Pre-Algebra
Worksheet 3 Solutions

1. What fraction of an hour are each of the following? Express your answers in lowest terms, and as mixed numbers when appropriate.

(a) 30 minutes

There are 60 minutes in an hour, so 30 minutes is

$$\frac{30}{60} = \frac{30}{60} \div \frac{30}{30} = \frac{1}{2}$$

(b) 45 minutes

$$\frac{45}{60} = \frac{45}{60} \div \frac{15}{15} = \frac{3}{4}$$

(c) 150 minutes

$$\begin{aligned} \frac{150}{60} &= \frac{150}{60} \div \frac{10}{10} \\ &= \frac{15}{6} \\ &= \frac{15}{6} \div \frac{3}{3} \\ &= \frac{5}{2} = 2\frac{1}{2} \end{aligned}$$

(d) 1 hour and 51 minutes

$$\begin{aligned} & 1\frac{51}{60} \\ = 1\frac{51}{60} &\div \frac{3}{3} \\ &= 1\frac{17}{20} \end{aligned}$$

(e) 30 seconds

There are 60 seconds in a minute, so (60×60) 3600 seconds in an hour

$$\begin{aligned} & \frac{30}{3600} \\ &= \frac{3}{360} \\ &= \frac{1}{120} \end{aligned}$$

2. What fraction of a dollar are each of the coins: the dollar coin, the quarter, the dime, the nickel and the cent?

A dollar is worth 100 cents.

(a) a dollar coin is worth $1/1 = \$1$

(b) a quarter is worth 25 cents. This is

$$\begin{aligned} &\frac{25}{100} \\ =&\frac{5}{20} \\ =&\frac{1}{4} \end{aligned}$$

A quarter is worth 1/4 of a dollar (which is why, of course, it's called a quarter).

(c) a dime is worth 10 cents This is

$$\begin{aligned} &\frac{10}{100} \\ =&\frac{1}{10} \end{aligned}$$

A dime is worth 1/10 of a dollar.

(d) a nickel is worth 5 cents. This is

$$\begin{aligned} &\frac{5}{100} \\ =&\frac{1}{20} \end{aligned}$$

A nickel is worth 1/20 of a dollar

(e) One cent is worth 1/100 of a dollar.

3. Convert the following mixed numbers to fractions in lowest terms.

(a) $1\frac{2}{3}$

Using the formula in Section 2.3, $1\frac{2}{3}$

$$= \frac{(1)(3)+1}{3} = \frac{5}{3}$$

(b) $-3\frac{3}{4}$

$$= -\frac{(3)(4)+3}{4} = -\frac{15}{4}$$

(c) $5\frac{5}{8}$

$$= \frac{(5)(8)+5}{8} = \frac{40+5}{8} = \frac{45}{8}$$

(d) $2\frac{1}{2}$

$$= \frac{(2)(2)+1}{2} = \frac{5}{2}$$

(e) $-4\frac{4}{6}$

$$= -\frac{(4)(6)+4}{6} = -\frac{28}{6} = -\frac{14}{3}$$

4. Put the following fractions in lowest terms.

(a) $\frac{16}{18}$

$$= \frac{8}{9}$$

(divide numerator and denominator by 2)

(b) $\frac{2}{3}$

This fraction is already in its lowest terms.

(c) $\frac{50}{100}$

$$= \frac{1}{2}$$

(divide numerator and denominator by 50. Or, in succession, by 2, 5 and 5).

(d) $\frac{51}{99}$

$$= \frac{17}{33}$$

(divide numerator and denominator by 3).

(e) $\frac{11}{121}$

$$= \frac{1}{11}$$

(divide numerator and denominator by 11).

(f) $\frac{5}{6}$

This fraction is already in its lowest terms.

5. Fred's rent is \$600 per month, and his paycheck is \$1500 per month. What fraction of his paycheck goes to rent? Express your answer as a fraction in lowest terms.

As a fraction, Fred's rent is $\frac{600}{1500}$

$$\begin{aligned} &= \frac{600}{1500} \div \frac{100}{100} \\ &= \frac{6}{15} \\ &= \frac{2}{5} \end{aligned}$$

6. You can compare two fractions and find which has the greater value if they are both expressed over the same denominator. In the following pairs of fractions, identify which one is greater:

(a) $\frac{3}{5}$ and $\frac{4}{5}$

$\frac{3}{5} < \frac{4}{5}$

(b) $-\frac{3}{5}$ and $\frac{4}{5}$

A negative number is always less than a positive one, so $-\frac{3}{5} < \frac{4}{5}$

(c) $\frac{3}{5}$ and $-\frac{4}{5}$

A negative number is always less than a positive one, so $\frac{3}{5} > -\frac{4}{5}$

(d) $-\frac{3}{5}$ and $-\frac{4}{5}$

Here, both numbers are negative. $-\frac{4}{5}$ is further to the left on the number lines so it is the smaller number. $-\frac{3}{5} > -\frac{4}{5}$

7. Explain in your own words what it means to say that the rational numbers are "densely packed". Sketch an example of what this means on a number line.

Between any two rational numbers, no matter how close in value, you can always find a third rational number whose value lies in between.

8. Arnold eats $\frac{3}{7}$ of a cake and Bob eats $\frac{2}{5}$ of a cake.

(a) Who ate more cake?

Which is larger, $\frac{3}{7}$ or $\frac{2}{5}$? First express these fractions using a common denominator

$$\frac{3}{7} \times \frac{5}{5} = \frac{15}{35} \text{ and } \frac{2}{5} \times \frac{7}{7} = \frac{14}{35}$$

$$\frac{15}{35} > \frac{14}{35}$$

so $\frac{3}{7} > \frac{2}{5}$

(b) Find an amount of cake that is in between the amounts that Arnold and Bob ate.

$$\frac{14}{35} \times \frac{2}{2} = \frac{28}{70}$$
$$\frac{15}{35} \times \frac{2}{2} = \frac{30}{70}$$

The rational number $\frac{29}{70}$ is between $\frac{3}{7}$ and $\frac{2}{5}$.

9. A first grade class took a poll to find out their favorite ice cream. 1/4 chose chocolate, 1/4 chose vanilla and 1/2 chose strawberry. 2 kids are lactose intolerant and can't eat ice cream. If there are 22 kids in the class, how many kids liked each flavor?

If two children didn't make a choice, then $22 - 2 = 20$ children responded.

$$20 \times \frac{1}{4} = \frac{20}{1} \times \frac{1}{4} = \frac{20 \cdot 1}{1 \cdot 4} = \frac{20}{4} = 5$$

5 children chose chocolate. Repeating the calculation for the other flavors shows that 5 children also chose vanilla, but 10 chose strawberry.