Dynamics of Gas and Grains II

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Outline

How do we get planetesimals?

- Intermediate particles' orbits decay quickly
- Hurdle: $1\ \mathrm{cm}$–$1\ \mathrm{km}$ ($E_{\mathrm{bind}} \ll E_{\mathrm{kinetic}}$)
- Solar wind time limit
- Growth must proceed quickly
- Difficult to accomplish with gravity

Overview of Drag Instability

- Environment
    - Disk with particles and gas
    - Settled particles
    - Drag coupling slows particles coherently
- Mechanism
    - $\rho \quad \uparrow$
    - $L \quad \uparrow$
    - $\dot{r} \quad \downarrow$
    - $\rho \quad \uparrow$

Application of Drag

- Minimal drift, particle-dominated limit
- Growth rate $\approx 0.49(\Sigma_\mathrm{p}/S_1)^{-2/5}\nu_*^{-1/5}$
    - $S_1 \sim 0.28\ \mathrm{g/cm^2}$
    - $\nu_* \sim 2.3 \times 10^{-5}$
- Exponential growth in inward-drifting frame
- Competitive with collisions

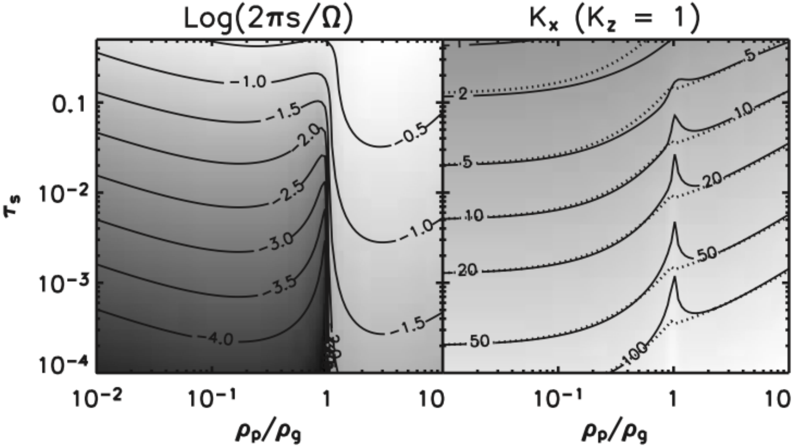
Streaming Instability — Background

- Gas dragging particles ⇒ particles affected by gas
- Two mixed fluids
- No vertical differentiation
- No self-gravity
- No turbulence
- Incompressible gas
- Stopping times

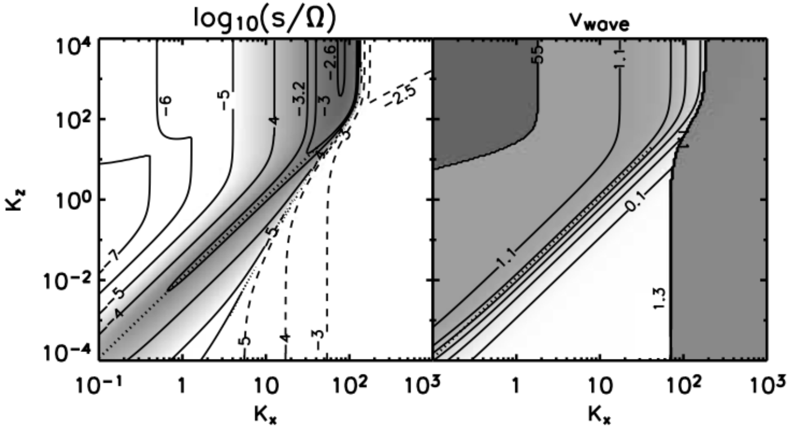
$$\tau_s = \Omega_K \times \begin{cases} \frac{\rho_s a}{\rho_g c_g}, & a < \frac{4}{9}\lambda_{\mathrm{mfp}} \text{ (Epstein)} \\ \frac{\rho_s a}{\rho_g c_g}\left(\frac{4a}{9\lambda_{\mathrm{mfp}}}\right), & a < \frac{4}{9}\lambda_{\mathrm{mfp}} \text{ (Stokes)} \end{cases}$$

Features of Streaming

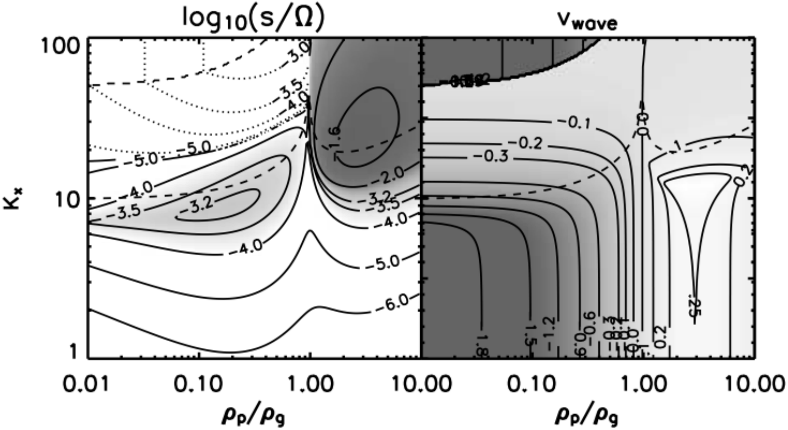
- 6-th order dispersion relation
    - 3 quickly decaying modes
    - 2 epicycles
    - 1 secular mode
- Growth rate dependence
    - $\rho_\mathrm{p}/\rho_\mathrm{g} \to 0, \infty \quad \downarrow$
    - $\rho_\mathrm{p}/\rho_\mathrm{g} \to 1 \quad \downarrow$



