Turbulence and Magnetic Fields in Star Formation[1]

Eve C. Ostriker

Department of Astronomy
The University of Maryland
College Park, MD 20742-2421

Abstract. Magnetized turbulence is believed to play an important role in the evolution of star-forming clouds in our Galaxy. I summarize the properties of these clouds, and outline the distinctions between various regimes of turbulence. I then present arguments and supporting calculations that in such clouds, MHD turbulence with mean magnetic field strengths of a few $\times 10\mu$G is able to (1) preserve clouds against self-gravitating collapse for > 10 Myr, (2) yield velocity power spectra $\sim v(k) \propto k^{-1/2}$ comparable to observations, and (3) generate very inhomogeneous density structure, in which lumps often coincide with discontinuities in velocities and magnetic field directions.

INTRODUCTION

Magnetic fields and turbulence are thought to be important in many stages of star formation as observed "close up" in our Galaxy [1]. In the late stages of star formation, MHD turbulence in the form of a saturated Balbus-Hawley instability may be one of the primary mechanisms driving accretion in the ionized portions of circumstellar disks (see chapter by Stone). Large-scale magnetic fields are also considered to be necessary in accelerating and collimating the jets and outflows that are a natural byproduct of the accretion process [2]. On yet larger scales than individual star/disk/outflow systems, magnetic fields are believed to play an important role in mediating turbulent motions, supporting against gravitational collapse, and encouraging the growth of condensations within the molecular clouds where stars form [3–5]. This paper focuses on the interplay between magnetic fields, turbulence, and gravity on this largest scale, in particular summarizing ideas about what pa-

---

[1] To appear in: *Star Formation, Near and Far*, Eds. S. Holt & L. Mundy (AIP Press, 1997)

rameters control the physics, and describing some results of numerical studies relevant to this problem.

Cloud properties

Well-observed star-forming regions in our galaxy range from the dark clouds which form low-mass stars in a distributed fashion (typical cloud mass $M \sim 10^4\ M_\odot$, typical cloud size $L \sim 10$ pc), to the giant molecular cloud complexes (GMCs) which form both high- and low- mass stars, primarily in clusters (cloud $M \sim 10^5\ M_\odot$, $L \sim 40$ pc) [6–8]. Strong molecular cooling leads to temperatures in the range $T = 10 - 50$ K (corresponding sound speeds are $c_\mathrm{s} = 0.2 - 0.4$ km s^{-1}). These clouds are quite inhomogeneous, with mean number densities in the range $n = 25 - 100$ cm^{-3}, but volume filling factors of only ~ 0.05 [9,10]; structure is observed down to the smallest resolved scales [11]. Star-forming clouds show a hierarchy of density structure: complex > cloud > clump ($n \sim 10^3$) > core ($n \sim 10^4$) > star. At typical temperatures and mean cloud densities, the thermal Jeans length $L_\mathrm{J} = c_\mathrm{s}\sqrt{\pi/G\rho}$ is in the range $2 - 5$ pc, which is much smaller than size of the whole cloud.

For structures of all scales within molecular clouds, the turbulent component of the linewidth is at least as large as the thermal component, with the former strongly dominating in all but centers of low-mass dense cores [6,12–14]. The larger clumps and clouds (and larger dense cores) have turbulence consistent with virial velocities $v \sim (GM/L)^{1/2}$ (a few $\times$ km s^{-1}) implying they are self-gravitating, and also show a linewidth increasing with size as approximately $\sigma_v \propto L^{0.5}$ [15–18]. The lower-mass clumps are not self-gravitating, and are thought to be pressure-confined [10,19,20].

