Math 135: Intermediate Algebra
Homework 10 Solutions
December 18, 2007

Homework from: Akst & Bragg, "Intermediate Algebra through Applications", 2006 Edition, Pearson/Addison-Wesley
Subject: Linear Systems, Radical Equations
8.1: 1-17, 35-47, 69-75 (odd numbers)
8.2: 1-17, 27, 31, 35-39, 45, 49, 51 (odd numbers)
8.4: 1-17, 21-25, 39-43 (odd numbers)

8.1, page 669

1. $x^2 = 16$

Look at each side of this equation. To solve for x, we have to take the square root of both sides of the equation. Remembering that square roots can be both positive and negative, we have:
$\sqrt{x^2} = \pm x$ (left hand side)
$\sqrt{16} = \pm 4$ (right hand side)
Taking the square root of both sides of the equation we have:
$\pm x = \pm 4$
There are four possible combinations: $+x = +4$ (1), $+x = -4$ (2), $-x = +4$ (3), and $-x = -4$ (4). Note that (1) and (4) are the same, and that (2) and (3) are the same. So the answers are $x = +4$ or $x = -4$, i.e.
$\boxed{x = \pm 4}$

3. $y^2 = 24$
$y = \pm\sqrt{24}$
Note that we can factor 24: $24 = 6\times 4$. 4 is a perfect square (2^2), so the final answer is $\boxed{y = \pm 2\sqrt{6}.}$

5. $a^2 + 25 = 0$. Subtract 25 from both sides to get
$a^2 = -25 = 25 \times -1$. So $a = \pm 5\sqrt{-1} = \boxed{\pm 5i}$

7. $4n^2 - 8 = 0$
$4n^2 = 8$, $n^2 = 2$, $\boxed{n = \pm\sqrt{2}}$

9. $\frac{1}{6}y^2 = 12$
$y^2 = 12 \times 6 = 72 = 2 \times 36$. So $\boxed{y = \pm 6\sqrt{2}}$

11. $3x^2 + 6 = 21$
Divide both sides by 3; $x^2 + 2 = 7$; Subtract 2 from both sides; $x^2 = 5$; $\boxed{x = \pm\sqrt{5}}$

13. $8 - 9n^2 = 14$
Add $9n^2$ to both sides. $8 = 14 + 9n^2$; $9n^2 = 8 - 14 = -6$
$n^2 = -\frac{6}{9} = -\frac{2}{3}$

$\boxed{n = \pm i\sqrt{\frac{2}{3}}}$

15. $(x - 1)^2 = 48$

$(x - 1)^2 = 48$; $x - 1 = \sqrt{48} = \sqrt{3 \times 16} = \pm 4\sqrt{3}$; $\boxed{x = 1 \pm 4\sqrt{3}}$

17. $(2n + 5)^2 = 75$

$(2n + 5) = \pm\sqrt{75} = \pm\sqrt{3 \times 25} = \pm 5\sqrt{3}$

$2n = -5 \pm 5\sqrt{3}$; $\boxed{n = \frac{-5\pm5\sqrt{3}}{2}}$

35. Fill in the blank to make a perfect square: $x^2 - 12x + [\ ]$

To get the term needed to make a perfect square, divide the coefficient of x by 2 and square it. The coefficient is -12, $-12/2 = -6$, $(-6)^2 = 36$. So the number needed to fill in the blank is $\boxed{36}$.

37. $n^2 + 7n + [\ ]$

$\boxed{n^2 + 7n + \frac{49}{4}}$

39. $t^2 - \frac{4}{3}t + [\ ]$

$\boxed{t^2 - \frac{4}{3}t + \frac{4}{9}}$

41. Solve by completing the square: $x^2 - 8x = 0$

$x^2 - 8x = 0$

$x^2 - 8x + 16 = 16$

$(x - 4)^2 = 16$

$x - 4 = \pm 4$

$\boxed{x = 0 \text{ or } x = 8}$

43. $n^2 - 3n = 4$

$n^2 - 3n + \frac{9}{4} = 4 + \frac{9}{4}$

$(n - \frac{3}{2})^2 = \frac{24}{4}$; $n - \frac{3}{2} = \pm\frac{5}{2}$; $n = \frac{8}{2}$ or $n = -\frac{2}{2}$

