Pre-Algebra Worksheet 2 Factors: Answers

1. Which of the following statements are true?
   (a) 3 is a factor of 18. (b) 3 is a multiple of 18. (c) 18 is a multiple of 3. (d) 27 has 7 as a factor. (e) 35 has 5 as a factor. (f) 12 has -3 and -2 as factors.

   Answer:
   (a) 3 is a factor of 18: $3 \times 6 = 18$

   (b) 3 is not a multiple of 18 because 3 divided by 18 is not a whole number with no remainder

   (c) 18 is a multiple of 3 because 18 is divisible by 3 with no remainder: $18 \div 3 = 6$

   (d) 7 is not a factor of 27: $27 \div 7 = 3$ with remainder 6

   (e) 5 is a factor of 35: $35 \div 5 = 7$ with no remainder

   (f) -3 and -2 are both factors of 12: $12 \div -3 = -4$, and $12 \div -2 = -6$.

2. A *perfect number* is equal to the sum of all of its positive factors other than itself (all factors, not just prime factors). For example, 6 is perfect because its positive factors are $1, 2, 3, 6$, and $1 + 2 + 3 = 6$. The next perfect number after 6 is between 20 and 30. What is it?

   Answer:

   Let's look at the factors of the numbers from 20 to 30, excluding the number itself as a factor:

   20: factors 1,2,4,5,10: sum of factors $1+2+4+5+10 = 22$

   21: factors 1,3,7: sum $= 11$

22: factors 1,2,11: sum=14

23: factors 1; sum = 1 (23 is a prime number)

24: factors 1,2,3,4,6,8,12: sum = 35

25: factors 1,5: sum = 6

26: factors 1,2,13: sum = 16

27: factors 1,3,9: sum = 13

28: factors 1,2,4,7,14: sum = 28

29: factors 1: sum = 1 (29 is a prime number)

30: factors 1,2,3,5,6,10,15: sum = 42

It's clear from the above that 28 is a *perfect number*.

3. Find the prime factorizations of the following numbers:
(a) 24 (b) 64 (c) 29 (d) 120 (e) 81 (f) 51

Answer:

$24 = 2\times2\times2\times3$

$64 = 2\times2\times2\times2\times2\times2$

29 is a prime number

$120 = 2\times2\times2\times3\times5$

$81 = 3 \times 3 \times 3 \times 3 \times 3$

$51 = 3 \times 17$ (check for divisibility by 3: 5+1 =6, so 51 is divisible by 3)

4. Find all of the primes between 1 and 30.

   Answer:

   First, let's eliminate all of the obvious numbers.

   (a) 1 is not a prime number: see notes

   (b) No even number greater than 2 is prime, because it is divisible by 2. This leaves 2,3,5,7,9,11,13,15,17,19,21,23,25,27,29 as candidates

   (c) no number with its last digit 5 (except 5 itself of course) is prime because it is divisible by 5. This removes 15 and 25 from consideration

   (d) check for divisibility by 3 by adding the integers. 9 is divisible by 3; 15 is ($1 + 5 = 6$) and so on. Note that in the list under (b), every third number, starting at 3, is divisible by 3 (3,9,15,21,27) (and every fifth number is divisible by 5) - this is just the result of multiplication

   (e) removing the numbers divisible by 3 and 5 from the list leaves us with 2,3,5,7,11,13,17,19,23,29

   (f) So the first few prime numbers are 2,3,5,7. Try dividing the rest by 7 (nope), 11 (nope) and 13 (nope). Dividing by bigger primes would leave you with answers less than 1, so you're done. The prime numbers between 1 and 30 are 2,3,5,7,11,13,17,19,23 and 29.

5. Find the Greatest Common Factors of the following pairs of numbers, first using Method #1, and then using the Euclidean Algorithm: (a) (10,15) (b) (21,49)

Answer:

First, let's use Method #1 (prime factorization):

(a)
$10 = 2 \times 5$
$15 = 3 \times 5$

The largest subset of numbers common to both is 5. Thus the greatest common factor (GCF) of 10 and 15 is 5.

(b)
$21 = 3 \times 7$
$49 = 7 \times 7$

The GCF is 7.

Now let's use Euclid's method:

(a)
$15 \div 10 = 1$, remainder 5
$10 \div 5 = 2$, remainder 0

so the GCF of 15 and 10 is 5.

(b)
$49 \div 21 = 2$, remainder 7
$21 \div 7 = 3$, remainder 0

so the GCF of 21 and 49 is 7.

6. Find the Least Common Multiple of the number pairs in the previous problem

Answer:

Use the factors from Method #1:

(a)
10, 15: $2\times$ 5 and 3 $\times$ 5: $2\times3\times5 = 30$. The LCM of 10 and 15 is 30

(b)
21, 49: $3\times7\times7 = 147$. The LCM is 147

7. 48 boxes are to be stacked in a rectangular array a boxes wide, b boxes deep, and c boxes high. Find integers a, b, c that are as nearly equal to one another as possible. This will make a compact array.

   Answer:

   Let's look at the factors. If you pile the boxes in an array measuring $a \times b \times c$, the total number of boxes is $a \times b \times c$. The factors of 48 are: $2\times2\times2\times2\times3$. How do you combine these to make three numbers as close in size as possible? Make the factors $48 = 4\times4\times3$. The array of boxes in 4 wide, 4 deep and 3 high (or 4 wide, 3 deep and 4 high).