Unfortunately, the magnetic field strengths and directions within molecular clouds are less well-known than other cloud properties, because of the difficulty in making, and sometimes interpreting, the relevant observations [21,22]. Observational studies of magnetic fields in star-forming regions have been thoroughly reviewed by [23]. Field strength observations for star-forming clouds are mostly based on Zeeman splitting in HI or OH, with the result that measured line-of-sight field strengths fall in the range $B = 10 - 30\mu$G [21,24,25]. In several objects, detailed comparisons of mean Alfvén speed $v_\mathrm{A} = B/\sqrt{4\pi\rho}$, turbulent velocity dispersion, and virial speeds suggest approximate equipartition between magnetic, kinetic, and gravitational energies [26]. Optical polarization of background stars traces the averaged plane-of-sky field direction, assuming absorption by spinning dust grains aligned perpendicular to the field. Maps of optical polarization suggest smoothly-varying field directions in the lower-density gas [27,28]. More recently, polarization studies have been extended to the far-infrared [29,30], where polarized emission is believed to come from warm dust grains; these maps also imply large-scale correlations in the mean field orientation (perpendicular in the plane of the sky to the direction of polarized emission).

Regimes of turbulence

Taken together, the observations of star-forming regions (and molecular clouds more generally) suggest that their underlying dynamics lies in a novel regime of magnetohydrodynamic (MHD) turbulence. The basic state in this regime is characterized by (1) highly supersonic motions up to $\sigma_v/c_\mathrm{s} \sim 10$, (2) a strongly magnetized, compressible medium with $v_\mathrm{A} \sim \sigma_v$, and (3) self-gravity stronger than thermal pressure forces over a wide range of scales within the cloud ($L >> L_\mathrm{J}$). This basic state is far from the regimes of turbulence that have heretofore received most attention; thus, there are few predictions about how spectra of velocity, magnetic field, and density perturbations should develop, how the turbulence should decay in time, etc.

The most familiar regime of turbulence assumes an incompressible, unmagnetized fluid; in the classical Kolmogorov picture, the dynamics consists of a conservative energy cascade from large to small scales such that an eddy's velocity v scales with its size L as $v \propto L^{1/3}$, corresponding in wavenumber space to $v \propto k^{-1/3}$ hence an energy spectrum $dE/dk \propto k^{-5/3}$ [31]. Also well-studied by fluid dynamicists [32,33], but perhaps more familiar to cosmologists (e.g. [34]) than to the interstellar medium community, is Burgers turbulence. Burgers turbulence consists of the development of discontinuities in the velocity and density fields in pressure-free, unmagnetized, flows; the spectral description of the shocks that form is $dE/dk \propto k^{-2}$ (i.e. $v(k) \propto k^{-1/2}$). The Burgers model approximately describes the early supersonic phase seen in numerical simulations of compressible (unmagnetized) turbulence [35,36]. Most of the work on MHD turbulence has concentrated on the case of an incompressible medium, with different energy spectra predicted depending on on the system's dimensionality and the level of nonlinearity in the perturbations [37–39].

Sustained turbulence of any sort requires a "driver" – i.e. a source of energy to replace what is turned into heat (through viscous dissipation at the "bottom" of a cascade, or locally in shocks). In massive molecular clouds, a considerable stock of available energy exists in the gravitational potential. In clouds where stars are already forming, the outflows from young stars are very effective at returning this energy to the cloud, and may act as a feedback element in self-regulated star-formation [40,41]; it is also possible that local instabilities can tap into this gravitational store during the early contraction of a cloud, and that turbulence may be excited by energy (in shear and compressive motions) carried into the cloud from the exterior and partially trapped, cf. [42].

Because of the basic distinction between a conservative, incoherent energy cascade, and a collection of dissipative, coherent, local interactions (in shocks), the dynamics of star-forming regions might end up bearing more resemblance to compressible unmagnetized Burgers turbulence than to incompressible turbulence (magnetized or unmagnetized) [43]. Adding magnetic fields may, however, be expected to alter the dynamics of compressible, supersonic, self-gravitating fluid turbulence significantly, and indeed early MHD simulations showed many effects absent in unmagnetized simulations [44–46] (e.g. reduced shock strengths, strong transient matter compression by magnetic tension and pressure forces, reduction in contraction driven by self-gravity). Further discussion of these and more recent simulations appears below.

