

- a. Estimate the coordinates of each point.

NB: for coordinates where the last two digits are either 25 or 75, anything in between 10 and 40, in the former case, or 60 and 90 in the latter, are acceptable. For example, 475 could be anything from 460-490, as the graph is hard to read at this precision.

$$\begin{aligned} A &= (400, 350) \\ B &= (450, 400) \\ C &= (475, 600) \\ D &= (525, 725) \\ E &= (600, 500) \end{aligned}$$

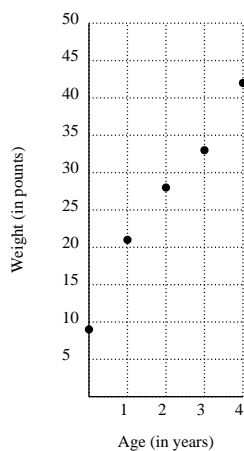
- b. Which students had a higher math score than verbal score?

Students C and D.

- c. What is the significance of the sum of the coordinates?

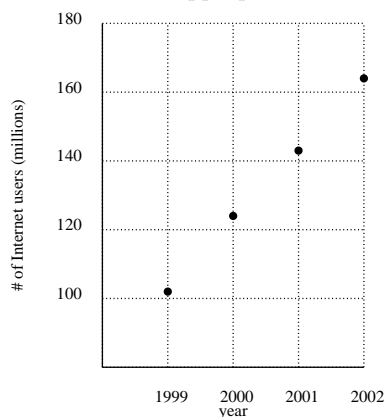
The sum is the total SAT score.

19. Plot information on the coordinate plane.



21. The table (not shown) shows the estimated number of Internet users (in millions) in the US for the years from 1999 to 2002.

a. Choose an appropriate scale and plot this information on the coordinate plane



b. Describe and trends you observe

The number of Internet users has been increasing almost linearly with time since 1999.

23. The graph shows the monthly average for the price of a gallon of regular unleaded gasoline (in cents) in the US for a 12-month period from August 2002 to July 2003

a. Estimate the average price for a gallon of regular unleaded gasoline in February.

165 cents.

b. In which months was the average price of a gallon of gasoline below \$1.50?

August through January.

c. Between which two consecutive months was the greatest increase in the price per gallon? What was the increase in price?

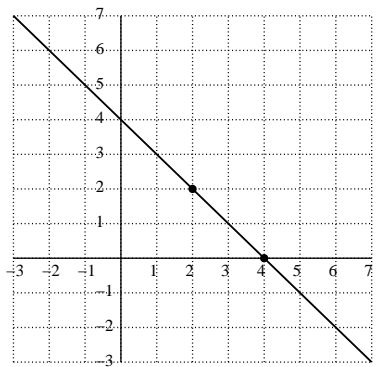
Between January and February, the increase in price was  $165 - 145 = 20$  cents.

### 3.2: Problems 1-11, 25-29, 49-57

Find the slope of the line that passes through the points. Then plot the points and sketch the line that passes through them

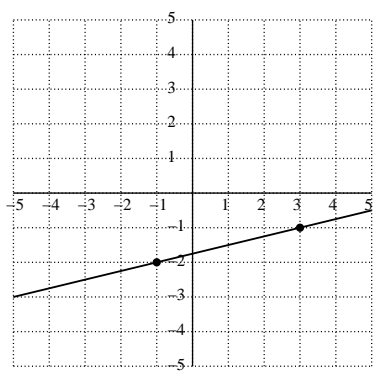
1.

$$m = \frac{2-0}{2-4} = -1$$



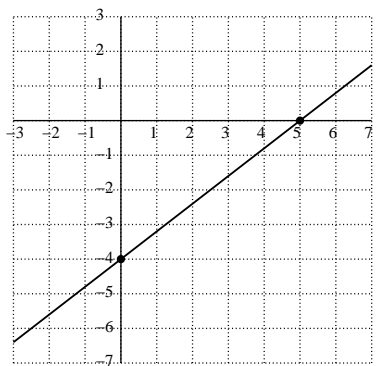
3.

