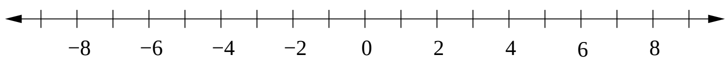
Final Exam Solutions
Math 135: Intermediate Algebra

1. Solve and graph the solution: $1 - 2x > -1$ and $3x + 4 > 1$.

   Solution:

$$\begin{aligned} 1 - 2x > -1 &\quad \text{and} \quad 3x + 4 > 1 \\ -2x > -2 &\quad \text{and} \quad 3x > -3 \\ x < 1 &\quad \text{and} \quad x > -1 \end{aligned}$$

   So the solution is $-1 < x < 1$, and the graph is



2. Find the equation of the line perpendicular to $2x - y = 2$ that passes through the point $(1, 2)$. Graph the line.

   Solution:

   First we find the slope of the line given:

$$\begin{aligned} 2x - y &= 2 \\ -y &= -2x + 2 \\ y &= 2x - 2 \end{aligned}$$

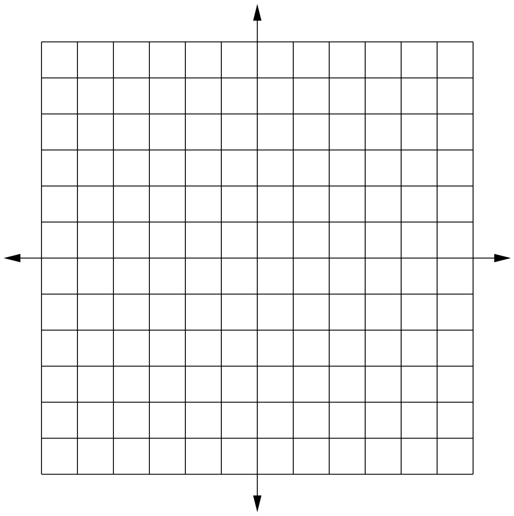
   So the slope is $m = 2$. The perpendicular line slope is $m_\perp = -\frac{1}{2}$. To find the equation of the perpendicular line, we'll use slope-intercept form and plug in to get b:

$$\begin{aligned} y &= -\frac{1}{2}x + b \\ 2 &= -\frac{1}{2}(1) \\ 2 &= -\frac{1}{2} + b \\ \frac{5}{2} &= b \end{aligned}$$

   So the perpendicular line is

$$y = -\frac{1}{2}x + \frac{5}{2} \qquad (1)$$

   To graph, since we know the slope is $-\frac{1}{2}$, we can just start at $(1, 2)$ then go over 2 and down 1, to $(3, 1)$, and again, to $(5, 0)$. Connecting these points gives the line:



3. A coin collector has a collection that includes buffalo nickels worth \$15 each and Eisenhower dollars worth \$5 each. If the collection contains 22 coins and has a total value of \$190, how many of each type of coin does it contain?

   Solution:

   Let b be the number of buffalo nickels. Since there are 22 coins in total, there must be $22 - b$ Eisenhower dollars. Making a chart:

Type of coin	Number of coins	Value per coin	Total value
Buffalo nickel	b	15	$15b$
Eisenhower dollar	$22 - b$	5	$5(22 - b)$

   The total value is \$190, so we can write an equation:

$$\begin{aligned} 15b + 5(22 - b) &= 190 \\ 15b + 110 - 5b &= 190 \\ 10b + 110 &= 190 \\ 10b &= 80 \\ b &= 8 \end{aligned}$$

   So there are $b = 8$ buffalo nickels and $22 - b = 14$ Eisenhower dollars.

4. A window pane in the shape of a right triangle has a hypotenuse that is 3 meters long. If the window's base is 2 meters greater than its height, find the base and height of the window.

   Solution:

   Let the h be the height of the window, so the base is $h + 2$, as shown below.

   Using the Pythagorean Theorem, we know that $\text{hypotenuse}^2 = \text{base}^2 + \text{height}^2$, so we can write the equation

$$h^2 + (h + 2)^2 = 3^2$$

$$
\begin{aligned}
h^2 + h^2 + 4h + 4 &= 9 \\
2h^2 + 4h - 5 &= 0 \\
h &= \frac{-4 \pm \sqrt{4^2 - 4(2)(-5)}}{2(2)} \\
&= \frac{-4 \pm \sqrt{56}}{4} \\
&= -1 \pm \frac{2\sqrt{14}}{4} \\
&= -1 \pm \frac{\sqrt{14}}{2}
\end{aligned}
$$

The height must be positive, which is possible only if we choose the positive root. Thus, the height is $h = -1 + \sqrt{14}/2$ meters. The base is 2 meters greater, so it is $h + 2 = 1 + \sqrt{14}/2$ meters.

