Pre-Algebra Lecture 6

Today we will discuss **Decimals and Percentages**.

Outline:

1. Decimals
2. Ordering Decimals
3. Rounding Decimals
4. Adding and subtracting Decimals
5. Multiplying and Dividing Decimals
6. Percentages
7. Percents of a number
8. Percents of an increase/decrease
9. Extra Problems

1 Decimals

Decimals represent a whole unit or a part of a whole number. A decimal number usually means there is a *decimal point* in the number. The decimal point is the most important part - it is exactly to the *right* of the units position and tells us what each position means.

Consider the decimal

$$17.591$$

The number to the left of the decimal point is the *whole number* (17 for this example).
As we move further left, every number place gets 10 times bigger.
As we move further right, every number place gets 10 times smaller.
So

$17.\mathbf{5}91$ The first digit on the right of the decimal point means tenths (1/10)

$17.59\mathbf{9}1$ The second digit on the right means hundredths (1/100)

$17.591\mathbf{1}$ The third digit on the right means thousandths (1/1000)

Another way to think about a decimal is as a whole number plus some tenths, and some hundredths etc... So we could re-write the number 17.591 as

$$17.591 = 17 + \frac{5}{10} + \frac{9}{100} + \frac{1}{1000} \tag{1}$$

So 17.591 is 17 and 5 tenths, 9 hundredths and 1 thousandth.
We refer to the number of digits after the decimal point as the *decimal places*. So 17.591 has 3 decimal places.

- **Example:** What is 13.76?

  Solution:

  On the left side, the ”13” is the whole number part.
  There are two digits on the right side - the ”7” is in the ”tenths” position, and the 6 is the ”hundredths” position.
  We can represent the decimal number in the following way:

Thousands	Hundreds	tens	units	.	tenths	hundredths	thousandths	ten-thousandths
		1	3	.	7	6		

  So, 13.76 is ”13 and 7 tenths and 6 hundredths”.

Let’s try another example.

- **Example:** Write 9 and 6 thousands as a decimal.

  Solution:
  9 is the whole number part so we know to write $9.$
  6 thousands is written as 0.006

Thousands	Hundreds	tens	units	.	tenths	hundredths	thousandths	ten-thousandths
			9	.	0	0	6	

  So, 9 and 6 thousands is 9.006.

Now you try the following:

1. Read the following decimal: 574.405
2. Write the decimal 7 thousand and five and two thousand two ten-thousandths.

2 Converting Decimals to Fractions

To illustrate how to convert decimals to fractions, let us start with 2.34 as a example.
When converting a decimal to a fraction, these steps should be followed:

1. You should start by splitting the decimal up into 2 parts: the part on the left of the decimal point and the part on the right.

The decimal 2.34 has 2 parts: the ”2” which is the number of whole units, and ”.34” which is the fractional part.

2. Identify the *number of decimal places* the fractional part has and convert it into a fraction with a denominator of 1 followed by THAT number of zeros.

In this case, there are 2 decimal places, so we put 100 into the denominator. So we can write

$$0.34 = \frac{34}{100} \rightarrow 2.34 = 2\frac{34}{100}$$

3. Lastly, simplify the fraction.

We simplify a fractional part by finding the common factor between the top and bottom and dividing through:

$$\frac{34}{100} = \frac{17 \times 2}{50 \times 2} = \frac{17}{50}$$

So the decimal 2.34 can be written as the fraction $2\frac{17}{50}$.

Now, you try the following:

Write the following as fractions:

1. 1.025
2. 0.220
3. 4.666

3 Ordering Decimals

To compare decimals, we start from the **left to right** and compare the digits in the corresponding places.

For example, compare 67.84 and 67.29.

$\mathbf{67}.84$ We see that the first 2 digits are the same

$\mathbf{67}.29$

$67.\mathbf{8}4$ However 8 is greater than 2

$67.\mathbf{2}9$

so

$$67.84 > 67.29$$

What about 5.6 and 5.637?

It is helpful to write zeros after 5.6 so that the decimals have the same number of decimal places. So 5.600 is the still the same number as 5.6.

