Turbulence in GMCs

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Seminar in theoretical astrophysics

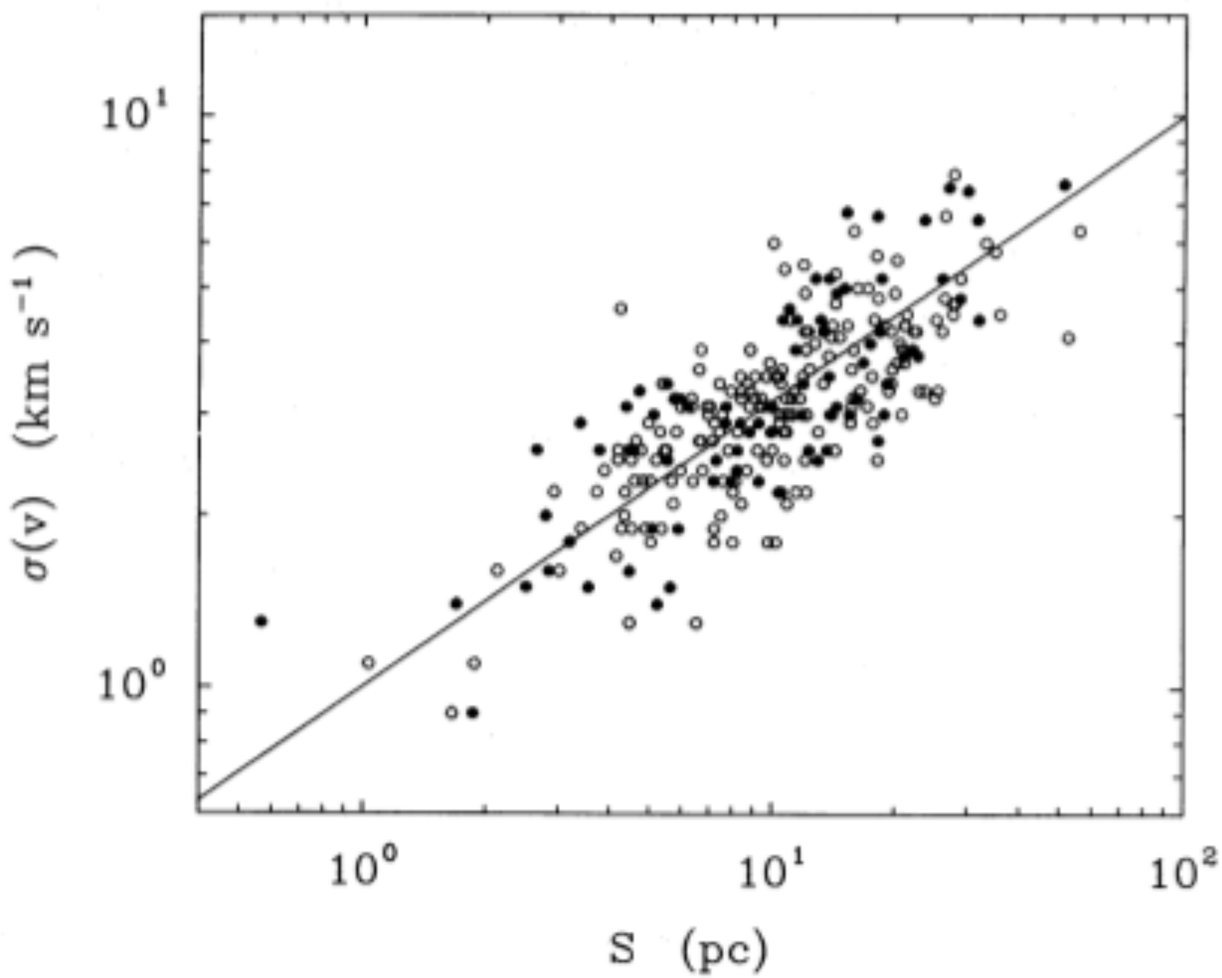
Overview

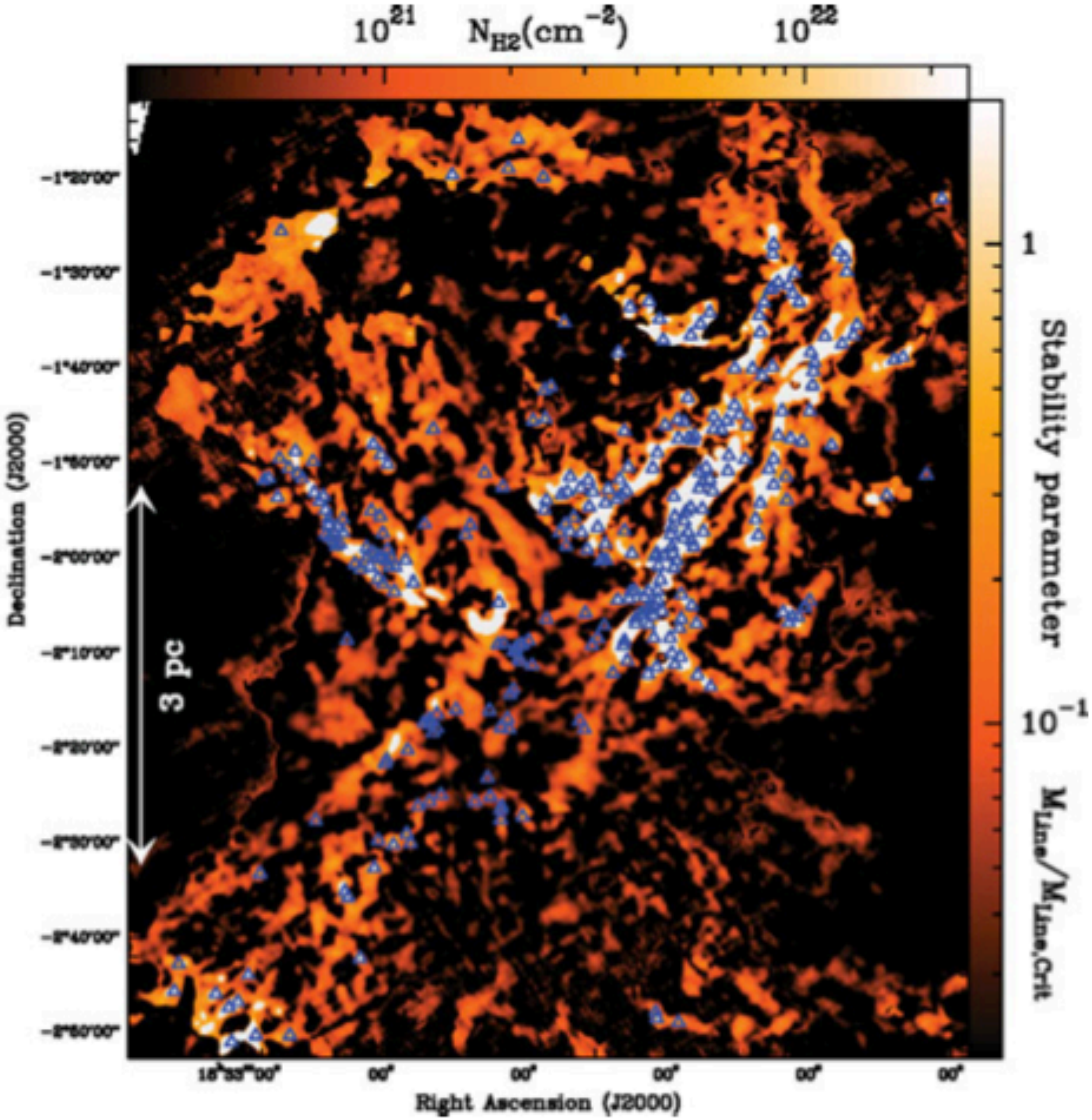
- Observed properties
- Turbulence models in different regimes
- Numerical simulations
- Turbulent dissipation timescales
- Density structure

Observed turbulence properties

- Contain the most of the mass in the molecular ISM. Diameter greater than 10 pc and masses greater then 10^5 solar masses, minimum local density 100 cm^{-3}
- Generally supersonic turbulence in a large range of scales (sonic scale is 0.03 pc)

Solomon et al., 1987





10^{21}
$N_{H2}(cm^{-2})$
10^{22}
Stability parameter
1
10^{-1}
$M_{Line}/M_{Line,Crit}$
Declination (J2000)
Right Ascension (J2000)
3 pc

What is turbulence?

