Protostellar Disks: Accretion Processes

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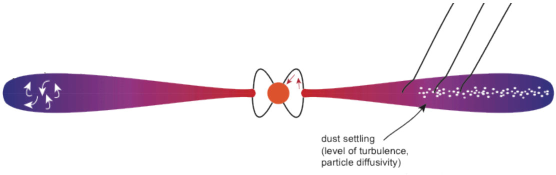
Introduction

- Modelling Protostellar Disks
- Angular Momentum Transport
- Accretion by the Magnetorotational Instability:
  - Ideal Case
  - Resistive Case and Ionization Structure
  - Other non-ideal effects
  - Non-linearities
- Accretion by Hydrodynamic Instabilities

Protostellar Disks


dust settling
(level of turbulence,
particle diffusivity)

- Masses around $0.01 - 0.1 M_\odot$ and sizes around $10 - 100 AU$
- Thin Disks (Minimum Mass Solar Nebula):
  - $\Sigma(r) \approx 1700 \left(\frac{r}{1 \text{ AU}}\right)^{-3/2} \text{gcm}^{-2}$
  - $\frac{h}{r} = \frac{c_s}{\Omega r} \approx 0.03 \left(\frac{r}{1 \text{ AU}}\right)^{1/4}$
- Cool and Dusty: $T(r) \approx 280 \left(\frac{r}{1 \text{ AU}}\right)^{-1/2} \text{K}$
- Magnetic fields between $10^{-2} - 1G$

Mass Accretion

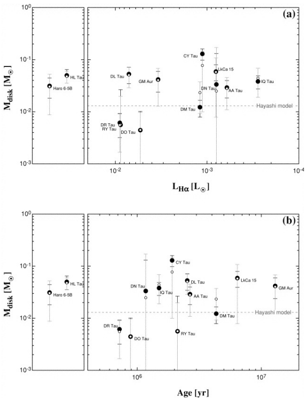


Figure: (Kitamura, 2002)

- Disks last for $\sim 1 - 10$Myr
- Must accrete or disperse disk mass in this time
- Accretion rates $\sim 10^{-9} - 10^{-7} M_\odot yr^{-1}$
- Disk evolves, accretes mass onto protostar by
- Loss of mass and angular momentum (photoevaporation, disk braking, disk winds)
- Angular Momentum Transport

Angular Momentum Transport

- Local turbulence creates viscoscity $\nu = \alpha \frac{c_s^2}{\Omega}$ related to the local stress

$$W_{r\phi} = \left[\delta v_r \delta v_\phi - \frac{B_r B_\phi}{4\pi\rho} \right]_\rho$$
$$W_{r\phi} = \alpha c_s^2$$

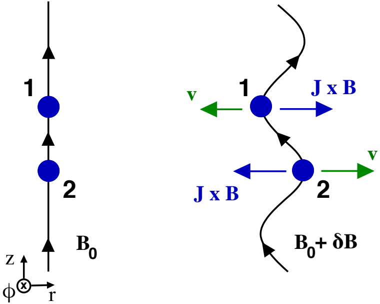
- This viscoscity drives disk evolution

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{r} \frac{\partial}{\partial r} \left[\sqrt{r} \frac{\partial}{\partial r} \left(\nu \Sigma \sqrt{r} \right) \right]$$
$$\dot{M} = 6\pi r^{1/2} \frac{\partial}{\partial r} (2\Sigma \nu r^{1/2})$$

- Accretion and diffusion outwards if $\frac{\partial(\nu\Sigma)}{\partial \Sigma} < 0$
- Most internal methods of accretion require sustained turbulence

MHD Turbulence

- Can create turbulence by:
    - Self-gravity (Wendy)
    - Hydrodynamic Instabilities
    - Magnetorotational Instability (MRI)



- In Ideal MHD $\frac{\partial \mathbf{B}}{\partial t} = \nabla \times [\mathbf{v} \times \mathbf{B}]$
- Differential rotation creates tension along field lines
- Excites turbulence, drives some mass inwards, angular momentum outwards
- Excited if $\frac{\mathrm{d}}{\mathrm{d}r}\left(\Omega^2\right) < 0$

Complications

- In reality, there are non-ideal effects

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left[\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B} - \frac{\mathbf{J} \times \mathbf{B}}{e n_e} + \frac{(\mathbf{J} \times \mathbf{B}) \times \mathbf{B}}{c \gamma \rho_i \rho} \right].$$