THE INTERNAL DYNAMICS OF SELF-GRAVITATING, MAGNETIZED CLOUDS

If we consider an idealized piece of a gas cloud cloud of uniform density n, temperature T, magnetic field B, and linear size L, we can define characteristic timescales which describe the relative importance of gravity, thermal pressure, and magnetic pressure/tension. Respectively, we define

$$t_{\mathrm{G}} \equiv \frac{L_J}{c_s} = \left(\frac{\pi}{G\rho}\right)^{1/2} = 10^7 \text{ yr} \left(\frac{n}{10^2 \text{ cm}^{-3}}\right)^{-1/2}, \tag{1}$$

$$t_{\mathrm{T}} \equiv \frac{L}{c_s} = 5\times 10^7 \text{ yr} \left(\frac{L}{10 \text{ pc}}\right)\left(\frac{T}{10 \text{ K}}\right)^{-1/2}, \tag{2}$$

and

$$\begin{aligned} t_{\mathrm{M}} &\equiv \frac{L}{v_A} = \frac{L}{B/\sqrt{4\pi\rho}} \\ &= 8\times 10^6 \text{ yr} \left(\frac{L}{10 \text{ pc}}\right)\left(\frac{n}{10^2 \text{ cm}^{-3}}\right)^{1/2}\left(\frac{B}{10\mu\mathrm{G}}\right)^{-1}. \end{aligned} \tag{3}$$

Quiescent cloud stability

A uniform cloud with just thermal pressure is susceptible to gravitational instabilities on scales where $t_{\mathrm{T}} > t_{\mathrm{G}}$. Star-forming molecular clouds have $t_{\mathrm{T}} >> t_{\mathrm{G}}$, implying they would be unstable by Jeans's original criterion if they were unmagnetized and quiescent. Since the Galactic star formation rate would be much too high if all the cold molecular clouds fragmented and collapsed to make stars in a time $\sim t_{\mathrm{G}}$, the simplest Jeans analysis is clearly insufficient as a prescription for when gas is converted to stars (at least in present-day spiral galaxies like our own).

If we now allow for nonzero magnetic field strengths, we have the result that gravitational instabilities perpendicular to the mean magnetic field direction are suppressed by magnetic pressure on scales where $t_{\mathrm{M}} < t_{\mathrm{G}}$ (in the limit $t_{\mathrm{M}} << t_{\mathrm{T}}$) [47]. This criterion is equivalent to having the column density satisfy $N_H < 4\times 10^{21} \text{ cm}^{-2}(B/10\mu\mathrm{G})$; thus $B \sim 25\mu\mathrm{G}$ would make observed clouds stable to Jeans modes with $\hat{k} \perp \hat{B}$. Such an initially quiescent cloud would tend to "pancake" along the direction of the uniform magnetic field, however, because there are no magnetic forces parallel to field lines to oppose gravity. We would require a ratio of B/N_H a factor of π larger to prevent fragmentation of a cold "pancake" threaded by uniform fields [48,49]; the critical cases of magnetized cloud equilibria have the same central flux-to-mass ratio [50,51].

Turbulence in magnetized clouds – quasilinear theory

In a time-dependent magnetic field, matter experiences restoring forces due to magnetic pressure and tension; this allows supersonic oscillatory motions without "collisions," hence reducing shock losses and permitting longer-lived turbulence. Arons & Max [52] early argued that the supersonic motions observed in molecular clouds may owe their existence to Alfvén-type (transverse) MHD waves which have $\delta B_\perp/B_0 = \delta v_\perp/v_{\rm A}$ perpendicular to the mean magnetic field $\mathbf{B_0}$.