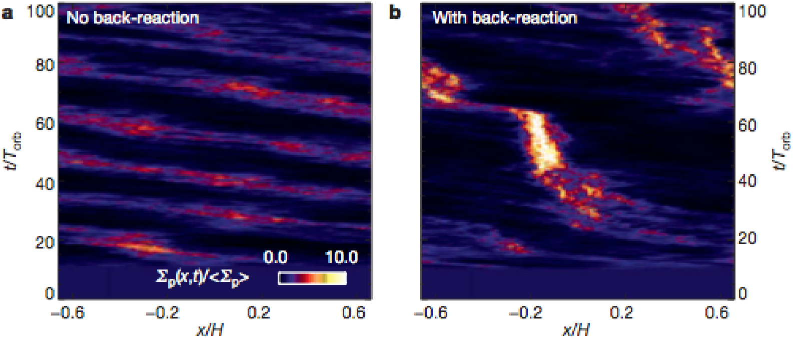
Best growth obtained varying radial wavenumber. (Youdin & Goodman 2005)



Growth and wave speed for $\rho_p/\rho_g = 0.2$, $\tau_s = 0.01$. (Youdin & Goodman 2005)



Growth and wave speed for $\tau_s = 0.01$, $K_z = 1$. (Youdin & Goodman 2005)



(Johansen et al. 2007)

Streaming Summary

- Growth faster than diffusion: $K \lesssim 2\pi\sqrt{s\eta}\Omega\alpha$
- Maximum growth: $(k/2)(V_{\mathrm{max}} - V_{\mathrm{min}})$
- Typical conditions: $\sim 10^{18}$–10^{20} g

Formation of Vortices

- Need turbulence
- Coriolis force at large scales
- Instability or mergers
- Shear $\Rightarrow$ anticyclonic
- Grow to size of disk thickness and velocity of sound speed
- Separated radially

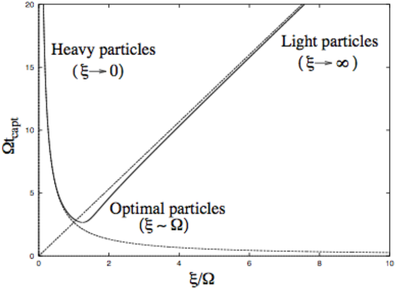
Particle Capture

- Balance of Coriolis, centrifugal, and friction forces
- Elliptical vortex (aligned with shear), $q = a/b$
- Light particles:

$$t_{\text{capt}} = \frac{4\xi}{3\Omega^2} \frac{q(q-1)^2}{(q-2)(2q+1)}$$

- Heavy particles:

$$t_{\text{capt}} = \frac{1}{\xi} \frac{2q(q-1)}{(q-3)(2q+1)}$$



Capture times. (Chavanis 2000)

Particle Capture

- Optimal trapping:

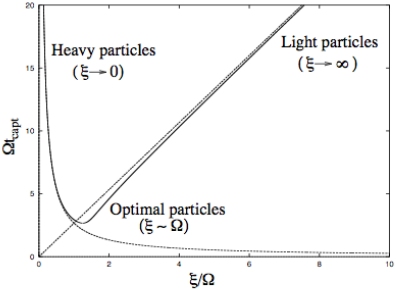
$$\xi_{\mathrm{opt}} \approx \left(\frac{3(q-2)}{2(q-1)(q-3)}\right)^{1/2}\Omega$$

$$t_{\mathrm{capt}}^{\mathrm{opt}} \approx \left(\frac{8(q-1)^3 q^2}{3(q-3)(2q+1)^2(q-2)}\right)^{1/2}\frac{1}{\Omega}$$

- Condition for any trapping

$$q > 3$$

$$-\frac{5}{2}\Omega < \omega < -\frac{3}{2}\Omega$$



Capture times. (Chavanis 2000)

Mass Accumulation

- Particles concentrated by turbulence
- Particles brought in by shear
    - Vortex mass

$$\dot{M} = \frac{3}{2}\Sigma_{\mathrm{p}}\Omega R^2 \left(f(\xi)\right)^2$$

$$M \to \frac{3}{2}\left(\Omega t_{\mathrm{life}}\right)\Sigma_{\mathrm{p}} R^2 \left(f(\xi)\right)^2$$