- Magnetic field drifts due to diffusion terms
    - Ohmic Diffusion: $\eta = \frac{c^2}{4\pi\sigma_c}$
    - Ambipolar Diffusion: $\frac{(\mathbf{J}\times\mathbf{B})\times\mathbf{B}}{c\gamma\rho_i\rho}$
    - Hall Diffusion: $\frac{\mathbf{J}\times\mathbf{B}}{e n_e}$
- Stronger coupling between magnetic field and fluid required for MRI
- With Diffusion, what regions of the disk accrete?
- When are non-ideal effects important and what effect do they have?

Ohmic Diffusion

- Magnetic Diffusion due to finite resistivity $\eta = \frac{c^2 m_e \gamma_e \rho}{4\pi e^2 n_e}$
- Important for low ionization fraction since resistivity increases with neutral fraction
- Suppresses MRI when resistive damping $\tau_\eta \sim \frac{\lambda^2}{\eta}$ is shorter than growth rate $\tau \sim \frac{\lambda}{v_A}$
- Equivalent to:

$$\mathrm{Re}_M = \frac{h v_A}{\eta} \lesssim 1$$

$$\frac{n_e}{n} = x \sim 5 \times 10^{-13} \left(\frac{h/r}{0.05}\right)^{-1} \left(\frac{v_A}{0.1 c_s}\right)^{-1}$$

Disk Ionization

- Thermal ionization, at typical densities $n_H \sim 10^{13} gcm^{-3}$, reaches $x \sim 10^{-13}$ for $T \gtrsim 10^3$K
- Protoplanetary disks are much colder than most astrophysical disks, does not hold beyond $r \sim 0.1$AU

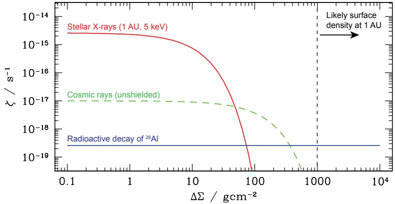


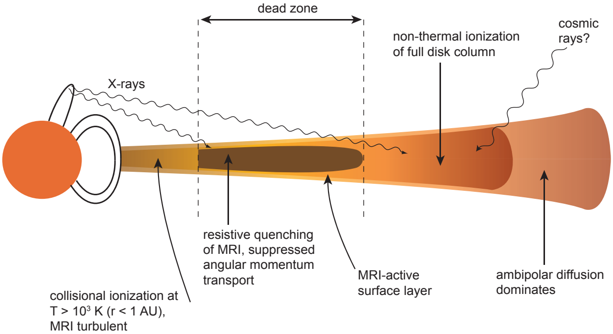
Figure: (Armitage, 2010)

Non-thermal sources of ionization dominate

- Radioactive Decay
- Cosmic Rays
- Protostellar X-rays

Dead Zones

- Leads to tiered structure of protostellar disks (Gammie, 1996)
  - Thermally ionized and MRI turbulent interior
  - Non-thermally ionized and MRI turbulent exterior
  - Intermediate region with thin active layer and mid-plane dead zone



Layered Accretion

- Accretion proceeds through the active layer onto the dead zone
- Uneven accretion leads to gravitational instability and heating to above $\sim 10^3$K
- Mass accreted onto dead zone rapidly accreted onto protostar (Variable/Bursty Accretion)
- Accretes sufficient mass onto the dead zone (Gammie, 1996)

$$M \approx 1.3 \times 10^{-3} \left(\frac{\alpha}{10^{-2}}\right)^2 \left(\frac{\Sigma_a}{100\mathrm{gcm}^{-2}}\right)^3 \kappa_0 \left(\frac{\delta t}{10^4 yr}\right) M_\odot$$

- But:
  - Results ignore Hall and Ambipolar Diffusion
  - Very sensitive to the exact opacity/recombination rate

