Pre-Algebra Lecture 8: Powers, Exponents and Square Roots

Outline:

1. Definitions of **powers** and **exponential** expressions.
2. Product Rule for exponential expressions
3. Quotients Rule for exponential expressions
4. Negative exponents
5. Power of a product or quotient of exponential expressions
6. Scientific notation
7. The square root of a number

1 Definitions of powers and exponential expressions

Today we will learn about **powers** and **exponential expressions**.

- **Powers**

  A product in which the factors are identical is called a power of that factor. For instance, $32 = 2 \times 2 \times 2 \times 2 \times 2$, so 32 is the fifth power of 2.

- **Exponential expressions**

  Exponential expressions are just a way to write powers in short form. The exponent indicates the number of times the base is used as a factor. So in the case of 32 it can be written as $2 \times 2 \times 2 \times 2 \times 2 = 2^5$, where 2 is the "base" and 5 is the "exponent".

  We read this expression as "two to the fifth power".
  In general, we will have that $a^n = a \times a \times a ... \times a$ (n times),
  where a will be the **base** and n the **exponent**, and we will read it as "a to the nth power."
  Other examples in which exponents are used to show powers are given below.
  - 4^2 is four to the second power (or 4 squared) and means 4×4, or 16.
  - 10^3 is ten to the third power (or 10 cubed) and means $10 \times 10 \times 10$, or 1000.
  - a^6 is a to the sixth power and means $a \times a \times a \times a \times a \times a$.
  A number raised to the first power is that number. For example, $10^1 = 10$. To show the factors of a number expressed using exponents, you write the number in "expanded form", also called factored form.

2 Product Rule for exponential expressions

The product of two exponential expressions with the same base will be give by

$$\mathbf{a^m \times a^n = a^{m+n}}$$

For example,

$3^2 \times 3^3 = 3^{2+3} = 3^5$

$c^3 \times c \times c^4 = (c^3 \times c) \times c^4 = c^{3+1} \times c^4 = c^{3+1+4} = c^8$

We can also use the product rule to simplify expressions. For instance,

$2u^3x^26u^2x^{11} = 2 \times u^3 \times x^2 \times 6 \times u^2 \times x^{11} = (2 \times 6) \times (u^3u^2) \times (x^2x^{11}) = 12u^{2+3}x^{2+11} = 12u^5x^{13}$.

We do not add the exponents of u and x, since these are different bases unless $u = x$.

- **Definition**
  If $a \neq 0$ then we define $a^0 = 1$

This definition is needed in order to add exponents consistently. For example,

$a^2 = a^{2+0} = a^2a^0$ implies that $a^0 = 1$.

3 Quotients Rule for exponential expressions

If a is a non-zero real number, then the quotient of two exponential expressions with the same base is

$$\mathbf{\frac{a^m}{a^n} = a^{m-n}}$$

Let's use this rule to simplify a few expressions.

$$\frac{40w^{16}}{5w^{14}} = \frac{40}{5}w^{16-14} = 8w^2$$

$$\frac{10a^7b^5c^4}{15a^2b^5c^3} = \frac{10}{15}a^{7-2}b^{5-5}c^{4-3} = \frac{2}{3}a^5b^0c^1 = \frac{2}{3}a^5c$$

4 Negative Exponents

If a is a non-zero real number then

$$\mathbf{a^{-n} = \frac{1}{a^n}}$$

Let's use this rule to simplify a few expressions leaving no negative exponents. For example,

$$\frac{3^2}{3^8} = 3^{2-8} = 3^{-6} = \frac{1}{3^6}$$

$$\frac{12x^3y^9z^2}{3x^4y^5z^5} = \frac{12}{3}x^{3-4}y^{9-5}z^{2-5} = 4x^{-1}y^4z^{-3} = \frac{4y^4}{xz^3}$$

5 Power of a product or quotient of exponential expressions

If a and b are real numbers, then

$$\mathbf{(a^m)^n = a^{mn}}$$
$$\mathbf{(ab)^m = a^m b^m}$$
$$\mathbf{\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}}$$

Let's see some examples.
$(2^3)^2 = (2^3) \times (2^3) = (2 \times 2 \times 2) \times (2 \times 2 \times 2) = 2^6 = 2^{2\times 3}$

$(4 \times 5)^3 = (4 \times 5)(4 \times 5)(4 \times 5) = 4 \times 4 \times 4 \times 5 \times 5 \times 5 = 4^3 5^3$

$(\frac{3}{5})^2 = \frac{3}{5} \times \frac{3}{5} = \frac{3\times 3}{5\times 5} = \frac{3^2}{5^2}$

6 Scientific Notation

A positive number is written in scientific notation if it is written as the product of a number a, where $1 \leq a < 10$, and an integer power n of 10, i.e. $a \times 10^n$. For instance, the numbers 3.12×10^3, 1.27801×10^{-6} are examples of numbers written in scientific notation.
Here is how to write a number in scientific notation.

1. Move the decimal point in the original number until the new number has a value between 1 and 10.

2. Count the number of decimal places the decimal point was moved in step 1. If the original number is 10 or greater, the count is positive. If the original number is less than 1, the count is negative.

3. Write the product of the new number in step 1 by 10 raised to an exponent equal to the count found in step 2.

Let's see some examples:

$39.37 = 3.937 \times 10^1$	The number of inches in a meter.
$0.0254 = 2.54 \times 10^{-2}$	The number of meters in an inch.
$150,000,000 = 1.5 \times 10^8$	The distance from the Earth to the Sun in kilometers.
$0.000000589 = 5.89 \times 10^{-7}$	Wavelength of yellow light from a sodium street lamp, in meters.

When we multiply numbers in scientific notation, we add the powers of ten. For example, there are 3.15×10^7 seconds per year, and light travels 3.0×10^5 kilometers per second, so the distance traveled by light in one year (1 light-year) is

$(3.15 \times 10^7) \times (3.0 \times 10^5) = (3.15 \times 3.0) \times 10^{7+5} = 9.45 \times 10^{12}$ kilometers.

7 The square root of a number

Let a and b be real numbers and assume that b is nonnegative. Then we write $\sqrt{b} = a$ (read "the square root of b is a") if $a^2 = b$. (Why must b be nonnegative?)

For example,

$\sqrt{4} = 2$ since $2^2 = 4$.

$\sqrt{\frac{1}{16}} = \frac{1}{4}$ since $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$.