Exam II Solutions

Math 135: Intermediate Algebra

1. Multiply: $(2x^4 - x^3 + 3)(x+5)$.

   Solution:

$$\begin{aligned}(2x^4 - x^3 + 3)(x+5) &= 2x^5 + 10x^4 - x^4 - 5x^3 + 3x + 15 \\ &= 2x^5 + 9x^4 - 5x^3 + 3x + 15\end{aligned}$$

2. Divide: $(4x^4 - 1) \div (x - 1)$

   Solution:

$$\begin{array}{r l}
 & 4x^3 + 4x^2 + 4x + 4 \\ \cline{2-2}
x - 1) & 4x^4 \qquad\qquad\qquad\quad - 1 \\
 & -\,4x^4 + 4x^3 \\ \cline{2-2}
 & \qquad 4x^3 \\
 & \qquad -\,4x^3 + 4x^2 \\ \cline{2-2}
 & \qquad\qquad 4x^2 \\
 & \qquad\qquad -\,4x^2 + 4x \\ \cline{2-2}
 & \qquad\qquad\qquad 4x - 1 \\
 & \qquad\qquad\qquad -\,4x + 4 \\ \cline{2-2}
 & \qquad\qquad\qquad\qquad 3
\end{array}$$

   So the solution is

$$4x^3 + 4x^2 + 4x + 4 + \frac{3}{x-1}$$

3. Factor: $x^2 - 3x - 18$

   Solution:

$$x^2 - 3x - 18 = (x+3)(x-6)$$

4. Solve: $10x^2 - 11x + 3 = 0$

   Solution:

$$\begin{aligned}10x^2 - 11x + 3 &= 0 \\ (5x-3)(2x-1) &= 0 \\ 5x - 3 = 0 \quad &\text{or} \quad 2x - 1 = 0 \\ 5x = 3 \quad &\text{or} \quad 2x = 1 \\ x = \frac{3}{5} \quad &\text{or} \quad x = \frac{1}{2}\end{aligned}$$

5. Solve: $(x+3)^2 = 5(2x+1)$

   Solution:

$$\begin{aligned}(x+3)^2 &= 5(2x+1) \\ x^2 + 6x + 9 &= 10x + 5\end{aligned}$$

$$\begin{aligned} x^2 - 4x + 4 &= 0 \\ (x-2)^2 &= 0 \\ x = 2 & \end{aligned}$$

6. A city planner is designing a park which will be 200 meters longer than it is wide. If the park is to be $240,000$ square meters in area, what are its length and width?

   Solution:

   Let the width be x. In that case the length is $x + 200$. The area of the park is its length times its width, so we can write an equation:

$$\begin{aligned} x(x+200) &= 240,000 \\ x^2 + 200x - 240,000 &= 0 \\ (x-400)(x+600) &= 0 \\ x - 400 = 0 \quad &\text{or} \quad x + 600 = 0 \\ x = 400 \quad &\text{or} \quad x = -600 \end{aligned}$$

   The width must be positive, so $x = 400$ is the correct solution. The park is 400 meters wide and $400 + 200 = 600$ meters long.

7. A man standing on the top of a 90 meter high cliff throws a rock straight upward at 15 meters per second. The rock goes up some distance, then falls back to the base of the cliff. Its height *relative to the height from which it was thown* after t seconds is given by $-5t^2 + 15t$ meters. Find:

   (a) How long after the rock is thrown does it pass the man standing on top of the cliff?

   (b) How long after the rock is thrown does it hit the ground at the base of the cliff?

   Solution:

   (a) To pass the man on top of the cliff, the rock must be at the same height from which it was thrown. In other words, its height above its starting position must be 0. Therefore we want

$$\begin{aligned} -5t^2 + 15t &= 0 \\ -5t(t-3) &= 0 \\ -5t = 0 \quad &\text{or} \quad t - 3 = 0 \\ t = 0 \quad &\text{or} \quad t = 3 \end{aligned}$$

   The solution $t = 0$ corresponds to when the rock is first released, at 0 seconds. It's not the one we want. We want the solution $t = 3$: the rock passes the man after 3 seconds.

(b) To hit the cliff bottom, the height must be -90 meters. Therefore we want

$$\begin{aligned}
-5t^2 + 15t &= -90 \\
-5t^2 + 15t + 90 &= 0 \\
t^2 - 3t - 18 &= 0 \\
(t-6)(t+3) &= 0 \\
t - 6 = 0 \quad &\text{or} \quad t + 3 = 0 \\
t = 6 \quad &\text{or} \quad t = -3
\end{aligned}$$

Since the rock can only hit the bottom after it is thrown, the positive solution, $t = 6$, is the one we want. The rock hits the ground 6 seconds after it was thrown.

8. If a bank account earns interest at an annual rate r, to figure out how much it is worth after a year you multiply its original value by $(r+1)$. For example, an account with a starting value of \$500 earning interest at a rate of 4% would be worth $500(0.04+1) = 500(1.04) = 520$ dollars at the end of a year.

   (a) Suppose an account with a starting value of \$1,000 were worth \$1,050 at the end of a year. Find the interest rate.

   (b) Suppose an account with a starting value of \$1,000 were worth \$1,210 at the end of *two* years. Find the interest rate. (Hint: remember that in the second year, the account earns interest on its value at the end of the first year, not its value at the beginning.)

   Solution:

   (a) For this part, we just have to solve the equation we were given:

$$\begin{aligned}
1000(r+1) &= 1050 \\
1000r + 1000 &= 1050 \\
1000r &= 50 \\
r &= 0.05
\end{aligned}$$

   So the interest rate is 0.05, or 5%.

   (b) The value of the account after one year is $1000(r+1)$. In the second year, the value gets multiplied by $(r+1)$ again, so its value becomes $1000(r+1)^2$. Therefore we have

$$\begin{aligned}
1000(r+1)^2 &= 1210 \\
1000(r^2 + 2r + 1) &= 1210 \\
1000r^2 + 2000r + 1000 &= 1210 \\
1000r^2 + 2000r - 210 &= 0 \\
100r^2 + 200r - 21 &= 0 \\
(10r - 1)(10r + 21) &= 0
\end{aligned}$$

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\begin{aligned}
10r - 1 &= 0 \quad \text{or} \quad 10r + 21 = 0 \\
10r &= 1 \quad \text{or} \quad 10r = -21 \\
r &= 0.1 \quad \text{or} \quad r = -2.1
\end{aligned}
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Since banks don't normally offer accounts with negative interest rates, the positive solution is the one we want. The interest rate is $r = 0.1$, or 10%.