$\mathbf{5}.\mathbf{6}00$ Again, we see that the first 2 digits are the same

$\mathbf{5}.\mathbf{6}37$

$5.6\mathbf{0}0$ But 3 is greater than 0

$5.6\mathbf{3}7$

so

$$5.6 < 5.637$$

You try the following:

Write in the order of least to greatest:

1. 4.16, 3.16, 4.26
2. 641.3, 641.2, 642.3
3. 5.6642, 566.42, 56.642, 5664.2

4 Rounding Decimals

Rounding decimals is important for making estimates. When we round decimals, we want to write a decimal *with fewer decimal places*.

When you have to round a number, you are usually told how to round it. It is simplest when you're told how many "decimal places" to round to, but you should also know how to round to a named "place", such as "to the nearest thousand" or "to the ten-thousandths place".

To round a number you should follow these steps:

1. Identity the target place.

2. Look at the digit one place to the right of the "target" place.
If the digit is a five or greater, you round the target digit up by one.
Otherwise, you leave the target as it is.

3. Then you replace any digits to the right with zeroes (if they are to the left of the decimal point) or else you delete the digits (if they are past the decimal point).

- **Example:** Round 26.158 to a whole number.

  We are told to round to the nearest whole number. So the target place is the "6".
  Now look to the digit to the *right* of the target, which is 1 in this case.
  Because 1 is less than 5, we round the 26.1 down to 26.

  Solution:

Chosen Decimal place	Digit to the right	Is it more or less than 5?	Round the number
2**6**.158	1	No	Down to 26

- **Example:** Round 17.189 to 2 decimal places.

  We are told to round to 2 decimal places. This means that we want to re-write the decimal with 2 places after the decimal point. So the target position is the hundredths column.
  So we look at the digit to the *right*, the thousandths column in this case, which has a 9.
  Because 9 is greater than 5 we round the target digit up to 9. so the answer is 17.19.

  Solution:

Chosen Decimal	Digit to the right	Is it more or less than 5?	Round the number
17.1**8**9	9	Yes	Up to 17.19

Now you try the following:
Round...

1. 4.76 to the nearest tenth
2. 0.316 to 1 decimal place.
3. 538.81 to the nearest whole number

5 Adding and Subtracting Decimals

To add decimals, follow these steps:

1. Write down the numbers, one under the other, with the decimal points lined up.

2. Add zeros so the numbers have the same length.

3. Then add normally, remembering to put the decimal point in the answer.

- **Example** Add $23.6 + 1.75 + 300.002$.

  Solution:

  Remember to add in the zeros so that the numbers have the same decimal places.

  Line the decimals up

  $$\begin{array}{r} 23.6 \\ 1.75 \\ +300.002 \end{array}$$

  Add the zero's

  $$\begin{array}{r} 23.600 \\ 1.750 \\ +300.002 \end{array}$$

  Add:

  $$\begin{array}{r} 23.600 \\ 1.750 \\ +300.002 \\ \hline 325.352 \\ \hline \end{array}$$

  Remember to check that your answer makes sense. We can estimate the sum: $23+2+300 = 325$. The estimated sum is close to the exact sum, so the answer of 325.352 is reasonable.

We subtract decimals using the same principle as adding them.

- **Example** What is the difference between 40.255 and 18.98.

  Solution:
  Remember to add in the zeros so that the numbers have the same decimal places.

Line the decimals up

$$\begin{array}{r} 40.255 \\ -18.98 \end{array}$$

Add the zero's

$$\begin{array}{r} 40.255 \\ -18.980 \end{array}$$

Subtract:

$$\begin{array}{r} 40.255 \\ -18.980 \\ \hline 21.275 \\ \hline \end{array}$$

Now you try the following:

1. 41.44+ 13.2
2. 86.2+31.7
3. 8.1 - 3.9
4. 0.963 - 0.72

6 Multiplying and Dividing Decimals

6.1 Multiplication of decimals

To multiply decimals, we just follow these steps:

1. Multiply normally, ignoring the decimal points.

2. Then put the decimal point in the answer - *it will have as many decimal places as the two*

original numbers combined.

In other words, just count up how many numbers are after the decimal point in both numbers you are multiplying, then the answer should have that many numbers after its decimal point.

For example, consider 0.02 multiplied by 1.1:

Start with:

$$0.02 \times 1.1$$

multiply without decimal points:

$$2 \times 11 = 22$$

0.02 has 2 decimal places and 1.1 has 1 decimal place, so the answer has 3 decimal places: 0.022.