$\boxed{n = \text{-1,4}}$

45. $x^2 - 4x - 2 = 0$

$x^2 - 4x = 2$; $x^2 - 4x + 4 = 2 + 4$

$(x - 2)^2 = 6$; $\boxed{x = 2 \pm \sqrt{6}}$

47. $a^2 + 7a = 3a - 4$

$a^2 + 7a - 3a + 4 = 0$

$a^2 + 4a + 4 = 0$; $(a + 2)^2 = 0$; $\boxed{a = -2}$

69. *In a search-and-rescue mission, a team maps out a circular search area from the last known location of a group of hikers. If the search region is 78.5 square miles, how far from the last known location, to the nearest mile, is the team searching?*

Area of circle = πr^2, where r is the radius of the circle. In this case, it's the maximum distance over which the search is made. The area over which the search is made is 78.5 square miles. So

$\pi r^2 = 78.5$

We take π to be approximately 3.14. So
$3.14r^2 = 78.5; r^2 = 78.5/3.14 \approx 75/3 = 25$
where $\approx$ means "is approximately equal to". So we have $r^2 = 25$, i.e. r = 5 miles (the negative root doesn't mean anything in this context). $\boxed{\text{The search is carried out up to 5 miles away from the last place where the campers were seen.}}$

71. *A student invests \$1000 in an account earning r percent interest compounded annually. After two years, the amount, A, in the account is given by*

$$A = 1000(1 + r)^2),$$

where r is in decimal form. What is the interest rate if the account has \$1102.50 after two years?

The equation for the total amount A in a bank account that started with amount P is
$A = P(1 + r)^n$
where r is the annual interest rate and n is the number of years the accounts has been open. For this problem we have $A = \$1102.5$, $P = \$1000$ and $n = 2$, so
$1102.5 = 1000(1 + r)^2; (1 + r)^2 = 1.1025$
Approximate this: $(1 + r)^2 \approx 1.1$, $(1 + r) \approx 1.05$, $r \approx 0.05$ or 5%.
$\boxed{\text{The interest rate is 5\%.}}$

73. *A sandbag is dropped from a hot-air balloon 2000 ft above the ground. The height, h, above the ground of the sandbag t seconds after it is dropped is given by the equation*

$$h = -16t^2 + 2000$$

a. *How long after it is released will the sandbag be 1000 ft above the ground?*

$h = -16t^2 + 2000$ is the height above the ground of an object dropped from a height of 2000 feet t seconds after it is dropped. The time it takes the object to fall to 1000 feet above the ground can be found by setting $h = 1000$:
$1000 = -16t^2 + 2000;\ 16t^2 = 1000;\ t^2 = 1000/16;\ t = \sqrt{1000/16} = \frac{10\sqrt{10}}{4} = 2.5\sqrt{10}$ $\approx 2.5 \times 3.1 = 7.8$ seconds. $\boxed{\text{It takes the sandbag 7.8 seconds to fall 1000 feet,}}$

b. *Is the time it takes the sandbag to reach the ground exactly equal to twice the length of time found in the solution to part (a)? Explain.*

To find the time it takes to fall all the way to the ground, set $h = 0$. Then $16t^2 = 2000$; $t = \sqrt{2000/16} = \frac{10\sqrt{20}}{4} = 2.5\sqrt{20} \approx 11$. So the time it takes to fall all the way to the ground is about $\boxed{\text{11 seconds}}$. This is **not** twice the time it takes to fall 1000 feet (halfway) because the object is **accelerating**, i.e. its speed is increasing as it falls. The time taken to fall distance d is $t \propto \sqrt{d}$, where $\propto$ means "is proportional to".

75. *A homeowner wants to build a rectangular patio using his house as one side. He decides to make the length of the patio 10 feet longer than the width.*

a. *What are the dimensions of the patio if the homeowner wants to enclose an area of 144 sq ft?*

Let w be the width of the patio and l the length. We know that $l = w + 10$. The *area* A of the patio is $A = \textit{length} \times \textit{width} = w(w + 10)$, and is 144 square feet. So
$w(w + 10) = 144$; $w^2 + 10w = 144$
Solve by completing the square: $w^2 + 10w + 25 = 144 + 25$; $(w + 5)^2 = 169$; $w = 13 - 5 = 8$ (the negative root has no meaning here). So the width is 8 feet and the length, which is 10 feet longer, is 18 feet. $\boxed{\text{The patio measures } 18{\times}8 \text{ feet.}}$

b. *If the fencing costs $14.95 per foot, how much will it cost to enclose the patio?*

The distance that needs to be fenced is 2× the width plus 1× the length (the other side is up against the house). This is 2×8 + 16 = 34 feet. The total cost is then \$14.95 × 34 = $\boxed{\$508:30}$. (A simple way to do this is to note that \$14.95 is nearly \$15:00. So multiply \$15 by 34, multiply \$0.05 by 34, and subtract.)