$$m = \frac{-2 - (-1)}{-1 - 3} = \frac{1}{4}$$



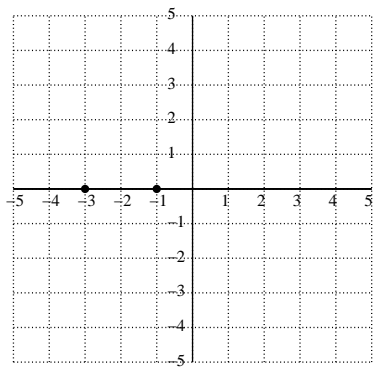
5.

$$m = \frac{0 - (-4)}{5 - 0} = \frac{4}{5}$$



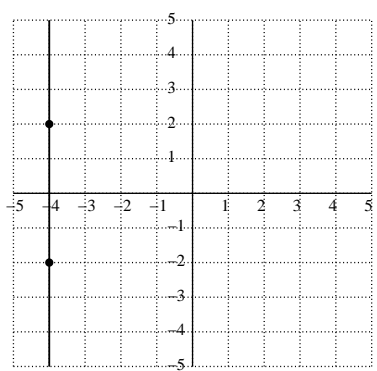
7.

$$m = \frac{0 - 0}{-3 - (-1)} = 0$$



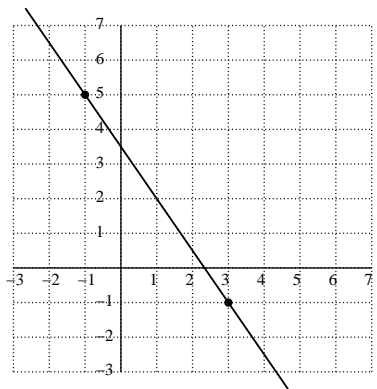
9.

$$m = \frac{-2-2}{-4-(-4)} = \text{undefined}$$



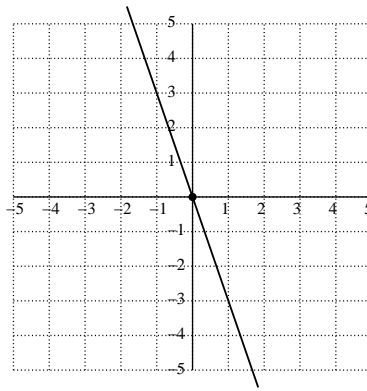
11.

$$m = \frac{-1-5}{3-(-1)} = -\frac{3}{2}$$

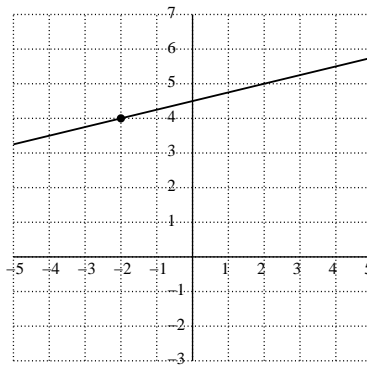


Graph the line using the given information

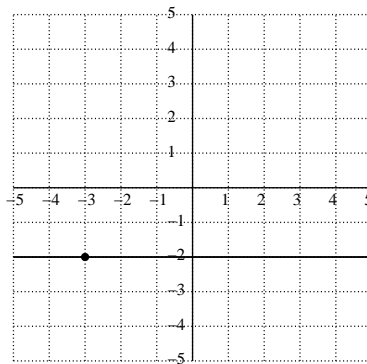
25. Passing through  $(0,0)$  and  $m = -3$



27. Passing through  $(-2, 4)$  and  $m = \frac{1}{4}$



29. Passing through  $(-3, -2)$  and  $m = 0$



49. According to some guidelines, the slope of a curb ramp adjoining a sidewalk and street cannot exceed  $\frac{1}{20}$ . Does the curb shown in the figure meet the guideline? Explain.

$m = \frac{\text{rise}}{\text{run}} = \frac{4}{84} = \frac{1}{21}$ . The slope of the ramp is  $\frac{1}{21}$ , which is less than the maximum allowed slope of  $\frac{1}{20}$ . So yes, the curb does meet the guideline.

51. Most states have a standard sales tax rate that is applied to purchases. The graph displays the amount of sales tax charged on taxable purchases in Massachusetts.

a. Is the slope of the line positive, negative, zero, or undefined?

The slope is positive.

b. Explain the significance of your answer to part (a) in terms of the amount of a purchase and the amount of sales tax charged.