- “Turbulence or turbulent flow is a flow regime characterized by chaotic and stochastic property changes.” (Wikipedia)
- Reynolds number

$$Re \propto \frac{v\nabla v}{\nu \Delta v} \propto \frac{l v_l}{\nu} \gg 1$$

Mathematical tools

- Structure functions $S_p(\mathbf{r}) =< |\mathbf{v}(\mathbf{x}) - \mathbf{v}(\mathbf{x}+\mathbf{r})|^p >$
- Autocorrelation functions

$$A(\mathbf{r}) =< \mathbf{v}(\mathbf{x}) \cdot \mathbf{v}(\mathbf{x}+\mathbf{r}) >=< |\mathbf{v}(\mathbf{x})|^2 > - \frac{S_2(\mathbf{r})}{2}$$

- Power spectrum $P_v(\mathbf{k}) = |\mathbf{v}(\mathbf{k})|^2$

Isotropic turbulence

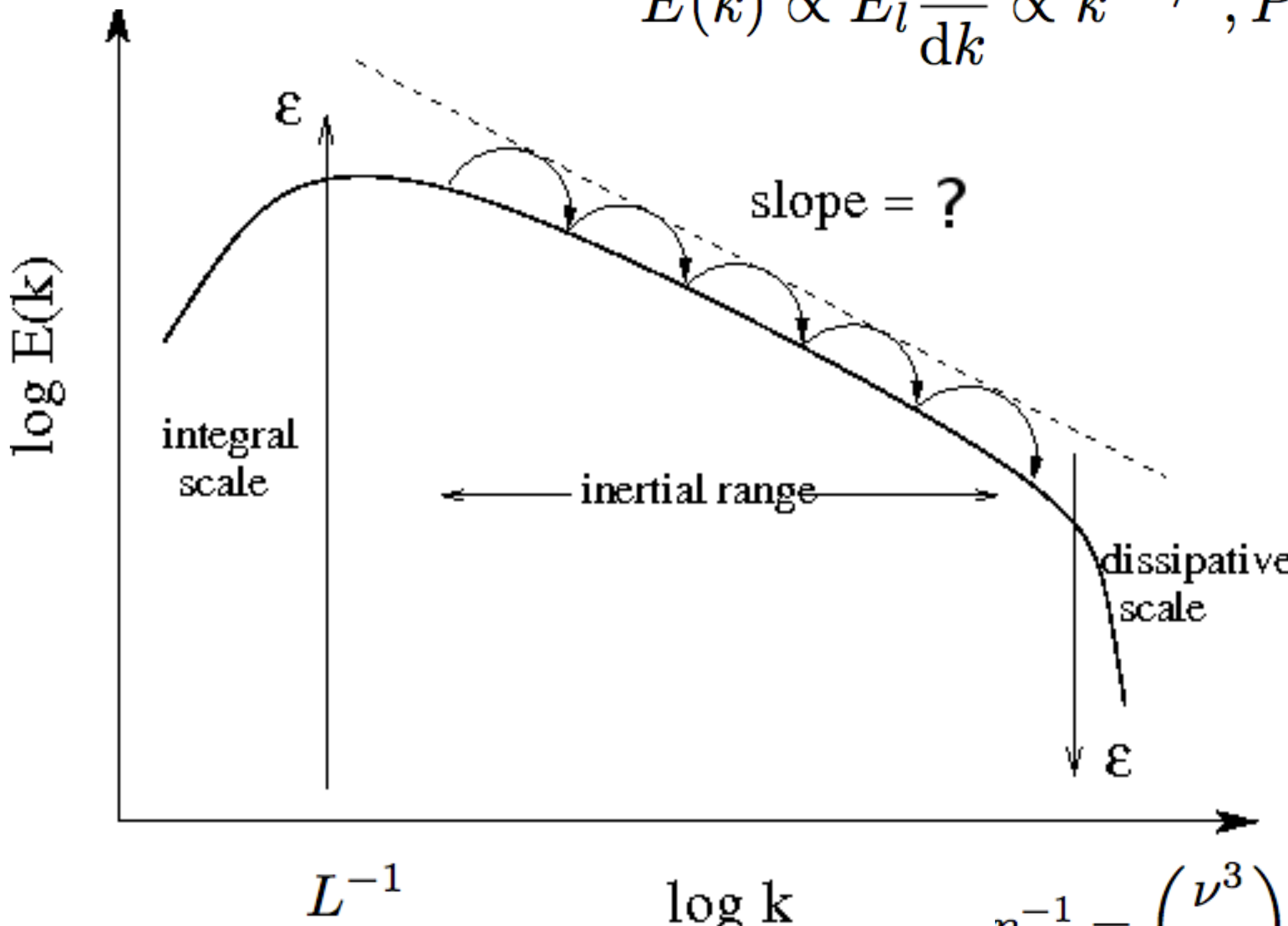
(A.N. Kolmogorov, 1941)