- **Example:** Calculate 4.56×3.

  Solution:
  We start with:

  $$4.56 \times 3$$

  and now multiply without decimal points:

  $$456 \times 3 = 1368$$

  We now need to add in the decimal point:

  $$\begin{array}{rl} 4.56 & \leftarrow \text{2 decimal places} \\ \times 3 & \leftarrow \text{0 decimal places} \\ \hline 13.68 & \leftarrow \text{2 decimal places} \\ \hline \end{array}$$

  So the answer is 13.68.

- **Example 2:** Calculate 0.03×0.21:
  Again the same rules apply for multiplying 2 decimals: multiply without decimal points:

  $$3 \times 21 = 63$$

  $$\begin{array}{rl} 0.03 & \leftarrow \text{2 decimal places} \\ \times 0.21 & \leftarrow \text{2 decimal places} \\ \hline 0.0063 & \leftarrow \text{4 decimal places} \\ \hline \end{array}$$

So the answer is 0.0063.
Now try this one:

- A part-time job at a deli pays $\$4.58$ per hour. How much would you earn in a 4 hour and 30 minute shift?

6.2 Division of decimals

We follow the same procedure for dividing a decimal by a whole number as dividing a whole number by a whole number.

- **Example:** Calculate 49.5 divided by 9.

  Solution:

  Ignore the decimal point for the moment and use Long Division:

$$\begin{array}{r} 055 \\ 9\overline{)495} \\ \hline 45 \\ -45 \\ \hline 0 \end{array}$$

  Now put the decimal point back in its original position: So the decimal point in the original number was

$$\begin{array}{r} 05.5 \\ 9\overline{)49.5} \end{array}$$

  The answer is 5.5.

But what if you want to divide by a Decimal Number?

The trick is to convert the number you are dividing by to a whole number first, by shifting the decimal point of both numbers to the right:

$$6.625 \div 0.5 \rightarrow 66.25 \div 5$$

What you are actually doing is multiplying BOTH numbers by 10. So

$$\frac{6.625}{0.5} \text{ is the same as } \frac{6.625 \times 10}{0.5 \times 10} = \frac{66.25}{5}$$

Now you are dividing by a whole number, and can continue as normal.

- **Example:** Calculate $d = \frac{0.0048}{0.15}$.

  Solution:
  We can re-write d as

$$d = \frac{0.0048 \times 100}{0.15 \times 100} = \frac{0.48}{15}$$

  You are now dividing by a whole number, so you can proceed: Ignore the decimal point for the moment and use Long Division:

$$\begin{array}{r} 0032 \\ 15\overline{)048} \\ -45 \\ \hline 30 \\ -30 \\ \hline 0 \end{array}$$

  *Put the decimal point in the answer directly above the decimal point in the dividend:

$$\begin{array}{r} 0.032 \\ 15\overline{)0.48} \end{array}$$

  So the answer is 0.032.

Now try this one:

- Andy has 5.8 pounds of coffee. He wants 0.25 pound bags. How many bags will he need?

7 Percentages

Percentage means parts per 100. When you say "Percent" you are really saying "per 100". So 50% actually means 50 out of 100. So 50% can be written as a fraction $\frac{50}{100}$. As we have learnt in the section on decimals, this is the same as the decimal 0.5. So, a Half can be written...

As a *percentage*: 50%
As a *decimal*: 0.5
As a *fraction*: $\frac{50}{100} = \frac{1}{2}$

So what percentage is 6 out of 100?
It is easy to see that because a percent is the amount out of 100, 6 out of 100 must be 6%.

Ok, then what percentage is 6 out of 300?

Now we want to convert the **fraction** $\frac{6}{300}$ to a **percent**. The easiest way to convert a fraction to a percentage is to *divide the top number by the bottom number and then multiply the result by 100* (and add the "$\%$" sign).

So we follow this procedure: We find the fraction $\frac{6}{300} = \frac{1}{50}$ and multiply by 100 to get $\frac{1}{50} \times 100 = 2\%$. (*Don't forget to write the $\%$ sign!*)

But $\frac{6}{300}$ can be written as the decimal 0.02.