Because Alfvén waves have nonzero perturbations perpendicular to $\mathbf{B_0}$ (the time-averaged field), they offer a mechanism to inhibit gravitational collapse along the mean field direction [1,53]. From quasilinear theory, time-averaged Alfvén waves yield a pressure force

$$\mathbf{F}_{wave} = -\frac{\nabla|\delta B_\perp|^2}{8\pi\rho}; \tag{4}$$

the corresponding polytropic wave pressure obeys $P_{wave} \propto \rho^{1/2}$ [54]. A pseudo-Jeans analysis with this quasilinear Alfvén wave pressure would then predict no mean-field collapse in a cloud provided

$$n_{\rm J} \equiv \frac{L}{L_J} \lesssim \frac{\delta B_\perp}{4c_s\sqrt{\pi\rho}}, \tag{5}$$

i.e.

$$\delta B_{rms} \gtrsim 18\mu{\rm G}\left(\frac{L}{20\ {\rm pc}}\right)\left(\frac{n}{100\ {\rm cm}^{-3}}\right) \tag{6}$$

and

$$\sigma_{v,\ wave} \gtrsim 2.3\ {\rm km\ s}^{-1}\left(\frac{L}{20\ {\rm pc}}\right)\left(\frac{n}{100\ {\rm cm}^{-3}}\right)^{1/2}. \tag{7}$$

Because the gas in clouds is only partially ionized, ambipolar diffusion acts to damp such waves, especially at short wavelengths [3,55,56]. Unless the background magnetic field were extremely strong (milligauss-level), perturbations of the amplitude in equation (6) would be quite nonlinear, and significant energy losses would be incurred due to the generation of compressive motions (regardless of the initial polarization of the Alfvén wave), which steepen to shocks [57,58]. In summary, analytic arguments (described in this section) suggest that MHD waves should be important to the dynamics of star-forming regions; the actual nonlinear amplitudes of such waves, however, necessitates direct numerical simulations (described in the next section) to investigate the nature of the resulting MHD turbulence, and to quantitatively discriminate among model parameters.

SIMULATIONS OF MAGNETIZED, TURBULENT CLOUDS

In order to understand the physics of star-forming gas clouds, and eventually to quantify how parameters such as the mean magnetic field strength may influence the star formation rate or IMF in a cloud or galaxy, detailed studies are required to characterize the nature of turbulence under a realistic range of conditions. In particular, methodical campaigns of numerical MHD simulations should help address many of the longstanding problems in cloud dynamics for which magnetic fields have been proposed as a solution. Eventually, such MHD simulations will be able to address many aspects of cloud dynamics in a fairly realistic fashion – incorporating, for example, the effects of cosmic-ray heating, radiative cooling, ion-neutral diffusion, and the interface with the surrounding medium. Even very simple cloud models (e.g. isothermal or adiabatic equation of state with periodic boundary conditions and negligible ion-neutral diffusion), however, can be used in the initial attack on such important questions as:

- Can nonlinear Alfvén-like waves prevent or delay cloud collapse? How much wave energy is needed?
- What sorts of velocity and density spectra/structures develop in magnetized, compressible media?
- How do dissipation rates of MHD turbulence depend on σ_v, v_A, and c_s? How much energy input would be required to prevent collapse of a self-graviating cloud?

As mentioned above, questions like these have been addressed in several plane-parallel and two-dimensional MHD simulations in the early 1990's [44–46]. More recently, Gammie & Ostriker [59] performed a survey of plane-parallel simulations over a wide range of parameters describing the relative levels of kinetic, magnetic, thermal, and gravitational energies in a model cloud; these computations are presently being extended to three dimensions [60]. In order to study cloud formation on large scales in a galactic disk, [61] have included parameterized shear flow, diffuse and localized heating, and cooling in a series of two-dimensional simulations. For additional related work on this topic, see the contributions to this Proceedings by Ballestros-Paredes, Balsara, Padoan, and Vazquez-Semadeni. Below I highlight some of the results that have been obtained so far.