- **Hypothesis 1**. For very high Reynolds numbers the small scale turbulent motions are statistically isotropic (i.e. no preferential spatial direction could be discerned).

- **Hypothesis 2**. For very high Reynolds numbers the statistics of small scales are universally and uniquely determined by the viscosity and the rate of energy dissipation.

$$\epsilon \propto \frac{v_l^3}{l}, v_l \propto l^{1/3}, E_l \propto \frac{v_l^2}{l} \propto l^{-1/3}$$

$$E(k) \propto E_l \frac{\mathrm{d}l}{\mathrm{d}k} \propto k^{-5/3}, P_v(k) \propto k^{-11/3}$$



$$\eta^{-1} = \left(\frac{\nu^3}{\epsilon}\right)^{-1/4}$$

MHD turbulence

(Goldreich & Sridhar, 1995)

- Critically balanced anisotropic cascade - the nonlinear mixing time perpendicular to the magnetic field and the propagation time along the magnetic field remain comparable for Alfven wavepackets at all scales.
- Since Alfven wave packets are exact solutions of incompressible MHD the interaction occurs only between wavepackets moving in opposite directions.
- The energy transfers produced by these collisions involve primarily k perpendicular (confirmed by both observations and numerics).

- As the energy cascade proceeds to smaller scales, turbulent eddies progressively become elongated along the large-scale field.
- The slope is the same in Kolmogorov model.

$$k_z V_A \propto v_l k_\perp$$

$$\epsilon \propto \frac{V_A^3}{L} \propto \frac{v_l^2}{t_{cas}}, t_{cas} \propto \frac{1}{k_z V_A}$$

$$k_z \propto k_\perp^{2/3} L^{-1/3}, v_l \propto V_A (k_\perp l)^{-1/3}$$

$$E(k_\perp) \propto k_\perp^{-5/3}$$

Supersonic turbulence (compressible flow)

- Some portion of the energy at a given scale must be directly dissipated via shocks, rather than cascading conservatively through intermediate scales until viscosity scale is reached.
- In the limit of zero pressure (Burgers, there are comprehensive analytical models!) the system will be completely driven by shocks

