Exam 1 Solutions

Math 135: Intermediate Algebra

1. Solve:

$$\frac{x-2}{3} - \frac{2-x}{5} = 3$$

Solution:

$$\begin{aligned}
15\left(\frac{x-2}{3} - \frac{2-x}{5}\right) &= 15\,(3) \\
5(x-2) - 3(2-x) &= 45 \\
5x - 10 - 6 + 3x &= 45 \\
8x - 16 &= 45 \\
8x &= 61 \\
x &= \frac{61}{8}
\end{aligned}$$

2. An investor put part of his money in a savings account at an annual return of 3% and the rest in a mutual fund at an annual return of 8%. He put a total of $5,000 into the two accounts. At the end of the year, the investor had $5,240. How much money did he invest in each of the two accounts?

Solution:

Let x be the amount of money he put in savings. Then $5000-x$ is the amount invested in the mutual fund. Making a table:

Investment	Amount	Interest Rate	Total Profit
Savings	x	0.03	$0.03x$
Mutual Fund	$5000-x$	0.08	$0.08(5000-x)$

He made a total profit of $240, so we can write an equation and solve for x:

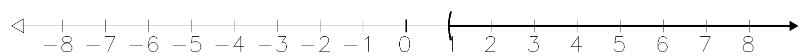
$$\begin{aligned}
0.03x + 0.08(5000-x) &= 240 \\
0.03x + 400 - 0.08x &= 240 \\
-0.05x + 400 &= 240 \\
-0.05x &= -160 \\
x &= 3200
\end{aligned}$$

Therefore the investor put $3,200 in savings and $1,800 in the mutual fund.

3. Solve and graph the solution on a number line: $2 - 6x < -4$.

Solution:

$$\begin{aligned} 2-6x &< -4 \\ -6x &< -6 \\ x &> 1 \end{aligned}$$

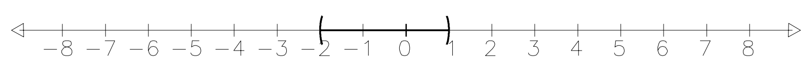


4. Solve, write your answer in interval notation, and graph the solution on a number line: $3x + 2 < 5$ and $8 - 2x < 12$.

Solution:

$$\begin{array}{rcl} 3x+2<5 & \text{and} & 8-2x<12 \\ 3x<3 & \text{and} & -2x<4 \\ x<1 & \text{and} & x>-2 \\ & -2<x<1 & \end{array}$$

In interval notation this is $(-2, 1)$.



5. The formula for converting a Fahrenheit temperature to a Celsius temperature is $C = \frac{5}{9}(F - 32)$. A certain chemical reaction can only take place at a temperature below 20 C. For what Fahrenheit temperatures can the reaction take place?

Solution:

The Celsius temperature must be less than 20 C, so the equation is:

$$\begin{aligned} C &< 20 \\ \frac{5}{9}(F-32) &< 20 \\ \frac{9}{5}\left[\frac{5}{9}(F-32)\right] &< \frac{9}{5}(20) \\ F - 32 &< 36 \\ F &< 68 \end{aligned}$$

The Fahrenheit temperature must be below 68 F.

6. (a) Find the slope and intercepts of the line $3x + y = 6$.

   (b) Graph the line.

Solution:

(a) Writing the line in slope intercept form:

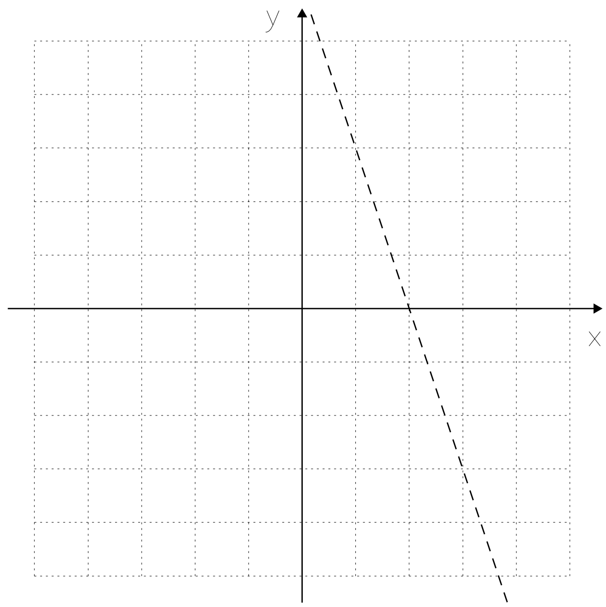
$$\begin{aligned} 3x + y &= 6 \\ y &= -3x + 6 \end{aligned}$$