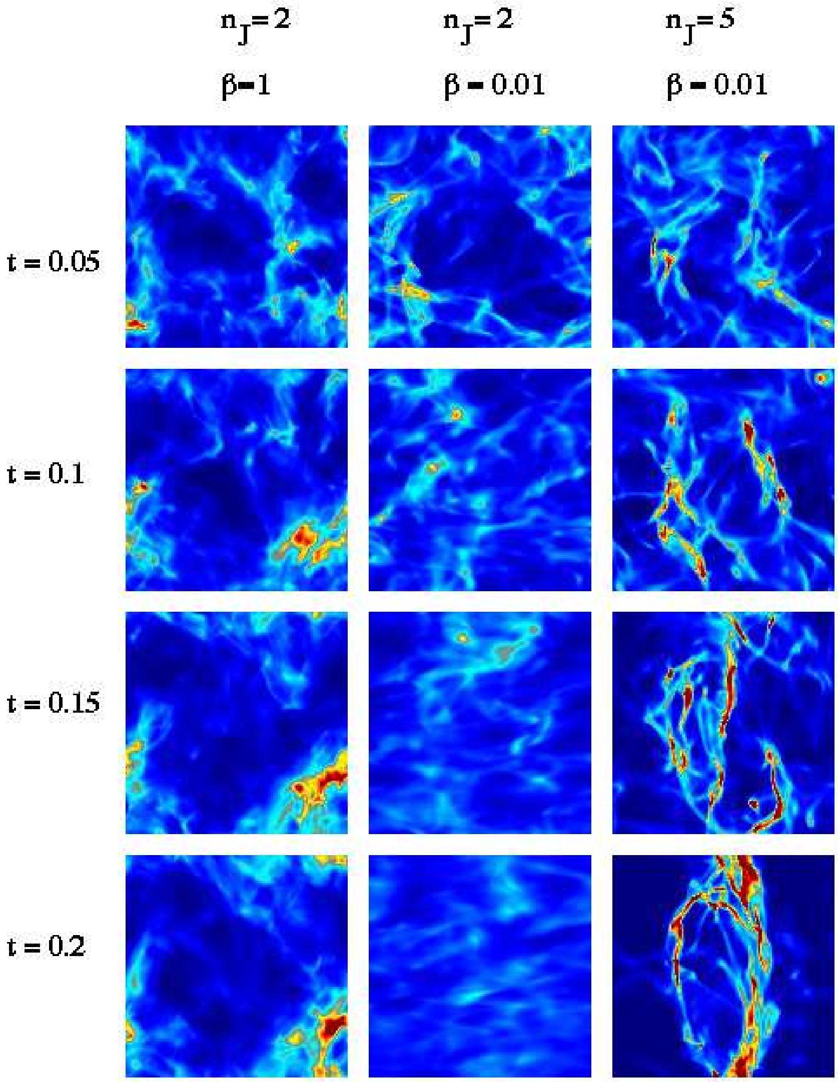

FIGURE 1. Evolution of the integrated line-of-sight density in three simulations with varying values of the mean magnetic field strength (described by $\beta = c_s^2/v_A^2 \equiv 4\pi\rho_0 c_s^2/B_0^2$) and size (described by the number of Jeans lengths $n_J \equiv L/L_J$). Time increases downward in each column from 0.05 to 0.2 sound crossing times, and the mean field runs right to left. All runs have uniform initial density, and initial wave energy $E_W = 100\bar{\rho}c_s^2L^3$ in random velocity perturbations. The first column has $\beta = 1$ and $n_J = 2$; the second column has $\beta = 0.01$ and $n_J = 2$; the third column has $\beta = 0.01$ and $n_J = 5$. Higher column density regions are brighter (except for the central peaks, which appear dark in this monochromatic representation of a color figure).

Simulation results on cloud support by MHD waves

As described above, Alfvén-type MHD waves are expected to be able to support clouds against collapse along the mean field direction, provided that the energy in the time-dependent, perturbed magnetic field is large enough compared to the gravitational energy of the cloud. Early two-dimensional simulations indeed showed weaker gravitational contraction in models with stronger magnetic fields [46]. The plane-parallel simulations of [59] verified that the prediction of equation (5) is in fact approximately correct quantitatively – marking the boundary between clouds that collapse rapidly (within $\sim t_{\rm G}$) and those that do not, for the case of initially turbulent clouds. For model clouds in which turbulence is continuously forced at scales smaller than the box, equation (5) marks the boundary between clouds that collapse and those that survive indefinitely [59].