In 1D almost obviously $P_v \propto k^{-2}$

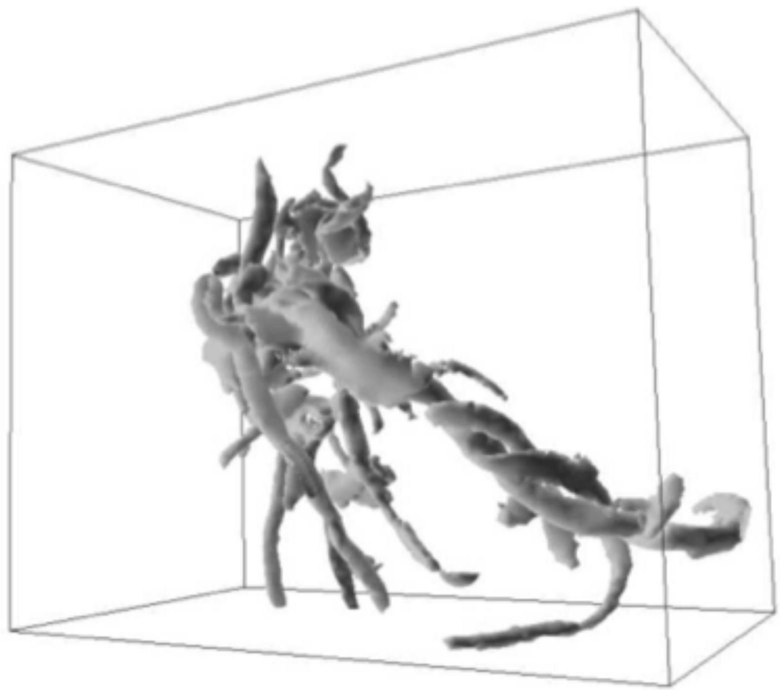
Problems

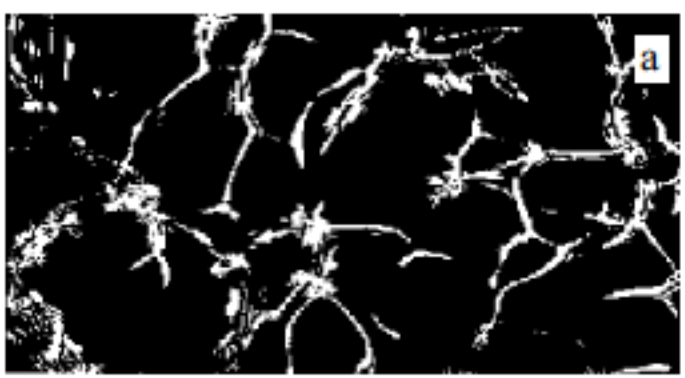
- No adequate model in the regime of strong compressibility and and moderate/strong magnetic fields
- Formation of coherent structures with probability one, deviations of high-order structure functions, non-gaussianity of velocity derivatives and increments pdfs, higher order statistics should be analyzed

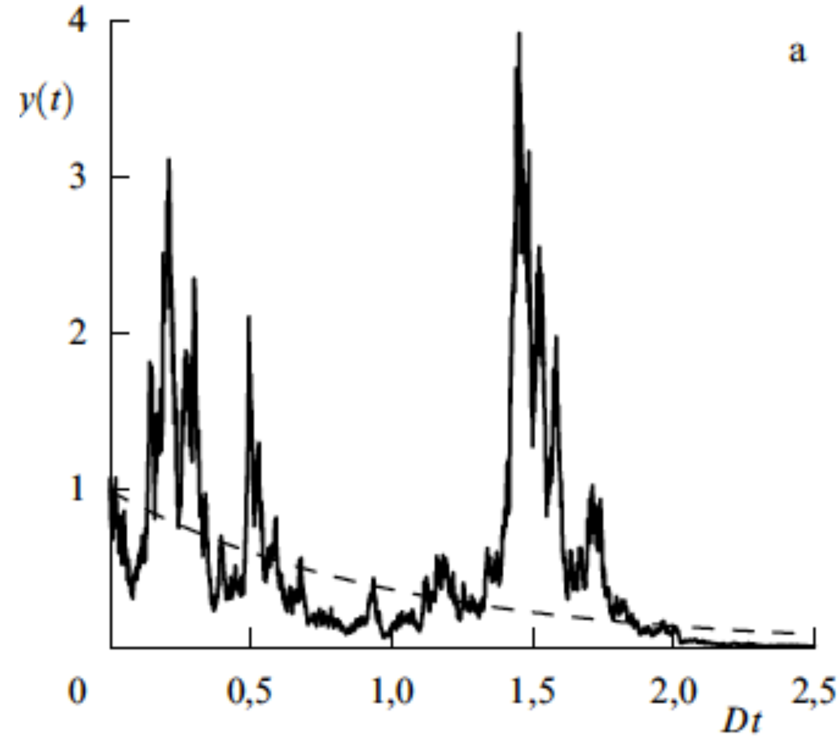
Intermittency 1

- Larger spatial and temporal velocity fluctuations at smaller scales.
- Energy is not evenly distributed in space and time by the cascade: the active sub-scales do not fill all the space

Intermittency 2







Solved for other problems, in particular for Schrodinger equation with random potential (Anderson localization, propagation of waves in turbulent media), but not for turbulence!