8.2, page 682

1. Solve $x^2 + 3x + 2 = 0$
Solve by factoring. The third term is +2, with factors +1,+2 or -1, -2. Since the middle term has numerical coefficient +3, and 2+1 = 3, the equation factors to
$(x + 2)(x + 1) = 0$, and the solutions are $\boxed{x = -1 \text{ or } x = -2.}$

3. $x^2 - 6x - 1 = 0$
This can't be factored, so use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

with a = 1, b = -6 and c = -1. So

$$x = \frac{6 \pm \sqrt{36 + 4}}{2} = \frac{6 \pm \sqrt{40}}{2} = \frac{6 \pm 2\sqrt{10}}{2}$$

$\boxed{x = 3 \pm \sqrt{10}}$ Alternately, you could complete the square:
$x^2 - 6x = 1$; $x^2 - 6x + 9 = 10$; $(x - 3)^2 = 10$; $x - 3 = \pm\sqrt{10}$;
$\boxed{\text{x} = 3 \pm \sqrt{10}.}$

5. $x^2 = x + 11$, or $x^2 - x - 11 = 0$
This won't factor. Use the quadratic formula, with $a = 1$, $b = -1$ and $c = -11$:

$$x = \frac{1 \pm \sqrt{1 + 44}}{2} = \frac{1 \pm \sqrt{45}}{2}$$

$\boxed{x = \frac{1 \pm 3\sqrt{5}}{2}}$

7. $x^2 - 4x + 13 = 8$
$x^2 - 4x + 13 - 8 = 0$; $x^2 - 4x + 5 = 0$
This won't factor, so use the quadratic formula:

$$x = \frac{4 \pm \sqrt{16 - 20}}{2} = \frac{4 \pm \sqrt{-4}}{2} = \frac{4 \pm 2i}{2}$$

$\boxed{x = \ 2 \pm i}$

9. $3x^2 + 6x = 7$; $3x^2 + 6x - 7 = 0$

$$x = \frac{-6 \pm \sqrt{36+84}}{6} = \frac{-6 \pm \sqrt{120}}{6} = \frac{-6 \pm 2\sqrt{30}}{6}$$

$\boxed{x = -1 \pm \frac{\sqrt{30}}{3}}$

11. $6t^2 - 8t = 3t - 4$; $6t^2 - 11t + 4 = 0$
$(3t-4)(2t-1) = 0$; 3t = 4 or 2t = 1; $\boxed{\text{t = 4/3 or t = 1/2}}$.

13. $2x^2 + 8x + 9 = 0$

$$x = \frac{-8 \pm \sqrt{64-72}}{4} = \frac{-8 \pm \sqrt{-8}}{4} = \frac{-8 \pm 2i\sqrt{2}}{4}$$

$\boxed{x = \frac{-4 \pm i\sqrt{2}}{2}}$

15. $1 - 5x^2 = 4x^2 + 6x$; $9x^2 + 6x - 1 = 0$

$$x = \frac{-6 \pm \sqrt{36+36}}{18} = \frac{-6 \pm \sqrt{72}}{18} = \frac{-6 \pm 6\sqrt{2}}{18}$$

$\boxed{x = \frac{-1 \pm \sqrt{2}}{3}}$

17. $2y^2 - 9y + 10 = 1 + 3y - 2y^2$; $4y^2 - 12y + 9 = 0$; $(2y-3)(2y-3) = 0$; $(2y-3)^2 = 0$
$\boxed{y = 3/2}$

27. $(x+6)(x+2) = 8$; $x^2 + 8x + 12 = 8$; $x^2 + 8x + 4 = 0$

$$x = \frac{-8 \pm \sqrt{64-16}}{2} = \frac{-8 \pm \sqrt{48}}{2} = \frac{-8 \pm 4\sqrt{3}}{2}$$

$\boxed{x = -4 \pm 2\sqrt{3}}$.