The initial results from studies underway in three dimensions [60] concur in showing that sufficiently strong MHD turbulence can support a cloud against gravitational collapse. Figure 1 shows the time evolution of three different model clouds with different levels of mean magnetic field (measured by $\beta \equiv c_{\rm s}^2/v_{\rm A}^2$) and mass (measured by the number of Jeans lengths $n_{\rm J}$ in each boundary of the simulated region), but all the same initial turbulent energy in random velocities. Comparing the two models with the same mass ($n_{\rm J} = 2$), but differing magnetization (weak field $\beta = 1$ and strong field $\beta = 0.01$), the figure shows that a strongly self-gravitating condensation forms in only the weakly-magnetized case. Comparing two models with the same magnetization ($\beta = 0.01$) but different gravitational potental energies (weak gravity $n_{\rm J} = 2$, strong gravity $n_{\rm J} = 5$), we see that only the $n_{\rm J} = 5$ cloud that has gravity strong compared to the level of magnetic perturbations ends up collapsing.

Simulation results on density structure

All the simulations cited have discovered highly inhomogeneous/filamentary structure growing in the model clouds as they evolve. For example, in the plane-parallel simulations of [59], typically 70-80% of the volume has density less than the mean, while the preponderance of the mass (75-80%) is in condensations with density above the mean value. The self-gravitating simulations show weak positive correlation between magnetic pressure and gas density, with the correlation increasing over increasing spatial scale [61,59]. On the other hand, non-self-gravitating, isothermal simulations show anticorrelation between magnetic and gas pressure because it is the magnetic field that "confines" the gas clumps [44,59]. In three-dimensional runs [60], strongly magnetized clouds are found to have density structures aligned preferentially perpendicular to the mean field. Models with low magnetization show rounder clumps than those with high magnetization (see Fig. 1). In general, high density regions are correlated with discontinuities of $\mathbf{v}$ and $\mathbf{B}$; low density regions have relatively straight field lines.

Simulation results on velocity/magnetic field perturbation spectra

Independent of the nature of the driving source of the turbulence (i.e. decay of a spectrum of perturbations imposed in the initial conditions or continuous energy injection; fluctuations imposed in just the velocity field, just the magnetic field, or both), the model cold, magnetized clouds evolve toward equipartition of energy in kinetic and fluctuating magnetic fields [61,59,60]. Because compressive motions damp more easily than transverse ones, the latter dominate the velocity field. Also independent of the spatial scale at which perturbations are imposed, the power spectra evolve toward distributions close to $v(k)^2 \propto B(k)^2/(4\pi\bar{\rho}) \propto k^{-2}$ [61,59,60]. The corresponding "linewidth-size" relation $\sigma_v \propto L^{0.5-0.7}$ has been directly verified for the plane-parallel case [59]. Both the plane-parallel and two-dimensional simulations have identified an "inverse cascade" of energy in growing power of δB and δv at large scale (provided that a mean magnetic field threads the box) [61,59].

Simulation results on dissipation rates

Dissipation rates are difficult to measure accurately in high dimensional computations because of the high resolution required. For the plane-parallel simulations [59], dissipation rates have been measured by equating the dissipation rate to the power injected in simulations which have reached a quasi-steady forced equilibrium. Given an supply of energy $\dot{E}_{\rm wave}$ injected on a scale λ into a box of size L, the dissipation time is computed to be

$$t_{\rm diss} \equiv \frac{E_{\rm wave}}{\dot{E}_{\rm wave}} = 0.5 \left(\frac{\lambda}{L}\right)^{1/2} \left(\frac{v_A}{c_s}\right)^{1/2} \frac{L}{\sigma_v}, \tag{8}$$

larger than the "eddy turnover time" by a factor $\sim (v_A/c_s)^{1/2}$. For typical GMCs, this result would imply that turbulence and support against collapse could be sustained indefinitely given a mechanical luminosity input of tens of $L_\odot$. How this result will change in three-dimensional simulations with more realistic physics (especially ambipolar diffusion) remains to be seen.