Numerical results

- Slope (of P_v) for incompressible MHD 3.5 – 3.67
- 3.5 – 4.0 for strongly compressible flows (3.95 in Kritsuk et al. (2007), 2048^3)
- In the limit of strong magnetic field results are consistent with Goldreich model.
- The power spectra below the driving wavenumber scale is nearly flat, so limited inverse cascades.
- For spatially localized forcing Nakamura & Li (2007) also found a break in the power spectrum, at wavelength comparable to the momentum injection scale. If v(l) continues to rise up to l ~ *L*, the overall scale of a system, this implies that turbulence is either (a) externally driven, (b) imposed in the initial conditions when the system is formed, or (c) driven internally to reach large scales.

From Beresnyak (2012)

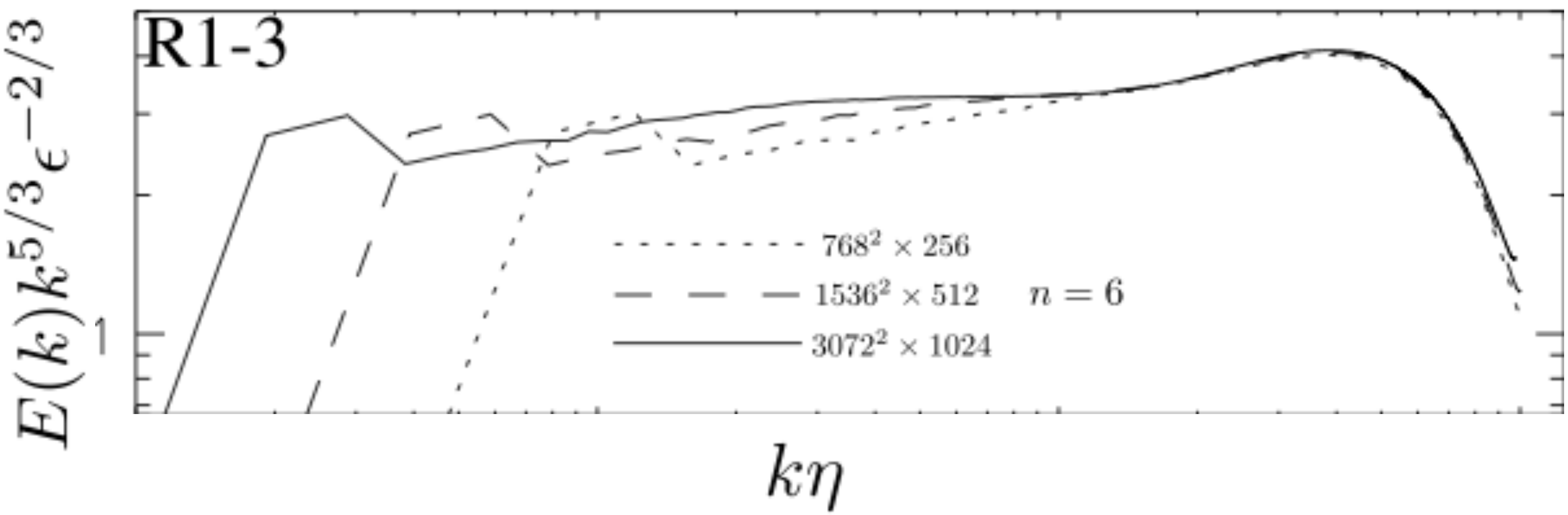


Table 1. Three-dimensional RMHD simulations

Run	$n_x \cdot n_y \cdot n_z$	Dissipation	$\langle\epsilon\rangle$	L/η
R1	$256 \cdot 768^2$	$-6.82 \cdot 10^{-14} k^6$	0.073	200
R2	$512 \cdot 1536^2$	$-1.51 \cdot 10^{-15} k^6$	0.073	400
R3	$1024 \cdot 3072^2$	$-3.33 \cdot 10^{-17} k^6$	0.073	800

Dissipation timescale

- Dimensional analysis gives $$\dot{E}_{turb} \propto \epsilon \frac{U^3}{l_0}$$
- For incompressible MHD $$\epsilon = \frac{1}{2}\frac{\dot{E}_{turb}}{E_{turb}}\frac{l_0}{U} \approx 0.6$$

0.6 – 0.7 for compressible (Stone, Ostriker & Gammie (1998)).

- Dissipation time can be estimated as

$$t_{diss} = \frac{E_{turb}}{|\dot{E}_{turb}|} \approx 0.5\frac{L}{\sigma_{los}} \approx 1Myr, U = \sqrt{3}\sigma_{los}$$

What can influence damping?