Larger purchases are charged more sales tax.

c. Calculate the slope of the line and explain what it represents in the context of the problem.

We find the slope by first picking two points on the graph, say  $(0, 0)$ , and  $(20, 1)$ , and then calculate the slope with the equation  $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 0}{20 - 0} = \frac{1}{20} = 0.05 = 5\%$ . Thus, if we let  $P$  represent the amount of the purchase, and  $T$  the amount of tax, then we have the equation  $T = 0.05P$ . The slope is just the percentage of your purchase that is taxed, i.e. the sales tax rate.

53. Two coffee shops sell a Kona coffee blend. The graph shows the cost of  $x$  pounds of coffee at each coffee shop.

a. What does the slope of each line represent?

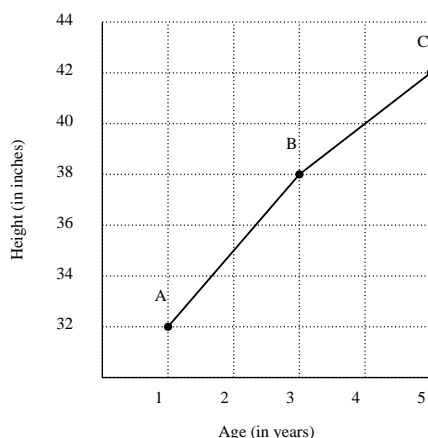
The slope is the price of coffee per pound.

b. Which coffee shop charges more for the Kona coffee? Explain.

Lets ask how much you pay for 5 pounds of coffee at both shops. According to the graph, shop A charges \$30, while shop B charges \$26. Since the cost line for shop A is always above the line for shop B, Shop A thus always charges more for the coffee.

55. The height of a child is recorded at each of her annual checkups. The chart shows her height at various ages.

a. Choose an appropriate scale and plot the points on the coordinate plane. Sketch the line segments AB and AC.



b. Compute the slopes.

For the segment AB we have  $m = \frac{38 - 32}{3 - 1} = \frac{6}{2} = 3$ . For the segment BC we have  $m = \frac{42 - 38}{5 - 3} = \frac{4}{2} = 2$ .

c. Was the child's rate of growth constant over the four years? Explain.

The rate of growth is given by the slope of the line segments AB and BC. The rate of growth was 3 inches/year for the first two years, and 2 inches/year for the last two years. The rate of growth was thus not constant over the four years.

**57.** Two long-distance telephone service providers charge a flat monthly fee plus a charge for each minute of a long-distance call. The graph shows the number of long-distance calling minutes and the monthly phone bill. Which provider charges the higher per minute fee? Explain.

The total monthly cost of each plan can be written in point-slope form:  $y = mx + b$ , where  $y$  is the cost,  $m$  is the per minute fee,  $x$  is the number of minutes, and  $b$  is the flat monthly fee. If we compute the slope of each line, we will thus know the per-minute fee for both plans.

The slope for plan A is can be obtained from the two points given in the graph:  $m = \frac{8.5-5}{50-0} = \frac{3.5}{50}$ . And the same can be done for plan B:  $m = \frac{11-7.5}{50-0} = \frac{3.5}{50}$ . The slopes are the same and thus they have the same per-minute fee.

### 3.3: Problems 7-39, 53-61

Determine if the given ordered pair is a solution of the given equation.

7.  $(0.5, -1.4)$ ,  $10y + 6x = 11$

$$\begin{array}{rcl} 10(-1.4) + 6(0.5) & \stackrel{?}{=} & 11 \\ -14 + 3 & \stackrel{?}{=} & 11 \\ -11 & \neq & 11 \end{array}$$

The point is not a solution to the equation.

9.  $(-0.1, 8)$ ,  $y = 8$ .

This point is a solution because the point has a  $y$ -value of 8, and the equation is  $y = 8$ .

Match each equation to its graph.

11.  $y = 2x - 1$ .

This equation has slope 2 and y-intercept -1. Graph b is thus the graph of this equation.

13.  $y = -2x - 1$ .

This equation has slope -2 and y-intercept -1. Graph d is thus the graph of this equation.

Find the  $x$ - and  $y$ -intercepts of the graph of each equation.