ACKNOWLEGEMENTS

I am grateful to my collaborators Charles Gammie and Jim Stone and for their comments on this manuscript, and for permission to include Figure 1 in this review.

REFERENCES

1. Shu, F.H., Adams, F.C. & Lizano, S. 1987, ARAA, 25, 23
2. Königl, A., & Ruden, S. P., in Protostars & Planets III, ed. E. Levy & J. Lunine (Tucson: University of Arizona Press), p. 641

3. McKee, C. F., Zweibel, E. G., Goodman, A. A., & Heiles, C. 1993, in Protostars and Planets III, ed. E. Levy & J. Lunine (Tucson: University of Arizona Press), p. 327
4. Elmegreen, B. G. 1991, in The Physics of Star Formation and Early Stellar Evolution, Eds. C. J. Lada & N. D. Kylafis (Dordrecht: Kluwer), p. 35
5. Mouschovias, T., in The Physics of Star Formation and Early Stellar Evolution, Eds. C. J. Lada & N. D. Kylafis (Dordrecht: Kluwer), p. 61
6. Blitz, L. 1993, in Protostars and Planets III, ed. E. Levy & J. Lunine (Tucson: University of Arizona Press), p. 125
7. Cernicharo, J. 1991, in The Physics of Star Formation and Early Stellar Evolution, Eds. C. J. Lada & N. D. Kylafis (Dordrecht: Kluwer), p. 287
8. Lada, E. A., Strom, K. M., & Myers, P. C. 1993, in Protostars and Planets III, ed. E. Levy & J. Lunine (Tucson: University of Arizona Press),
9. Pérault, M., Falgarone, E., & Puget J. L. 1985, A&A, 152, 371
10. Williams, J. P., Blitz, L. & Stark, A. A. 1995, ApJ, 451, 252
11. Falgarone, E., Puget, J. L., & Pérault, M. 1992, A&A, 257, 715
12. Benson, P. J., & Myers, P. C. 1989, ApJS, 71, 89
13. Caselli, P., & Myers, P. C., ApJ, 446, 665
14. Goodman, A. A., Barranco, J. A., Wilner, D. J., & Heyer, M., ApJ, submitted
15. Larson, R. B. 1981, MNRAS, 194, 809
16. Dame, T. M., Elmegreen, B. G., Cohen, R. S., & Thaddeus, P. 1986, ApJ, 305, 892
17. Solomon, P. M., Rivolo, A. R., Barrett, J., & Yahil, A. 1987, ApJ, 319, 730
18. Myers, P. C., & Goodman, A. A. 1988b, ApJ, 329, 392
19. Loren, R. B. 1989, ApJ, 338, 925
20. Bertoldi, F. & McKee, C. F. 1992, ApJ, 395, 140
21. Goodman, A. A., & Heiles, C. 1994, ApJ, 424, 208
22. Goodman, A. A., Jones, T. J., Lada, E. A., & Myers, P. C. 1995, ApJ, 448, 748
23. Heiles, C., Goodman, A. A., McKee, C. F.,& Zweibel, E. G. 1993, in Protostars and Planets III, ed. E. Levy & J. Lunine (Tucson: University of Arizona Press), p. 279
24. Crutcher, R. M., Troland, T. H., Goodman, A. A., Heiles, C., Kazes, I., & Myers, P. C., ApJ, 407, 175
25. Troland, T. H., Crutcher, R. M., Goodman, A. A., Heiles, C., Kazes, I., & Myers, P. C. 1996, ApJ, 471, 302
26. Myers, P. C., & Goodman, A. A. 1988a, ApJ, 326, L27
27. Moneti, A., Pipher, J. L., Helfer, H. L., McMillan, R. S., & Perry, M. L., ApJ, 282, 508
28. Goodman, A. A., Bastien, P., Menard, F., & Myers, P. C. 1990, ApJ, 359, 363
29. Davidson, J. A., Schleuning, D., Dotson, J. L., Dowell, C. D., & Hildebrand, R. H. 1995, in Astronomical Society of the Pacific, Airborne Astronomy Symposium on the Galactic Ecosystem: From Gas to Stars to Dust, Volume 73, p 225
30. Hildebrand, R. H., Dotson, J. L., Dowell, C. D., Platt, S. R., Schleuning, D.,