- Alfven waves moving in the same direction, no turbulent cascade. Inbalance in the flux of up and downward propagating packets (Maron & Goldreich (2001)).

Density structure

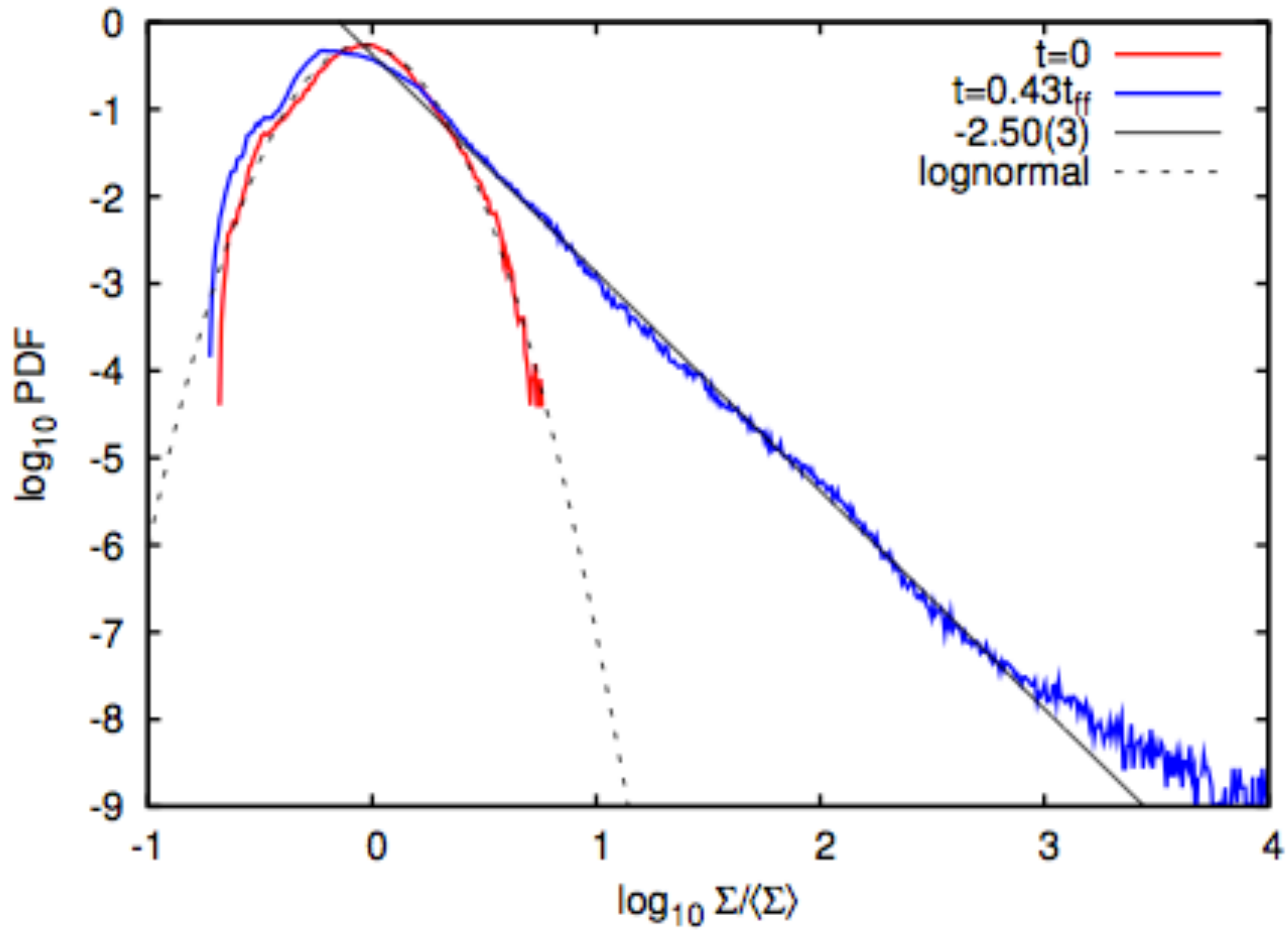
- Supersonic motion in a wide range of scales. Crusial for star formation theory.
- Log-normal distribution for almost all regimes (Ostriker, Stone & Gammie 2001; Ostriker 2003, Nordlund & Padoan 1999, Klessen 2000) with power-law tales.