How many and what kind of roots does each equation have? Use the determinant to find out.
35. $x^2 + 2x + 4 = 0$. Here $a = 1$, $b = 2$, and $c = 4$.
So the determinant $b^2 - 4ac$ = 4 - 16 = -12. Since the determinant is non-zero the equation has two solutions, and since it is negative they are both complex. This equation has $\boxed{\text{two complex solutions.}}$

37. $4x^2 - 12x = -9$; $4x^2 - 12x + 9 = 0$. The discriminant is 144 - 144 = 0, so the equation has $\boxed{\text{one real solution.}}$

39. $6x^2 = 2 - 5x$; $6x^2 + 5x - 2 = 0$.
The discriminant is $25 + 48 = 73$, so the equation has $\boxed{\text{two real solutions}}$.

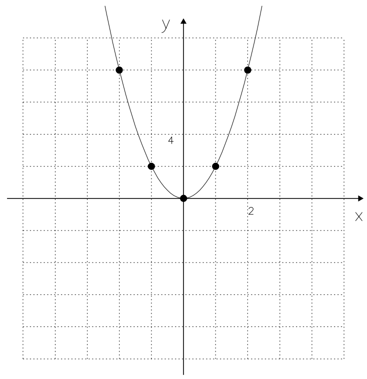
45. *The height h of a stone t seconds after it is thrown straight downward with an initial velocity of 20 ft per second from a bridge 800 ft above the ground is given by the equation* $h = -16t^2 - 20t + 800$. When is the stone 300 feet above the ground?
$h = -16t^2 - 20t + 800$. Substitute $h = 300$:
$300 = -16t^2 - 20t + 800$; $16t^2 + 20t - 800 + 300 = 0$;
$16t^2 + 20t - 500 = 0$; (to make it easier divide through by 4 to get):
$4t^2 + 5t - 125 = 0$; $(4t + 25)(t - 5) = 0$; t= -25/4 or $t = 5$. The negative solution has no meaning for this problem (though it's interesting to think about what it means), so the answer is:
$\boxed{\text{The stone reaches 300 feet from the ground 5 seconds after it is thrown.}}$

49. *A travel company offers special pricing on a weekend getaway trip,. For each person who buys a ticket, the price is reduced by \$5. The regular price of a ticket is \$300. To determine the revenue, R, for selling x tickets, the company uses the equation* $R = x(300 - 5x)$. *If the company wants to keep the reduced ticket price above \$200, how many tickets must it sell in order to generate revenue of \$3520?*
$x(300 - 5x) = 3520$; $300x - 5x^2 = 3520$; $5x^2 - 300x + 3520 = 0$
This will be easier to handle if you divide through by 5: $x^2 - 60x + 704 = 0$. You can get the factors of 704 easily just by keeping dividing by 2. This gives:
$(x - 44)(x - 16) = 0$. The solutions are $x = 44$ and $x = 16$. Both of these will give revenue of \$3520. Which should the company use? If $x = 16$, the price of each ticket is \$3520/16 = \$220. If x = 44, the price of each ticket is \$3520/44 = \$80. Since the company wants to keep the ticket price above \$200, $\boxed{\text{it must sell 16 tickets to generate revenue of \$3520.}}$

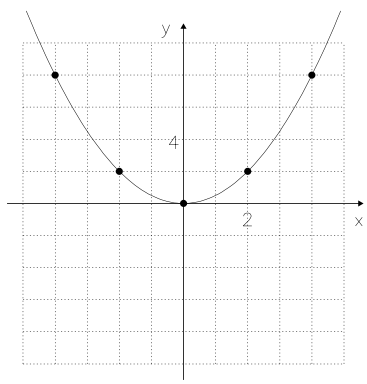
51. *Identity theft occurs when a person's name, social security number or other identifying information is used without his or her knowledge to commit fraud or other crimes. The number of reports, R, in thousands, of identity theft for each year from 2000 to 2003 can be approximated by the equation* $R = -0.5t^2 + 64t + 29$, *where t is the number of years after 2000. In what year were there approximately 215,000 reports of identity theft?* R is 215 (remember R is in thousands), so $215 = -0.5t^2 + 64t + 29$; $-0.5t^2 + 64t - 186 = 0$; $0.5t^2 - 64t + 186 = 0$; $t^2 - 128t + 372 = 0$. Note that this doesn't have integer solutions, but it almost does: if you factor 372 as 3×124, you get $(t-3)(t-124) = 0$, which expands out to $t^2 - 127t + 372 = 0$. This is OK because the equation is an approximation to the data, as stated in the problem. The solutions are t = 3 or t = 124. Obviously $t = 3$ is the relevant solution - we only have data for 2000 to 2003, and solution $t = 124$ corresponds to year 2124, well in the future. So $\boxed{\text{there were 215,000 identity theft reports in 2003.}}$ Incidentally, the sort of equation used here is an *empirical equation* - that is, one whose form is close to the form of the data but for which there is no theoretical support. These are pretty common in economics etc.