(The  $y$ -intercept is the point where  $x = 0$  in the equation, and similarly, the  $x$ -intercept is the point where  $y = 0$  in the equation. We will use these facts to answer the following questions.)

15.  $y = x - 4$

$$\begin{array}{ll} x = 0 : & y = 0 - 4 \\ & y = -4 \\ & y - \text{intercept} = (0, -4) \end{array}$$

$$\begin{array}{ll} y = 0 : & 0 = x - 4 \\ & x = 4 \\ & x - \text{intercept} = (4, 0) \end{array}$$

**17.**  $y = \frac{1}{3}x + 3$

$$\begin{aligned}x = 0 : \quad & y = 0 + 3 \\& y = 3 \\& y - \text{intercept} = (0, 3)\end{aligned}$$

$$\begin{aligned}y = 0 : \quad & 0 = \frac{1}{3}x + 3 \\& -3 = \frac{1}{3}x \\& -9 = x \\& x - \text{intercept} = (-9, 0)\end{aligned}$$

**19.**  $4x + 2y = -8$

$$\begin{aligned}x = 0 : \quad & 2y = -8 \\& y = -4 \\& y - \text{intercept} = (0, -4)\end{aligned}$$

$$\begin{aligned}y = 0 : \quad & 4x = -8 \\& x = -2 \\& x - \text{intercept} = (-2, 0)\end{aligned}$$

**21.**  $5x - 2y = 10$

$$\begin{aligned}x = 0 : \quad & -2y = 10 \\& y = -5 \\& y - \text{intercept} = (0, -5)\end{aligned}$$

$$\begin{aligned}y = 0 : \quad & 5x = 10 \\& x = 2 \\& x - \text{intercept} = (2, 0)\end{aligned}$$

**23.**  $8y - 6x + 2 = 0$

$$\begin{aligned}x = 0 : \quad & 8y + 2 = 0 \\& 8y = -2 \\& y = -\frac{2}{8} = -\frac{1}{4} \\& y - \text{intercept} = (0, -\frac{1}{4})\end{aligned}$$

$$\begin{aligned}y = 0 : \quad & -6x + 2 = 0 \\& -6x = -2 \\& x = \frac{2}{6} = \frac{1}{3} \\& x - \text{intercept} = (\frac{1}{3}, 0)\end{aligned}$$



**25.**  $10y = -15$

$$\begin{aligned} x = 0 : \quad 10y &= -15 \\ y &= -\frac{15}{10} = -\frac{3}{2} \\ y - intercept &= (0, -\frac{3}{2}) \end{aligned}$$

$$y = 0 : \quad 0 \neq -15$$

because there is no solution, there is no  $x$ -intercept

**27.**  $-3x = -18$

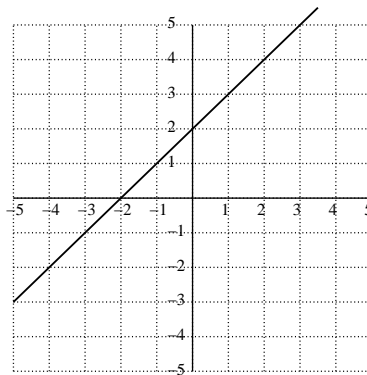
$$x = 0 : \quad 0 \neq -18$$

because there is no solution, there is no  $y$ -intercept

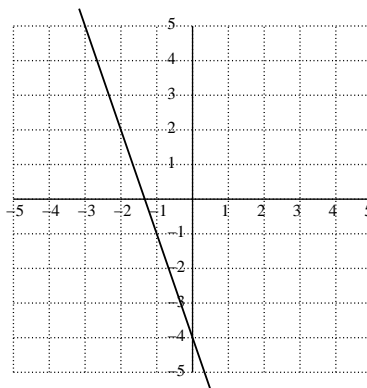
$$\begin{aligned} y = 0 : \quad -3x &= -18 \\ x &= \frac{-18}{-3} = 6 \\ x - intercept &= (6, 0) \end{aligned}$$