$$f_{V,M} = \frac{1}{\sqrt{2\pi\sigma_x^2}} \exp\left[\frac{-(x \pm |\mu_x|)^2}{2\sigma_x^2}\right] \qquad \mu_x = \sigma_x^2/2$$

$$\rho_{\rm med} = \bar{\rho}\exp(\mu_x)$$

- 3D Hydro sims Padoan, Jones & Nordlund (1997) $\mu_x \approx 0.5 \ln(1 + 0.25\mathcal{M}^2)$
- MHD (Ostriker, Stone and Gammie) – confirmed the increasing mean density with increasing turbulence, but no one-to-one relation with Mach number.

Kritsuk et al. (2011)



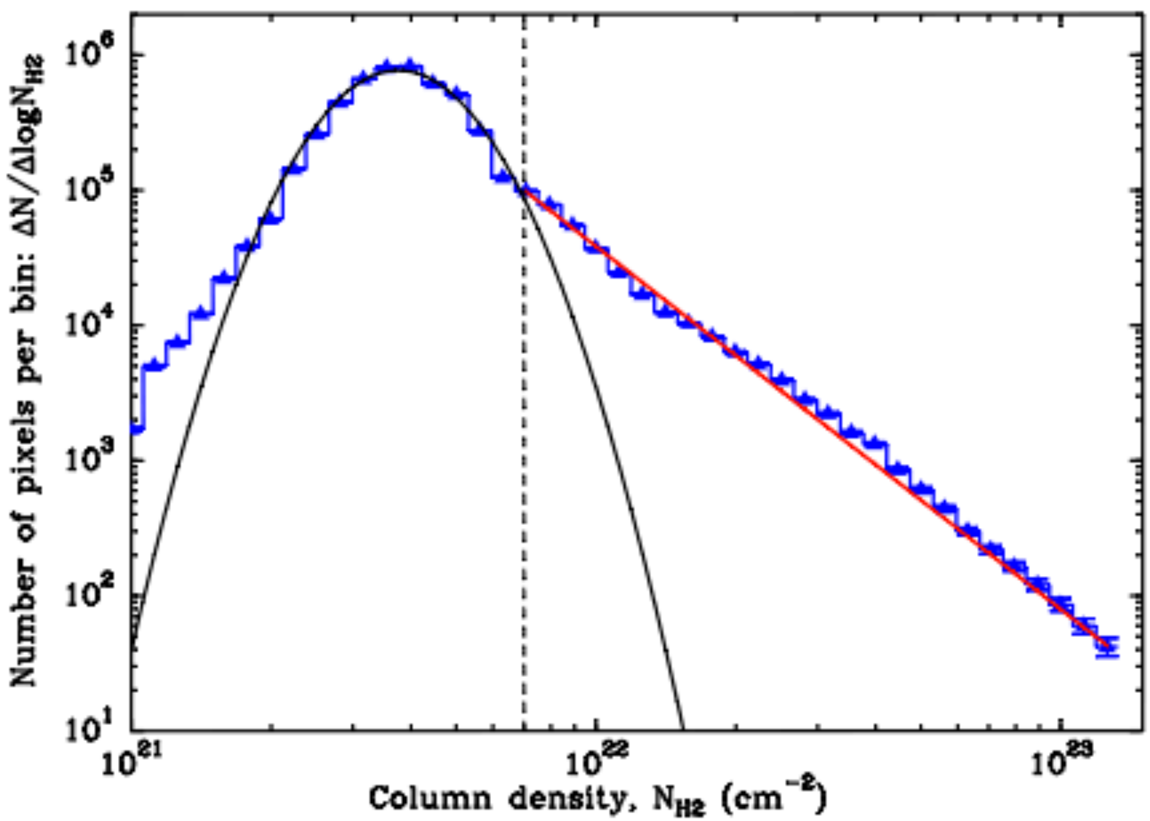


Fig. 9. Column density PDF of filaments in the Aquila star-forming molecular cloud (André et al 2011). The $N_{\rm H}$ values are inferred from the submillimeter dust emission measured by Herschel/SPIRE. Note the log-normal and power-law parts.