8.4, page 710

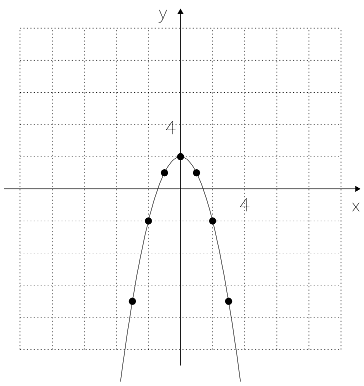
1. $y = f(x) = 2x^2$
$f(-2) = 8$; $f(-1) = 2$; $f(0) = 0$; $f(1) = 2$; $f(2) = 8$
The points are (-2,8), (-1,2), (0,0), (1,2), (2,8).



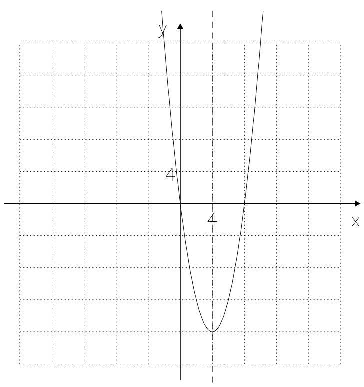
3. $y = f(x) = 0.5x^2$
$f(-4) = 8$; $f(-2) = 2$; $f(0) = 0$; $f(2) = 2$; $f(4) = 8$
The points are (-4,8), (-2,2), (0,0), (2,2), (4,8).



5. $y = f(x) = 2 - x^2$
$f(-3) = -7$; $f(-2) = -2$; $f(-1) = 1$; $f(0) = 2$; $f(1) = 1$; $f(2) = -2$; $f(3) = -7$
The points are (-3,-7), (-2,-2), (-1,1), (0,2), (1,1), (2,-2), (3, -7).

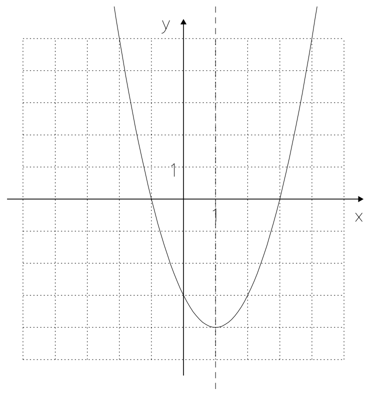


7. $y = f(x) = x^2 - 8x$
The x-coordinate of the vertex is at $x = -b/2a$, where we take $y = f(x) = ax^2 + bx + c$. So in this case $a = 1$, $b = -8$, $c = 0$. The x-coordinate of the vertex is then $x = 8/2 = 4$. The y-coordinate is given by $y = f(4) = 16 - 32 = -16$. So the coordinates of the vertex are $\boxed{\text{(4,-16)}}$. The axis of symmetry is the vertical line through the vertex, given by $\boxed{x = 4}$.
The x-intercepts are where $y = f(x) = 0$. So $x^2 - 8x = 0$; $x(x - 8) = 0$; $x = 0$ or $x = 8$. So the x-intercepts are at $\boxed{\text{(0,0) and (8,0)}}$.
The y-intercept is where $x = 0$. Here $y = f(x) = x^2 - 8x = 0$ when $x = 0$. So the y-intercept is at $\boxed{\text{(0,0)}}$. Here is the graph:

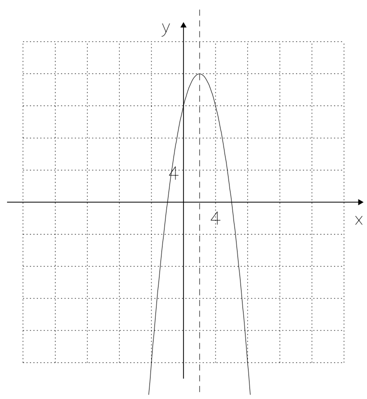


9. $y = f(x) = x^2 - 2x - 3$
The x-coordinate of the vertex is at $x = -b/2a$, where we take $y = f(x) = ax^2 + bx + c$. So in this case $a = 1$, $b = -2$, $c = -3$. The x-coordinate of the vertex is then $x = 2/2 = 1$. The y-coordinate is given by $y = f(1) = 1 - 2 - 3 = -4$. So the coordinates of the vertex are $\boxed{\text{(1,-4)}}$. The axis of symmetry is the vertical line through the vertex, given by $\boxed{x = 1}$.
The x-intercepts are where $y = f(x) = 0$. So $x^2 - 2x - 3 = 0$; $(x - 3)(x + 1) = 0$; $x = 3$ or $x =$ -1. So the x-intercepts are at $\boxed{\text{(3,0) and (-1,0)}}$.

The y-intercept is where $x = 0$. Here $y = f(x) = x^2 - 2x - 3 = -3$ when $x = 0$. So the y-intercept is at $\boxed{\text{(0,-3)}}$. Here is the graph:



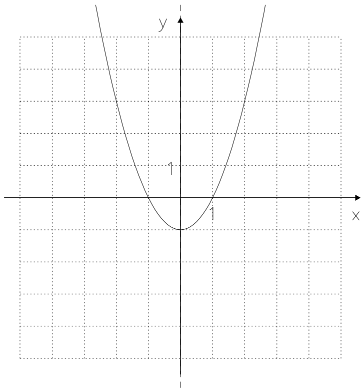
11. $y = f(x) = -x^2 + 4x + 12$
The x-coordinate of the vertex is at $x = -b/2a$, where we take $y = f(x) = ax^2 + bx + c$. So in this case $a = -1$, $b = 4$, $c = 12$. The x-coordinate of the vertex is then $x = 4/2 = 2$. The y-coordinate is given by $y = f(2) == 4 + 8 + 12 = 16$. So the coordinates of the vertex are $\boxed{\text{(2,16)}}$. The axis of symmetry is the vertical line through the vertex, given by $\boxed{x = 2}$.
The x-intercepts are where $y = f(x) = 0$. So $-x^2 + 4x + 12 = 0$; $(x - 6)(x + 2) = 0$; $x = 6$ or $x =$ -2. So the x-intercepts are at $\boxed{\text{(6,0) and (-2,0)}}$.
The y-intercept is where $x = 0$. Here $y = f(x) = -x^2+4x+12 = 12$ when $x = 0$. So the y-intercept is at $\boxed{\text{(0,12)}}$. Here is the graph:



13. $y = f(x) = x^2 - 1$
The x-coordinate of the vertex is at $x = -b/2a$, where we take $y = f(x) = ax^2 + bx + c$. So in this case $a = 1$, $b = 0$, $c = -1$. The x-coordinate of the vertex is then $x = 0$. The y-coordinate is given by $y = f(0) == -1$. So the coordinates of the vertex are $\boxed{\text{(0,-1)}}$. The axis of symmetry is the vertical line through the vertex, given by $\boxed{x = 0}$.

The x-intercepts are where $y = f(x) = 0$. So $x^2 - 1 = 0$; $x^2 = 1$; $x = \pm 1$. So the x-intercepts are at $\boxed{\text{(1,0) and (-1,0)}}$.

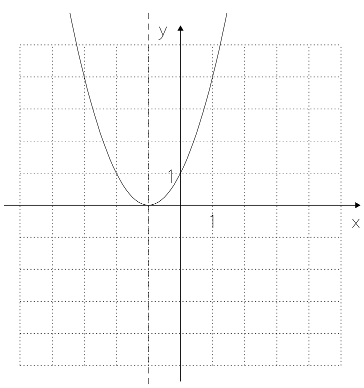
The y-intercept is where $x = 0$. Here $y = f(x) = -1$ when $x = 0$. So the y-intercept is at $\boxed{\text{(0,-1)}}$.
Here is the graph:



15. $y = f(x) = (x+1)^2 = x^2 + 2x + 1$
The x-coordinate of the vertex is at $x = -b/2a$, where we take $y = f(x) = ax^2 + bx + c$. So in this case $a = 1$, $b = 2$, $c = 1$. The x-coordinate of the vertex is then $x = -1$. The y-coordinate is given by $y = f(0) = 0$. So the coordinates of the vertex are $\boxed{\text{(-1,0)}}$. The axis of symmetry is the vertical line through the vertex, given by $\boxed{x = -1}$.

