Exam 3 Solutions

Math 135: Intermediate Algebra

1. Solve: $2x^2 - 4x - 1 = 0$

   Solution:

$$\begin{aligned} 2x^2 - 4x - 1 &= 0 \\ x &= \frac{4 \pm \sqrt{(-4)^2 - 4(2)(-1)}}{2(2)} \\ &= \frac{4 \pm \sqrt{24}}{4} \\ &= \frac{4 \pm 2\sqrt{6}}{4} \\ &= 1 \pm \frac{\sqrt{6}}{2} \end{aligned}$$

2. Consider the function $y = \frac{1}{2}x^2 - 2x + 1$.

   (a) Graph this function.

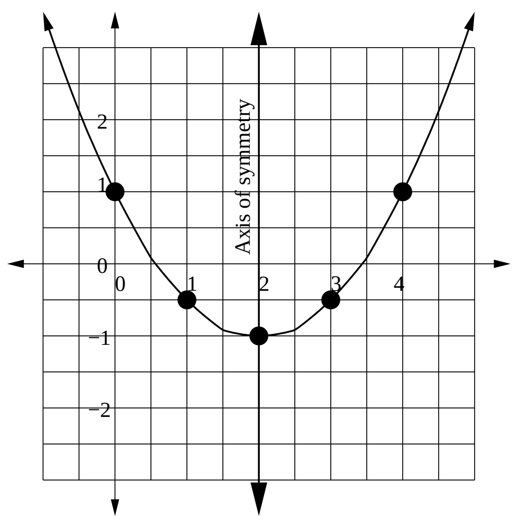
   (b) Find the coordinates of the vertex and mark it on your graph.

   (c) Find the axis of symmetry and draw it on your graph.

   Solution:

   (a) Picking a few points to graph:

x	y
0	$y = \frac{1}{2}(0)^2 - 2(0) + 1 = 1$
1	$y = \frac{1}{2}(1)^2 - 2(1) + 1 = -\frac{1}{2}$
2	$y = \frac{1}{2}(2)^2 - 2(2) + 1 = -1$
3	$y = \frac{1}{2}(3)^2 - 2(3) + 1 = -\frac{1}{2}$
4	$y = \frac{1}{2}(4)^2 - 2(4) + 1 = 1$



   (b) The vertex is at $x = -\frac{b}{2a}$, so $x = -\frac{(-2)}{2(1/2)} = 2$. The corresponding y is $y = \frac{1}{2}(2)^2 - 2(2) + 1 = -1$, so the vertex is $(2, -1)$.

   (c) The axis of symmetry is the vertical line passing through the vertex. This is $x = 2$, and it is drawn in the graph shown for part (a).

3. Evaluate: $81^{-3/4}$

   Solution:

$$81^{-3/4} = \frac{1}{81^{3/4}} = \frac{1}{3^3} = \frac{1}{27}$$

4. Solve:

$$\frac{2}{x-2} + \frac{4}{x+1} = 3$$

   Solution:

$$\begin{aligned}
(x-2)(x+1)\left(\frac{2}{x-2} + \frac{4}{x+1}\right) &= 3(x-2)(x+1) \\
2(x+1) + 4(x-2) &= 3(x^2 - x - 2) \\
2x + 2 + 4x - 8 &= 3x^2 - 3x - 6 \\
0 &= 3x^2 - 9x \\
0 &= 3x(x-3) \\
x = 0 \quad &\text{or} \quad x = 3
\end{aligned}$$

   Checking:

$$\begin{array}{rl}
x = 0 & x = 3 \\
\frac{2}{0-2} + \frac{4}{0+1} & \frac{2}{3-2} + \frac{4}{3+1} \\
= \frac{2}{-2} + \frac{4}{1} & = \frac{2}{1} + \frac{4}{4} \\
= -1 + 4 & = 2 + 1 \\
= 3 & = 3
\end{array}$$

   Both solutions check.

5. Solve: $2x - y = 7$ and $3x + 4y = -6$.

   Solution:

$$\begin{aligned}
4(2x - y) &= 4(7) && \text{(Multiply first equation by 4)} \\
8x - 4y &= 28 \\
+\underline{(3x + 4y)} &= +\underline{(-6)} && \text{(Add second equation)} \\
11x &= 22 \\
x &= 2 && \text{(Solution for } x\text{)} \\
2(2) - y &= 7 && \text{(Substitute } x \text{ back into first equation)} \\
4 - y &= 7 \\
-y &= 3 \\
y &= -3
\end{aligned}$$

6. A farmer is planning to fence a pasture for cows. The pasture will be a square x meters by x meters. Each month the farmer expects to make \$0.10 per square meter of pasture from his cows, to spend \$10 per meter of fencing for upkeep, and to pay fixed costs of \$5000 regardless of how big the pasture is.

   (a) Write down a function giving the farmer's net monthly income in terms of x.

   (b) Graph the function.

