Pre-Algebra Homework 2 Factors: Solutions

1. Find all of the primes between 70 and 100.

   Answer:

   First, reject the obvious non-prime numbers. None of the even numbers can be a prime because they can be divided by two. None of the numbers whose last digit is 5 is prime because it divides by 5. Every third odd number (check them of course) can be divided by 3 (see Worksheet 2). So: for the odd numbers between 70 and 100:

   75, 85, 95 are divisible by 5

   75, 81, 87, 93, 99 are divisible by 3

   This leaves 71, 73, 77, 79, 83, 89, 91, 97. Every 7th odd number divides by 7 - these numbers are 77 and 91 ($77 = 7 \times 11$ and $91 = 7 \times 13$.) The next prime number is 11, but $97 \div 11 = 8$ plus remainder 9, i.e. the quotient is smaller than 11. So there are no more prime number factors. The prime numbers between 70 and 100 are **71, 73, 79, 83, 89, 97**.

2. *Goldbach's Conjecture* says that every even number greater than 2 is the sum of two primes, though they might be the same prime, and there might be more than one way to write a number as the sum of two primes. For example, $4 = 2 + 2$, $6 = 3 + 3$, $8 = 3 + 5$, and $10 = 5 + 5 = 3 + 7$. "Conjecture" is a fancy word for "guess": nobody knows whether Goldbach's Conjecture is true, but on the other hand nobody has ever found an exception. Check the Conjecture for even numbers up to 20.

   Answer:

   Work this out by subtracting primes one after the other and checking the answer to see if it too is prime. So let's look at the numbers between 12 and 20.

$12 = 2+10 = 3+9 = 5+7 = 7+5$
2 and 10 are not primes, 3 and 9 are not primes, but 5 and 7 are primes (and 5+7 is the same as 7+5). So

$12 = 5 + 7$

and likewise

$14 = 3 + 11 = 7 + 7$

$16 = 3 + 13 = 5 + 11$

$18 = 5 + 13 = 7 + 11$

$20 = 3 + 17$

so at least as far as 20, this Conjecture is correct.

3. Find the Greatest Common Factors of the following pairs of numbers, first using Method #1, and then using the Euclidean Algorithm: (a) (16,48) (b) (42,63) (c) (21,16) (d) (52,39)

   Answer:

   (a)
   $16 = 2\times2\times2\times2$
   $48 = 2\times2\times2\times2\times3$

   so the Greatest Common Factor (GCF) is $2\times2\times2\times2 = 16$

   (b)
   $42 = 2\times3\times7$
   $63 = 3\times3\times7$

so the GCF is $3 \times 7 = 21$

(c)
$21 = 3 \times 7$
$16 = 2 \times 2 \times 2 \times 2$

the GCF is 1, i.e. there isn't one

(d)
$52 = 2 \times 2 \times 13$
$39 = 3 \times 13$

CGF = 13

Now use Euclid's algorithm:

(a)
16, 48: $48 \div 16 = 3$, remainder 0
so GFC = 16

(b)
42, 63: $63 \div 42 = 1$, remainder 21
$42 \div 21 = 2$, remainder 0
So GCF = 21

(c)
21, 16: $21 \div 16 = 1$, remainder 7
$16 \div 7 = 2$, remainder 2
$7 \div 2 = 3$, remainder 1
$2 \div 1 = 2$, so we can't go further

(d)
39,52: $52 \div 39 = 1$, remainder 13
$39 \div 13 = 3$, remainder 0

so the GCF $= 13$

4. Find the Least Common Multiple of the number pairs in the previous problem

   Answer

   Use the prime factorization from the previous problem:

   (a)
   16,48: $16 = 2\times2\times2\times2$; $48 = 2\times2\times2\times2\times3$
   LCM $= 2\times2\times2\times2\times3 = 48$
   The lowest common multiple is 48

   (b)
   42,63; $42=2\times3\times7$; $63=3\times3\times7$
   LCM $= 2\times3\times3\times7 = 126$

   (c)
   16,21: $16=2\times2\times2\times2$; $21=3\times7$
   LCM $= 2\times2\times2\times2\times3\times7=336$

   (d)
   39,52: $39=3\times13$; $52=2\times2\times13$
   LCM $= 2\times2\times3\times13 = 156$

5. A mother wishes to divide 6 chocolate bars evenly among 4 children. What is the smallest total number of pieces needed, into how many pieces must each bar be broken, and how many pieces does each child receive?