The x-intercepts are where $y = f(x) = 0$. So $(x+1)^2 = 0$; $x = -1$. So the x-intercept is at $\boxed{\text{(-1,0)}}$.

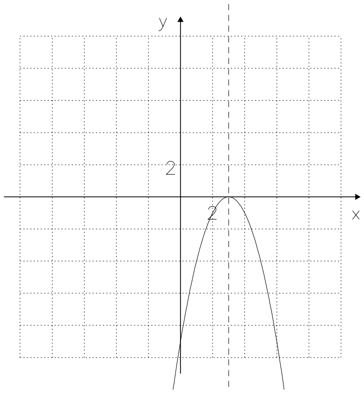
The y-intercept is where $x = 0$. Here $y = f(x) = 1$ when $x = 0$. So the y-intercept is at $\boxed{\text{(0,1)}}$.
Here is the graph:



17. $y = f(x) = -x^2 + 6x - 9$
The x-coordinate of the vertex is at $x = -b/2a$, where we take $y = f(x) = ax^2 + bx + c$. So in this case $a = -1$, $b = 6$, $c = -9$. The x-coordinate of the vertex is then $x = 3$. The y-coordinate is given by $y = f(0) = 0$. So the coordinates of the vertex are $\boxed{\text{(3,0)}}$. The axis of symmetry is the

vertical line through the vertex, given by $\boxed{x = 3}$.
The x-intercepts are where $y = f(x) = 0$. So $(x-3)^2 = 0$; $x = 3$. So the x-intercept is at $\boxed{(3,0)}$.

The y-intercept is where $x = 0$. Here $y = f(x) = -9$ when $x = 0$. So the y-intercept is at $\boxed{(0,-9)}$.
Here is the graph:



21.

$f(x) = x^2+5$: opens **upward**; vertex is a **minimum** point; has **no** x-intercepts; has **one** y intercept

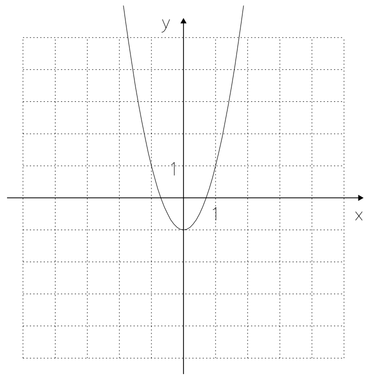
$f(x) = 1 - 4x + 4x^2$: opens **upward**; vertex is a **minimum** point; has **one** x-intercept and **one** y-intercept

$f(x) = 2 - 3x^2$; opens **downward**; vertex is a **maximum** point; has **two** x-intercepts and **one** y-intercept

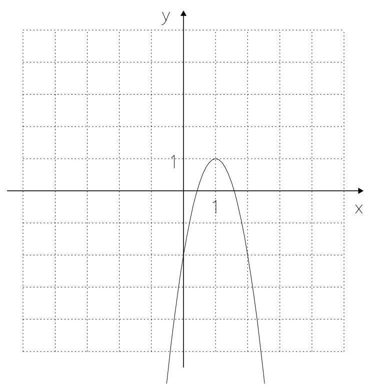
$f(x) = -2x^2 - 5x - 8$: opens **downward**; vertex is a **maximum** point; has **no** x-intercepts and **one** y-intercept

$f(x) = 4x^2 - 4x - 1$; opens**upward**; vertex is a **minimum** point; has **two** x-intercepts and **one** y-intercept.

23. $f(x) = 2x^2 - 1$. $\boxed{\text{The } \textit{domain} \text{ is } (-\infty, \infty) \text{ and the } \textit{range} \text{ is } [-1, \infty)}$.



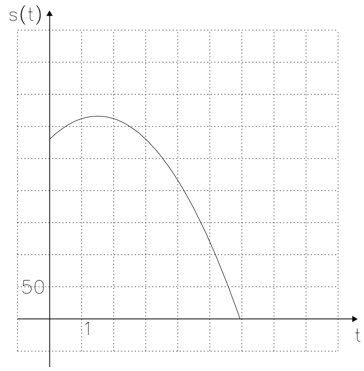
25. $f(x) = -3x^2 + 6x - 2$. $\boxed{\text{The } \textit{domain} \text{ is } (-\infty, \infty) \text{ and the } \textit{range} \text{ is } (-\infty,]}$.