So to convert from a **decimal** to a **percent**, we simply *multiply the decimal by 100*.

- **Example:** Convert 3/8 to a percentage

  Solution:
  First divide 3 by 8 using the usual division: $3 \div 8 = 0.375$
  Then multiply by 100: $0.375 \times 100 = 37.5$
  Add the "$\%$" sign: 37.5%
  Answer: $3/8 = 37.5\%$

To convert a **percent** to a **fraction**, first convert to a decimal (divide by 100), then use the steps for converting decimal to fractions.

- **Example:** If a candidate received 57 out of 300 votes, what percentage of the votes did he get?

  Solution: We can do this by converting "57 out of 300" to a decimal or a fraction. Because a percentage is the number of votes out of 100, we need to reduce the fraction to a fraction where the denominator is 100:

$$\begin{aligned} \frac{57}{300} &= \frac{57}{3 \times 100} && \text{Split the denominator into 3 and 100} \\ &= \frac{19}{100} && \text{57 divided by 3 is 19} \\ &= 0.19 \end{aligned}$$

  so he got 19 votes out of 100 or 0.19 which we know is 19%. So the candidate received 19% of the votes.

Now you try.
Write the following percents as decimals and fractions:

1. 37%
2. 0.5%
3. 101%

8 Percents of a number

In percent problems, 'of' means multiply.

Say, you need to find 16% of 1400, you must first convert the percentage "16%" to its decimal form; namely, the number "0.16". Then, since "sixteen percent OF fourteen hundred" tells you to multiply the 0.16 and the 1400, you get: (0.16)(1400) = 224. This says that 224 is sixteen percent of 1400.

Percentage problems usually work off of some version of the sentence "(this) is (some percentage) of (that)", which translates to "(this) = (some decimal) x (that)". You will be given two of the values, or at least enough information that you can figure two of them out. Then you'll need to pick a variable for the value you don't have, write an equation, and solve for that variable.

- **Example:** If a candidate received 15% of 300 votes, how many votes did she receive?

  Solution:
  The number of total votes is 300. Let us call n the number of votes the candidate received. We need to calculate n which is 15% of 300:

  $$\begin{aligned} n &= \frac{15}{100} \times 300 \qquad \text{Write the percent as a fraction or decimal} \\ &= 15 \times 3 \\ &= 45 \end{aligned}$$

  So the candidate received only 45 votes.

- **Example:** What percent of 30 is 20?

  Solution:
  We have the original number (30) and the comparative number (20). The unknown in this problem is the rate or percentage. Since the statement is "(Twenty) is (some percentage) of (thirty)", then the variable stands for the percentage, and the equation is:

  $$\begin{aligned} (x)30 &= 20 \quad \text{Divide both sides by 30} \\ \frac{30x}{30} &= \frac{20}{30} \\ x &= 0.667 \end{aligned}$$

  Since x stands for a percentage, remember to convert this decimal back into a percentage: $0.667 = 66.7\%$. So 20 is 66.7% of 30.

9 Percent of Increase or Decrease

Many percentage problems involve some kind of increase or decrease relative to some original value. *Always find the percentage of change relative to the original value.*

- **Example:** Suppose a certain item used to sell for 75 cents a pound, you see that it's been marked up to 81 cents a pound. What is the percent increase?

  Solution:
  First, I have to find the absolute increase: $81 - 75 = 6$.
  The price has gone up 6 cents. Now I can find the percentage increase over the original price. Note this language, "increase/decrease over the original", and use it to your advantage: it will remind you to put the increase or decrease over the original value, and then divide.

  This percentage increase is the relative change with respect to the original number, which is 75 in this case:

  $$\frac{6}{75} = 0.08$$

  so there was a 8% increase in price per pound.

- **Example:** Jerry bought a table for $\$480$ that originally cost $\$600$. What was the percent discount?

  Solution:
  Firstly, I have to find the absolute decrease: $600 - 480 = 120$.
  The price was reduced by $\$120$. Now I want to find what percent $\$120$ is of the original price.

  This percentage decrease is the relative change with respect to the *original*:

  $$\frac{120}{600} = 0.2$$

  To convert the decimal to a percent, we multiply by 100, so there was a $0.2 \times 100 = 20\%$ discount.