   Answer:

   Each child can have one whole bar of chocolate, with 2 left over ($6\div 4 = 1$, remainder 2). To share the 2 remaining bars equally among the four children, each must be broken into two equal halves. There are four halves, so each child gets one. Each child gets 1 whole bar plus one half bar, two pieces for each child, 8 pieces in all. If all the pieces have to be the same size, look at the least common multiple of 6 and 4: $6 = 2\times3$, $4=2\times2$. The LCM is $2\times2\times3 = 12$. She breaks each bar into two equal pieces, 12 pieces in all, and shares them among the 4 children, who get 3 pieces each.

6. Calculate the prime factors of the following numbers
(a) 1620
(b) 375
(c) 289

Answer

(a)
1620 obviously divides by 10, leaving 162. Sum the integers to get $1+6+2 = 9$, which is divisible by 3. 162 is an even number, so must divide by 2. So far we have as prime factors 2,5 ($10 = 2\times 5$), 3, and 2. Divide 162 by 2 to get 81, and 81 by 3 to get 27. This is still divisible by 3. $27\div 3 = 9$. $9\div 3 = 3$. So the prime factors of 1620 are: $1620 = 2\times 2\times 3\times 3\times 3\times 3\times 5$.

(b)
$375 = 5\times 75 = 5\times 5\times 15 = 5\times 5\times 5\times 3$ (and you can check that 375 is divisible by 3 by adding the integers).

(c)
Add the integers to get 19. So 289 is not divisible by 2 (it's not an even number), by 3, or by 5 (it doesn't end in a 5). Try 7: nope. Try 11: nope. 13: nope. 17; yes. $289 = 17\times 17$.

7. President Obama holds a 4th July celebration at the White House for 1289 people. If the invitations come in boxes of 25, each of which costs $40, what is the cost of the invitations?

Answer
First, divide 1289 by 25 to find out how many boxes are needed: $1289\div 25 =$ 51 with 14 left over. So you have to buy 52 boxes to have enough invitations, which cost $52\times$ \$40 = \$2080.

8. Use Method #2 at the end of the class notes:

$$LCM(a,b) = \frac{a\times b}{GCF(a,b)}.$$

where a and b are two numbers, to calculate the LCM of
(a) 13,4
(b) 10,30
(c) 9,15

Answer

(a)
GCF of 13 and 4 is 1. So

$$LCM(13, 4) = \frac{13 \times 4}{1}$$

LCM = 13×4/1 = 52

(b)
The GCF of 20 and 30 is 10. So the LCM = (20×30)/10 = 60

(c)

The GCF of 9 and 15 is 3. So the LCM = 9×15/3 = 45

9. Which of the following numbers divides by 3 with no remainder: (a) 246,105 (b) 17 (c) -27 (d) 178,316,166 (e) 29,629,630

Answer

In each case, sum the integers.

(a)
2+4+6+1+5 = 18; 1+8 = 9, which is divisible by 3

(b)
1+7=8, which is not divisible by 3

(c)
2+7 = 9, which is divisible by 3 (note we just need to check the absolute value)

(d)
1+7+8+3+1+6+1+6+6 = 39; 3+9 = 12; 1+2=3. So 178,316,166 is divisible by 3

(e) 2+9+6+2+9+6+3+0 = 37; 3+7 = 10, which is not divisible by 3.

So the numbers in (a),(c) and (d) are divisible by 3, and the others aren't.

10. A school PE coach organizes the children into teams to play lacrosse, basketball, relay racing, and baseball. What is the smallest number of kids at the school which will allow the school to put each child in a team to play every sport? (there are 9 players on a baseball team, 4 on a relay racing team, 5 on a basketball team and 10 on a lacrosse team).

    Answer

    We want the lowest common multiple of 9, 4, 5 and 10. Do it in pairs. The LCM of 9 and 4 is 36 (with factors 2, 2, 3, and 3). The LCM of 36 and 5 is 180 (with factors 2,2,3,3,5). The factors of 10 are 2×5. So the LCM of these four numbers (9, 4, 5 and 10) is 180. If you have 180 kids at the school, each of them can be on a team to play all four sports.