39. *A stone is thrown straight upward with an initial velocity of 48 ft per sec from a bridget 280 ft above a river. The height of the stone above the river t sec after it is thrown is given by the function* $s(t) = -16t^2 + 48t + 280$.

a. *Find the vertex of the graph*

The t value of the vertex is given by $t = -b/2a$. Here a = -16, b = 48, c = 280. So $t = -48/(2 \times -16) = 48/32 = 12/8 = 3/2$. To find the s value of the vertex, substitute $t = 3/2$ in $s(t) = -16t^2 + 48t + 280$, to get $s = -16 \times 9/4 + 48 \times 3/2 + 280$ or 316 feet 1.5 (3/2) seconds after the stone is thrown. The vertex of the graph is then at $\boxed{(3/2,\ 316)}$.



b. *Graph the function.*

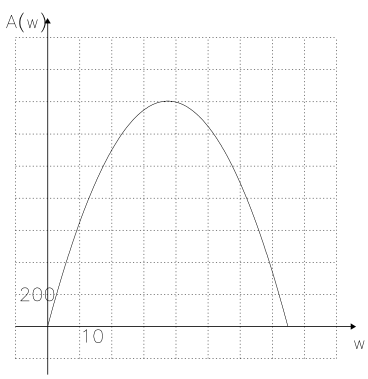
c. *When does the stone reach its maximum height? What is that height?*
The stone's maximum height is at the vertex of the graph, so $\boxed{\text{the stone reaches its maximum height of 316 ft 1.5 seconds after it is thrown.}}$

41. *A homeowner has 150 ft of fencing with which to build a rectangular exercise yard for her dog.*
a. *Express the length, l, of the exercise yard in terms of the width, w*

Since the total distance around the yard (called the *perimeter*) is $2l + 2w$ (think of starting at the top left corner and walking all the way round once), and she has 150 ft of fencing, we have $2l + 2w = 150$; $2(l + w) = 150$; $l + w = 75$; $\boxed{l = 75 - w}$

b. *Express in function notation the relationship between the area, $A(w)$, and the width, w, of the exercise yard.*
$Area = A = length \times width = l \times w$; $A = lw$. Since $l = 75 - w$, $A = w(75 - w)$; $\boxed{A(w) = 75w - w^2}$

c. *Graph this function*:



d. *What dimensions will maximize the area of the exercise yard? What is the maximum area?*

The maximum area occurs at the vertex of the function. The w value of the vertex is given by $w =$

$-b/2a$, where $a =$ -1, $b = 75$, $c = 0$. So $w = -75/(2 \times -1) = 75/2 = 37.5$. The corresponding value of the length, l, is given by $l =$ 75 - 37.5 = 37.5 ft. The corresponding area is then 37.5×37.5 sq ft = 1406.25 sq ft. Thus $\boxed{\text{the maximum area of an exercise yard with a perimeter of 150 ft. is } 37.5 \times 37.5 = 1406.25 \text{ sq. ft.}}$.
Notice something interesting? The maximum rectangular area for a given perimeter is a square. Have some fun proving this in the general case, i.e. for any value of the perimeter.

43. *A toy manufacturer determines that the daily cost, C, for producing x units of a toy dump truck can be approximated by the function $C(x) = 0.005x^2 - x + 100$.* (Parenthetical remark: another *empirical* function here).

a. How many toy dump trucks must the manufacturer produce per day in order to minimize the cost?
Note that the coefficient of x^2 is positive, so the function $C(x)$, a parabola, opens upwards and the vertex is at the minimum. So we need to find the coordinates of the vertex. The x coordinate of the vertex is $x = -b/2a$. Since $a = 0.005$, $b =$ -1 and $c = 100$, the x-coordinate of the vertex is at $x = 1/0.01 = 100$. So $\boxed{\text{the manufacturer must produce 100 units per day to minimize the cost.}}$

b. *What is the minimum daily cost?* This is the "y" coordinate of the vertex. $C(x) = 0.005x^2 - x + 100$. With $x = 100$, this becomes $C(100) = 50 - 100 + 100 = 50$. So $\boxed{\text{the minimum daily cost is \$50}}$.