Davidson, J. A., & Novak, G. 1995, in Astronomical Society of the Pacific, Airborne Astronomy Symposium on the Galactic Ecosystem: From Gas to Stars to Dust, Volume 73, p 97

31. Landau, L. D., & Lifschitz, E. M. 1987, Fluid Mechanics (New York:Pergamon)
32. Burgers, J. M. 1974, The Nonlinear Diffusion Equation (Reidel: Dordrecht)
33. Whitham, G. B. 1974, Linear and Nonlinear Waves (Wiley-Interscience: New York)
34. Gurbatov, S. N., Saichev, A. I., Shandarin, S. F. 1989, MNRAS, 236, 385
35. Porter, D. H., Pouquet, A., & Woodward, P. R. 1992, Phys. Rev. Lett. 68, 21
36. Porter, D. H., Pouquet, A., & Woodward, P. R. 1994, Phys. Fluids 6, 2133
37. Biskamp, D. 1993, Nonlinear Magnetohydrodynamics (Cambridge: Cambridge Univ. Press)
38. Sridhar, S., & Goldreich, P. 1994, ApJ, 432, 612
39. Goldreich, P., & Sridhar, S. 1995, ApJ, 438, 763
40. Norman, C., & Silk, J. 1980, ApJ, 238, 158
41. McKee, C. F. 1989, ApJ, 345, 782
42. Falgarone, E. & Puget, J. L. 1986, A&A, 162, 235
43. Passot, T., Pouquet, A., & Woodward, P. 1988, A&A 197, 228
44. Elmegreen, B. G. 1990, ApJ, 361, L77
45. Pouquet, A., Passot, T., & Leorat, J. 1991, in Fragmentation of Molecular Clouds and Star Formation, eds. E. Falgarone, F. Boulanger, & G. Duvert (Dordrecht: Kluwer), p. 101
46. Pouquet, A., Leorat, J., & Passot, T. 1991, in Advances in Turbulence 3, eds. A. V. Johansson & P. H. Alfredsson (Springer-Verlag: Berlin), p. 343
47. Chandrasekhar, S. & Fermi, E. 1953, ApJ, 118, 116
48. Nakano, T., & Nakamura, T. 1978, PASJ, 30, 671
49. Tomisaka, K., & Ikeuichi, S. 1983, PASJ, 35, 187
50. Mouschovias, T., & Spitzer, L. 1976, ApJ, 210, 326
51. Tomisaka, K., Ikeuchi, S., & Nakamura, T. 1988, ApJ, 335, 239
52. Arons, J. & Max, C. E. 1975, ApJ, 196, L77
53. Fatuzzo, M., & Adams, F. C. 1993, ApJ, 412, 146
54. McKee, C. F. & Zweibel, E. G. 1995, ApJ, 440, 686
55. Kulsrud, R. M., & Pearce, W. P. 1969, ApJ, 156, 445
56. Myers, P. C. & Khersonsky, V. K. 1995, ApJ, 442, 186
57. Zweibel, E. G., & Josafatsson, K. 1983, ApJ, 270, 511
58. Ghosh, S., & Goldstein, M.L. 1994, JGR, 99, 13351
59. Gammie, C. F., & Ostriker, E. C. 1996, ApJ, 466, 814
60. Gammie, C., Stone, J., & Ostriker, E. 1997, in preparation
61. Passot, T., Vázquez-Semadeni, E., & Pouquet, A. 1995, ApJ, 455, 536