5. Solve:

$$\frac{1}{x-1} + \frac{6}{x+1} = 3.$$

Solution:

$$
\begin{aligned}
\frac{1}{x-1} + \frac{6}{x+1} &= 3 \\
(x+1)(x-1)\left[\frac{1}{x-1} + \frac{6}{x+1}\right] &= (x+1)(x-1)(3) \\
1(x+1) + 6(x-1) &= 3(x^2 - 1) \\
x + 1 + 6x - 6 &= 3x^2 - 3 \\
7x - 5 &= 3x^2 - 3 \\
3x^2 - 7x + 2 &= 0 \\
(3x-1)(x-2) &= 0 \\
3x - 1 = 0 \quad &\text{or} \quad x - 2 = 0 \\
x = \frac{1}{3} \quad &\text{or} \quad x = 2
\end{aligned}
$$

6. A toy manufacturing company sells its toys for \$52 each. On a day where the company sells n toys, it costs the company $\$100 + n^2$ to produce the toys.

   (a) Graph the company's net profit versus the number of toys it sells on a given day. (Hint: try graphing for up to about 50 toys.)

   (b) What number of daily toy sales will give the company its maximum profit?

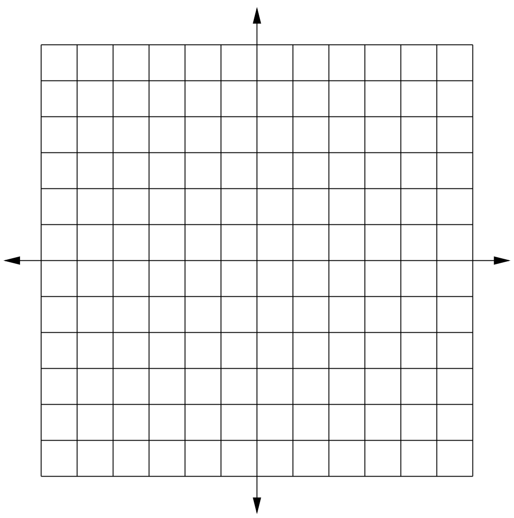
   (c) Find the minimum and maximum number of toys the company can sell in a day and still make a profit.

Solution:

(a) The company's net profit is its income minus its costs. Its income is $\$52n$, and its costs are $\$100 + n^2$, so its net profit is $-n^2 + 52n - 100$ dollars. To graph this, we'll pick some numbers

n	$-n^2 + 52n - 100$
0	-100
10	320
20	540
30	560
40	380
50	0

Graphing:



(b) This is a parabola, and we're trying to find its maximum. The maximum occurs at

$$n = -\frac{b}{2a} = -\frac{52}{2(-1)} = 26 \tag{2}$$

The company will have maximum profit from sales of 26 toys per day.

(c) To find the minimum and maximum values for which the company will make a profit, we set the profit equal to zero and solve:

$$\begin{aligned} -n^2 + 52n - 100 &= 0 \\ n^2 - 52n + 100 &= 0 \\ (n - 50)(n - 2) &= 0 \\ n = 50 \quad &\text{or} \quad n = 2 \end{aligned}$$

Thus, the company must sell more than 2 and less than 50 toys to make a profit. For exactly 2 or 50 toys it breaks even.

7. Evaluate: $64^{-4/3}$.

Solution:

$$64^{-4/3} = 4^{-4} = \frac{1}{4^4} = \frac{1}{256} \tag{3}$$

8. Solve: $3a - b = -1$ and $2b = a - 3$.

Solution:

$$\begin{aligned}
3a - b &= -1 \\
2b &= a - 3 \\
-a + 2b &= -3 \qquad \text{(rearranging second equation)} \\
2(3a - b) &= 2(-1) \qquad \text{(multiplying first equation by 2)} \\
6a - 2b &= -2 \\
+\underline{(-a + 2b)} &= +\underline{(-3)} \qquad \text{(adding third equation)} \\
5a &= -5 \\
a &= -1 \\
3(-1) - b &= -1 \qquad \text{(substituting into original equation)} \\
-3 - b &= -1 \\
-b &= 2 \\
b &= -2
\end{aligned}$$

So the solution is $a = -1$, $b = -2$.