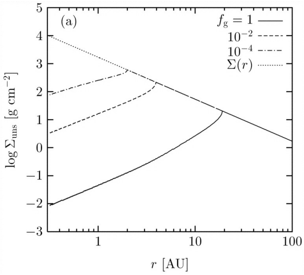
Uncertainties

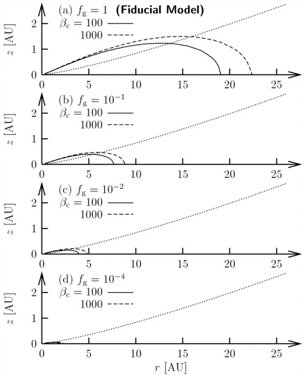
- x determined by balance between ionization and recombination
- Ionization (by X-rays) slightly uncertain
- Gas-phase recombination well known (though sensitive to metal abundance)
- Recombination onto grains dependent on both the fraction of dust and the size of dust grains

$$\frac{\dot{n}_{I,dust}}{\dot{n}_{I,gas}} \sim 20 \left(\frac{f_{\rm d}}{10^{-2}}\right) \left(\frac{x}{10^{-12}}\right)^{-1} \left(\frac{T}{100\ {\rm K}}\right) \left(\frac{a}{1\ \mu{\rm m}}\right)^{-1}$$

- Even if initial dust distribution is known, the rate of sedimentation is unknown

Uncertainties: Dust Fraction

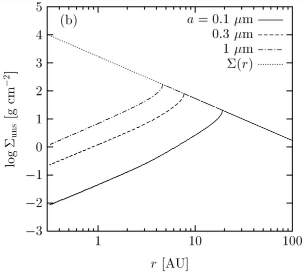




- Gammie, 1996 results assume $\Sigma_a \approx 100 \mathrm{gcm}^{-2}$, which is only true for small dust fractions

Figure: (Sano, Miyama, 2000)

Uncertainties: Dust Size



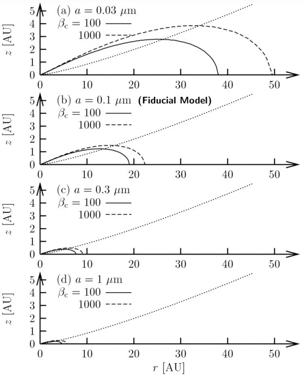


Figure: (Sano, Miyama, 2000)

- Significant active layer only for large dust sizes and small dust fractions
- This is true if disk has evolved significantly

Uncertainties: Disk Density

- Changes in disk surface density have less impact, but still uncertain

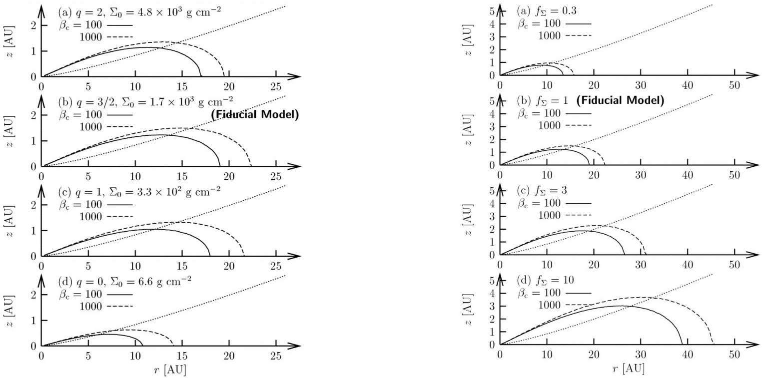


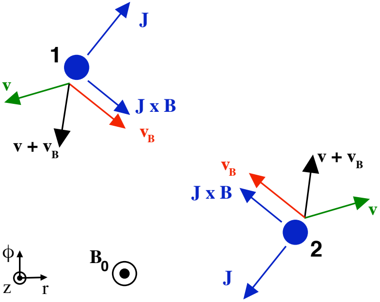
Figure: (Sano, Miyama, 2000)

Non-Ideal Effects: Ambipolar Diffusion

- Other effects don't destroy flux but let it drift wrt neutral fluid

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times\left[(\mathbf{v}+\mathbf{v}_\mathrm{B})\times\mathbf{B} - \eta(\nabla\times\mathbf{B})_\parallel\right]$$

$$\mathbf{v}_\mathrm{B} = \mathbf{v}_\mathrm{P} + \mathbf{v}_\mathrm{H}$$



Ambipolar diffusion (low density, high x) dominates when the field is frozen to ions, with a drift due to neutral drag

$$\mathbf{v}_\mathrm{P} = \frac{\mathbf{J}\times\mathbf{B}}{\mathbf{c}\gamma_\mathbf{i}\rho_\mathbf{i}\rho}$$