*Graph each equation*

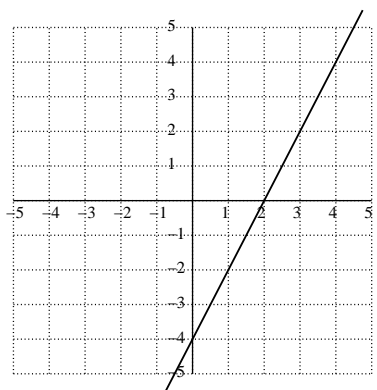
**29.**  $y = x + 2$



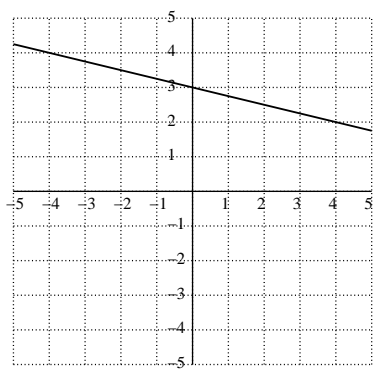
**31.**  $y = -3x - 4$



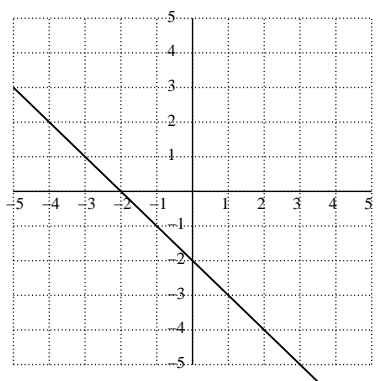
**33.**  $2x - y = 4$



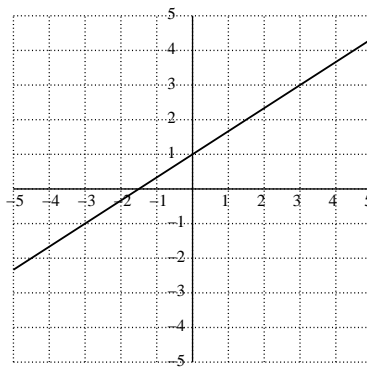
**35.**  $x + 4y = 12$



**37.**  $3x + 3y = -6$



**39.**  $2x - 3y = -3$

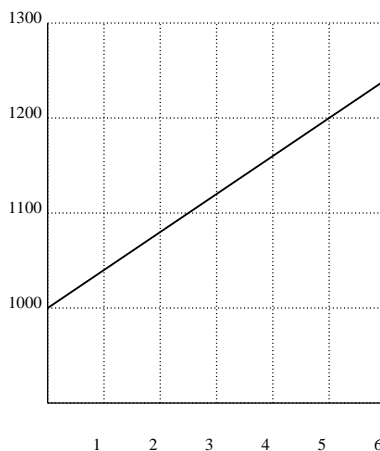


**53.** One thousand dollars is deposited into a savings account earning 4% simple interest annually. If no additional deposits are made, the amount in the account,  $A$ , after  $t$  years is given by:  $A = 40t + 1000$ .

**a.** Complete the table.

For the  $t$  values of 0, 2, 4, and 6, the corresponding  $A$  values are 1000, 1080, 1160, and 1240.

**b.** Choose an appropriate scale for the axes and then graph the equation.



**c.** What does the  $A$ -intercept represent in the context of the problem?

The  $A$ -intercept represents the amount of money in the account initially, when  $t = 0$ .

**d.** From the graph, find the number of years it takes for the amount in the account to grow to \$1200.

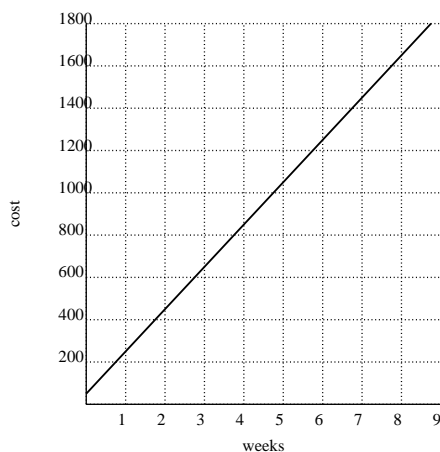
From the graph we can see that  $A = \$1200$  when  $t = 5$  years.