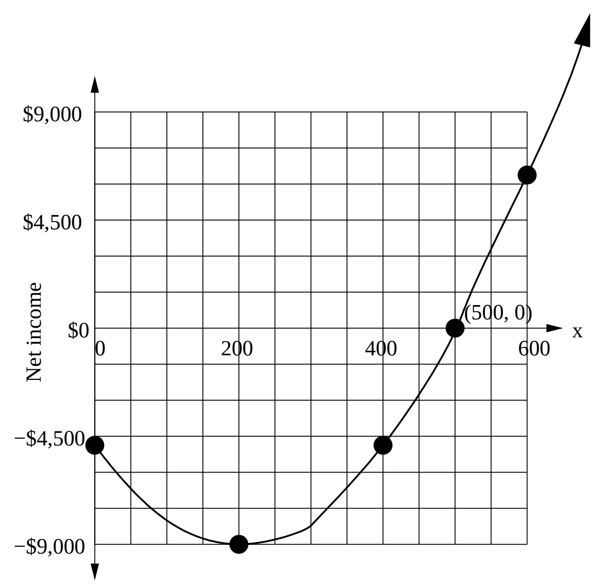
   (c) Find the minimum pasture size required for the farmer to make a profit. Indicate this point on your graph.

Solution:

(a) The pasture is a square of x by x meters, so the area of the pasture is x^2 square meters, and the farmer's total income is $0.1x^2$, since it's \$0.10 per square meter. The pasture has 4 sides of length x, so it's perimeter is $4x$ meters, which costs \$10 per meter, a total of $40x$, for upkeep. There is also the \$5000 fixed cost. Thus, the total net income is $0.1x^2 - 40x - 5000$.

(b) Picking a few points:

x	Income
0	-5000
200	-9000
400	-5000
600	7000



(c) For the farmer to earn a profit, his net income must be 0 or greater, so we solve

$$\begin{aligned} 0.1x^2 - 40x - 5000 &= 0 \\ x^2 - 40x - 50000 &= 0 \\ (x-500)(x+100) &= 0 \\ x = 500 \quad &\text{or} \quad x = -100 \end{aligned}$$

Since only the positive solution makes sense, we take $x = 500$. The pasture must be at least 500 meters in size for the farmer to make a profit. The corresponding point on the graph is $(500, 0)$.

7. A coast-to-coast airplane trip takes 5 hours heading East, with the wind, and 6 hours heading West, against the wind. If the trip is 3000 miles each way, find the speed of the wind and the speed of the plane in still air.

Solution:

Let v be the speed of the plane in still air and w be the speed of the wind. With the wind the speed is $v + w$, and against the wind it is $v - w$. Making a table:

Direction	Speed	Time	Distance
East	$v + w$	5	$5(v + w)$
West	$v - w$	6	$6(v - w)$

Since Distance = Speed × Time, we can write two equations:

$$\begin{aligned} 5(v+w) = 3000 \quad &\Rightarrow \quad v + w = 600 \\ 6(v+w) = 3000 \quad &\Rightarrow \quad v - w = 500 \end{aligned}$$

Solving:

$$\begin{aligned} v + w &= 600 \\ +\underline{(v - w)} &= +\underline{500} \qquad \text{(Adding the two equations)} \\ 2v &= 1100 \\ v &= 550 \\ 550 + w &= 600 \qquad \text{(Substituting } v \text{ into first equation)} \\ w &= 50 \end{aligned}$$

So the speed of the plane in still air is 550 miles per hour, and the speed of the wind is 50 miles per hour.

8. A coffee store owner has two types of coffee beans: a Kona bean and a Brazillian bean. He mixes them to make a breakfast blend, which is 40% Kona and 60% Brazillian, and a house blend, which is 10% Kona and 90% Brazillian. If a 10 ounce bag of the breakfast blend is \$10.50 and a 10 ounce bag of the house blend is \$8.25, find the price per ounce for pure Kona beans and for pure Brazillian beans.

Solution:

Let k and b be the price per pound of the Kona and Brazillian beans. A 10 ounce bag of the breakfast blend contains 4 ounces of Kona and 6 of Brazillian, while the house blend contains 1 ounce of Kona and 9 of Brazillian. We can therefore make a table:

	Kona			Brazillian			
Bl;end	Oz	Cost/oz	Cost	Oz	Cost/oz	Cost	Total cost
Breakfast	4	k	$4k$	6	b	$6b$	$4k + 6b$
House	1	k	k	9	b	$9b$	$k + 9b$

Since we are given the cost of a bag of each blend, we can write two equations:

$$\begin{aligned} 4k + 6b &= 10.5 \\ k + 9b &= 8.25. \end{aligned}$$

Now we just need to solve:

$$
\begin{array}{rcll}
4(k+9b) & = & 4(8.25) & \text{(Multiplying second equation by 4)} \\
4k+36b & = & 33 & \\
-\underline{(4k+6b)} & = & -\underline{10.5} & \text{(Subtracting first equation)} \\
30b & = & 22.5 & \\
b & = & 0.75 & \\
4k+6(0.75) & = & 10.5 & \text{(Substituting into first equation)} \\
4k+4.5 & = & 10.5 & \\
4k & = & 6 & \\
k & = & 1.5 &
\end{array}
$$

So the Kona beans cost \$1.50 per ounce, and the Brazillian beans cost \$0.75 per ounce.