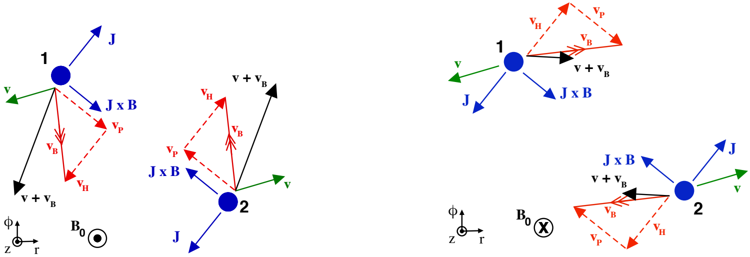
$$\gamma_i = \frac{\langle\sigma v\rangle_i}{m_i + m}$$

Non-Ideal Effects: Hall Diffusion

- Hall diffusion dominates when the field frozen to electrons alone and induces a drift due to the differential ion-electron motion

$$\mathbf{v}_{\mathrm{H}} = -\frac{\mathbf{J}}{\mathbf{e n_e}}$$

- The Hall effect depends on the field direction and can either reinforce or entirely suppress the MRI

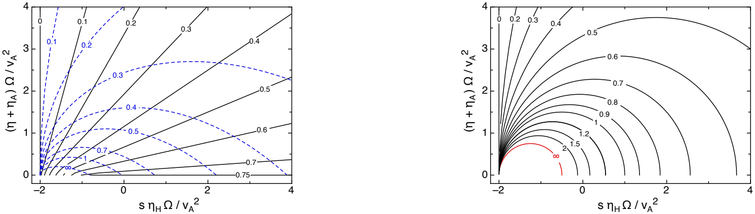


Non-Ideal Effects in Protoplanetary Discs

Figure: (Sano, Stone, 2002)

- Compare diffusion terms:
  - $\frac{O}{I} \equiv \frac{1}{Re_M}$
  - $\frac{A}{I} \equiv \frac{\Omega}{\gamma \rho_i}$
  - $\frac{H}{I} \equiv \frac{X}{2}$
- Assume equilibrium:
  - $\sigma_c = \frac{e^2 n_e}{m_e n_n \langle \sigma v \rangle_e}$
  - $\eta = \frac{c^2}{4\pi\sigma_c}$
  - $X = \frac{\eta\Omega}{2v_A^2}$
- Typical protoplanetary disks are Hall/Ohm dominated in inner regions and Ambipolar-dominated beyond $r \sim 20$AU
- Hall diffusion may be very important for mass accretion

Non-Ideal Effects: Linear Regime



Maximum growth rate (Black), wavenumber (Blue) and largest stable wavenumber (Right).

(Wardle, Salmeron 2012)

- For $\mathbf{B} = \mathbf{s}\mathbf{B}\hat{\mathbf{z}}$ $(s = \pm 1)$ under perturbations $\exp(\nu t - ikz)$
- Weakly coupled electron-ion-neutral plasma:
  - $\eta_{\rm A} = \dfrac{B^2}{4\,\pi\,\gamma_i\,\rho\rho_i}$
  - $\eta_{\rm H} = \dfrac{c\,B}{4\,\pi\,e\,n_e}$
- Pure ohmic and ambipolar diffusion tend to decrease the growth rate and increase the maximum wavelength of perturbations
- Hall Diffusion increases the maximum growth rate

Non-Ideal Effects: Linear Regime

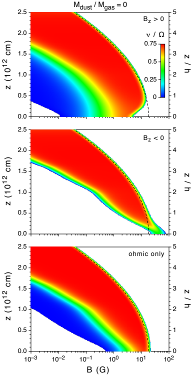
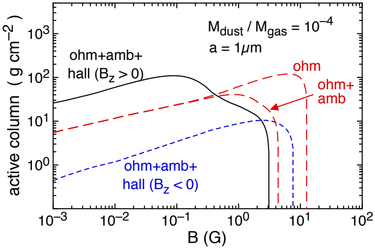


Figure: Maximum growth rate vs. height above mid-plane. (Wardle, Salmeron, 2012).