From this we immediately see that the slope is -3 and the y intercept is $(0, 6)$. To get the x intercept, set $y = 0$ and solve:

$$\begin{aligned} 0 &= -3x + 6 \\ 3x &= 6 \\ x &= 2 \end{aligned}$$

So the x intercept is $(2, 0)$.

(b) The graph is (each division is 1 unit):



7. A surveyor is laying out a new path on a college campus. She uses a coordinate system where x is the distance East of the central fountain (in meters) and y is the distance North of the fountain. The path runs between Abel Hall, which is at 10 meters North, -20 meters East, and Baker Hall, which is at 30 meters North, -30 meters East.

   (a) Find the distance between the two buildings.

   (b) Find the midpoint between them.

   (c) Plot the positions of the two buildings and the midpoint between them. Be sure to indicate on your graph which point is which.

Solution:

(a) The points are at $(-20, 10)$ meters and $(-30, 30)$ meters; note that East is the x direction and North is the y direction. Therefore the distance is

$$
\begin{aligned}
d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
&= \sqrt{[-30 - (-20)]^2 + (30 - 10)^2} \\
&= \sqrt{(-10)^2 + 20^2} \\
&= \sqrt{100 + 400} \\
&= \sqrt{500} \\
&= 10\sqrt{5}
\end{aligned}
$$

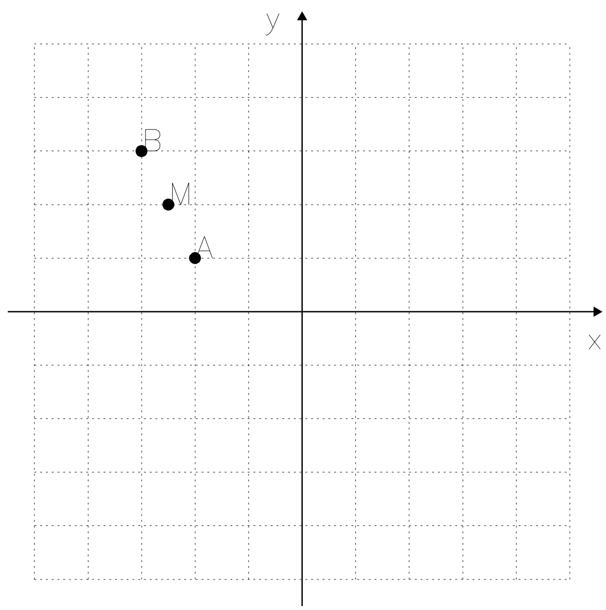
Therefore the distance is $10\sqrt{5}$ meters.

(b) The midpoint is

$$\begin{aligned} \text{midpoint} &= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \\ &= \left(\frac{-20-30}{2}, \frac{10+30}{2}\right) \\ &= \left(\frac{-50}{2}, \frac{40}{2}\right) \\ &= (-25, 20) \end{aligned}$$

The midpoint is at -25 meters East, 20 meters North.

(c) The graph is (each division is 10 m. A = Abel Hall, B = Baker Hall, M = midpoint):



8. (a) Find the equation of the line perpendicular to $y = \frac{2}{3}x$ that passes through the point $(2, 1)$.

(b) Graph the line.

Solution:

(a) The slope of the line we are given is $m = \frac{2}{3}$, so the slope of the perpendicular line is

$$m_{\text{perp}} = -\frac{1}{m} = -\frac{3}{2}$$

The equation is therefore $y = -\frac{3}{2}x + b$, and we need to find b for the point we are given. Plugging in:

$$\begin{aligned} 1 &= -\frac{3}{2}(2) + b \\ 1 &= -3 + b \\ b &= 4 \end{aligned}$$

So the equation is $y = -\frac{3}{2}x + 4$.

(b) The graph is (each division is one unit):