**55.** A day-care center charges \$50 enrollment fee plus \$200 per week for child-care services.

**a.** Express the amount  $A$  paid to the day-care center in terms of the number of weeks  $w$  of child-care services.

$$A = 200w + 50$$

**b.** Graph the equation on the coordinate plane.

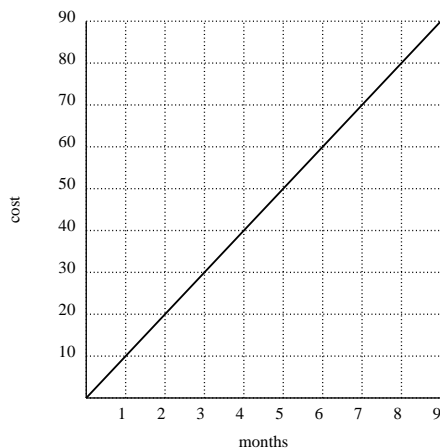


c. Explain why it makes sense to consider the portion of the graph that lies only in Quadrant I.

The problem discusses the amount of money it costs to send a child to day-care for a certain number of weeks. Since neither the cost, nor the number of weeks can be a negative number, the graph only makes sense in Quadrant I, where both the cost and number of weeks are positive.

57. A subscription for an on-line news magazine costs \$10 per month. Write an equation that represents the cost  $c$  of the subscription for  $m$  months. Then graph the equation.

$$c = 10m$$

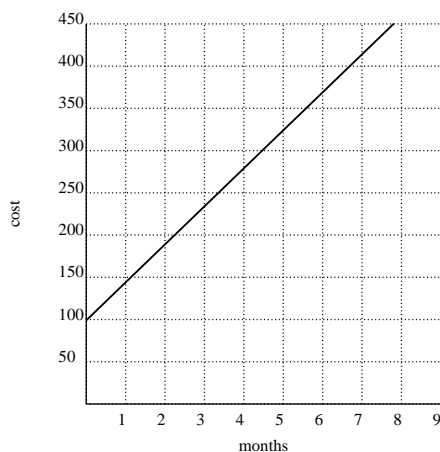


59. A local cable company charges \$99 to install a high-speed Internet connection plus a \$45 monthly service fee.

a. Express the cost  $c$  of a high-speed Internet connection in terms of the number of months  $n$  of service.

$$c = 45n + 99$$

b. Choose an appropriate scale and graph the equation on the coordinate plane.



c. From the graph, estimate the total cost for 6 months of high-speed Internet service.

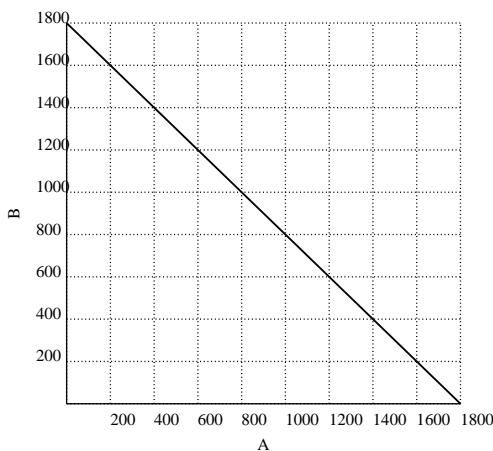
The cost for 6 months is  $\approx \$375$ , as estimated from the graph. (The actual value is \$369).

**61.** A college awards associate's degrees and bachelor's degrees. At last years commencement, the college awarded  $A$  associate's degrees and  $B$  bachelor's degrees. A total of 1800 degrees were awarded.

a. Translate this information into an equation

$$A + B = 1800.$$

b. Graph the equation.



c. Identify the  $A$ - and  $B$ -intercepts of the graph. What do they represent in terms of the number of degrees awarded?

The  $A$ -intercept is  $(1800, 0)$ , the  $B$ -intercept is  $(0, 1800)$ . This is the total number of degrees awarded, and so the  $B$ -intercept occurs where all the degrees are associate's, and the  $A$ -intercept is where all the degrees are bachelor's.

d. In a sentence or two, explain why only some of the points on the graph in Quadrant I are reasonable solutions of this problem.

There cannot be fractions on degrees awarded. So the only reasonable values are positive integers. For example, the coordinate  $(800, 1000)$  is both along the line and reasonable, while the coordinate  $(800.5, 999.5)$  is also along the line but is not reasonable.