- Hall diffusion does stabilise (destabilise) the disk compared to ohmic diffusion alone
- Up to 2 orders of magnitude difference in active layer column density

Non-Ideal Effects and Dust

- Effect of Hall Diffusion probably dwarfed by uncertainty in dust fraction
- No grains: Coupling can probably be maintained at midplane
- 1% mass in grains (early evolution), no significant active layer
- If grains remain small (turbulence), no significant active layer

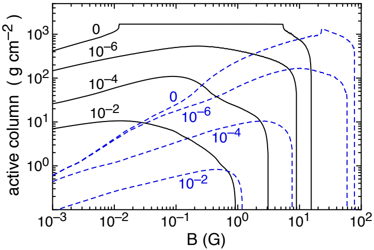


Figure: Active layer size at 1 AU for positive and negative magnetic fields and various dust mass fractions with $a = 1\mu m$. (Wardle, Salmeron, 2012)

Non-Ideal Effects: Simulations

- No guarantee that linear conditions will guarantee a steady state of MHD turbulence and outwards transport of angular momentum

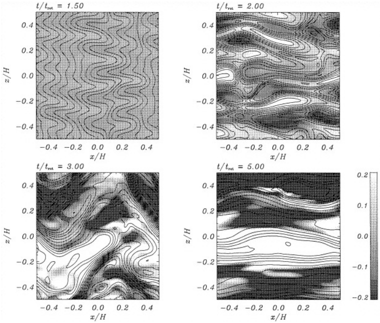


Figure: Radial Velocity and Magnetic Field. (Sano, Stone, 2002)

- Non-linear effects captured in 2-fluid simulations
- Small initial perturbations in gas pressure $\sim 10^{-6}$
- Angular Momentum Transport as in ideal MHD

Non-Ideal Effects: Simulations

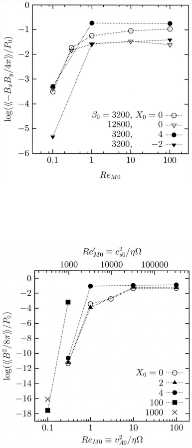


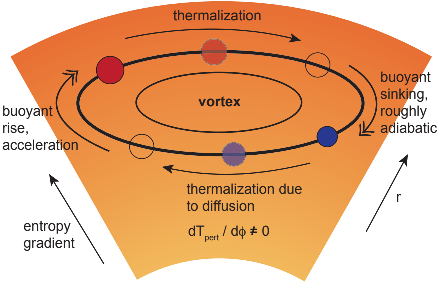
Figure: (Sano, Stone, 2002)

- Ohmic diffusion condition remains the same
- Hall diffusion marginally changes the saturation stress
- Hall diffusion has no effect on the critical Reynolds number
- But don't probe regime of hall domination $X\mathrm{Re}_M > 2$ and $\mathrm{Re}_M < 1$

Hydrodynamic Instabilities

- Angular Momentum Transport may be achieved with pure hydrodynamic instabilities:
  - Convection
  - Planet-Driven Evolution
  - Baroclinic Instability
- Likely to be subdominant to MHD turbulence
- Can be important in dead zones

Baroclinic Instability



- Radial entropy gradients and efficient cooling produce vorticity
- Particles moving inwards are cooler, drawn to lower orbits
- Efficient thermal diffusion heats them up along Keplerian orbits

Momentum Transport

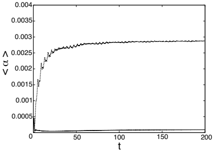


Figure: (Lesur, 2010)

- Vorticity if: Convectively unstable

$$N_r^2 = -\frac{1}{\Gamma\rho}\frac{\mathrm{d}P}{\mathrm{d}r}\frac{\mathrm{d}}{\mathrm{d}r}\ln\left(\frac{P}{\rho^\Gamma}\right) < 0$$

- Efficient cooling
- Significant initial perturbation (Subcritical)

- 2D simulations show growth of vorticity and weak angular momentum transport
- Not necessarily the same in 3D

Conclusions

- MRI turbulence important in mass accretion for protoplanetary disks
- Leads to layered disk structure, with accretion through a thin active layer
- Still large uncertainties concerning:
  - Exact Modelling of Disk (MMSN)
  - Dominant Ionization Sources
  - Recombination Rate and the large part played by dust grains
  - Behaviour of Hall Diffusion in the non-linear regime
- Baroclinic Instability possible source of additional accretion in dead zones

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