    - Heavy vs. light

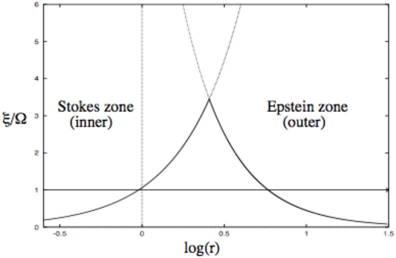
$$f(\xi) \approx \begin{cases} \left(\frac{\Omega}{\xi}\right)^{1/2}, & \text{light} \\ \frac{\xi}{\Omega}, & \text{heavy} \end{cases}$$

Application to Solar System

- Minimum mass nebula
- Stokes vs. Epstein

$$\frac{\xi}{\Omega} = \begin{cases} \frac{1913}{a^2\rho_s} r^{5/4}, & r < r_c \\ \frac{850}{a\rho_s} r^{-3/2}, & r > r_c \end{cases}$$

$$r_c = \left(\frac{4}{9}\frac{a}{1\ \mathrm{cm}}\right)^{4/11}\ \mathrm{AU}$$



$a = 30$ cm, $\rho_s = 2$ g/cm^3. (Chavanis 2000)

Application to Solar System

- Transition

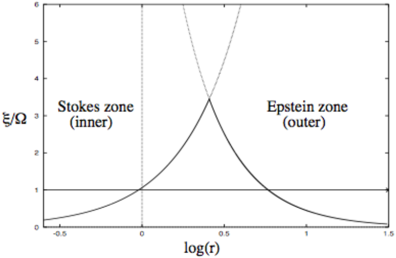
$$1.7\ \mathrm{AU} < r_c < 3.9\ \mathrm{AU}$$

- Interior optimum near $1\ \mathrm{AU}$

$$r_{in} = \left(\frac{a^2\rho_s}{1913}\right)^{4/5}$$

- Exterior optimum near $6\ \mathrm{AU}$

$$r_{out} = \left(\frac{850}{a\rho_s}\right)^{2/3}$$



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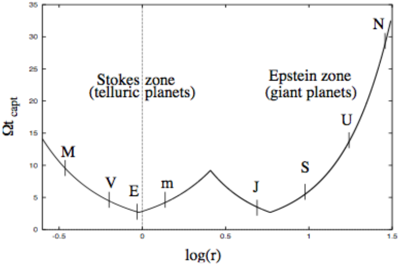

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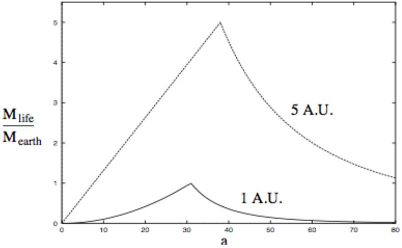

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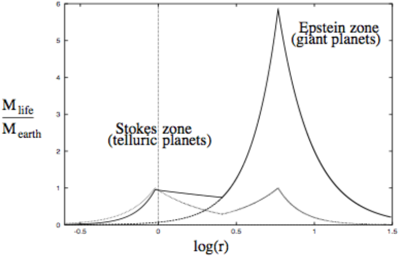

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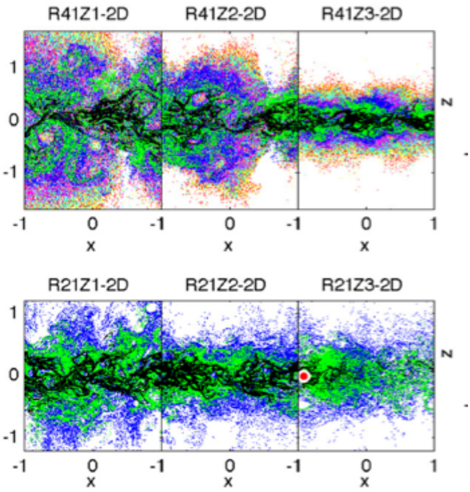
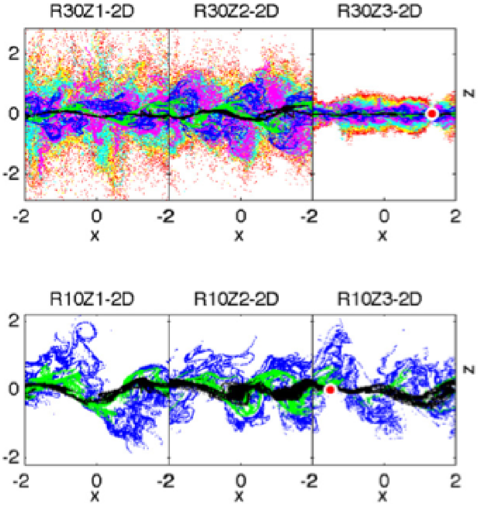
Simulation Considerations

- Hydrodynamic
- Grain sizes
- Disk structure
- 2D vs. 3D
- Pure hydro vs. MHD



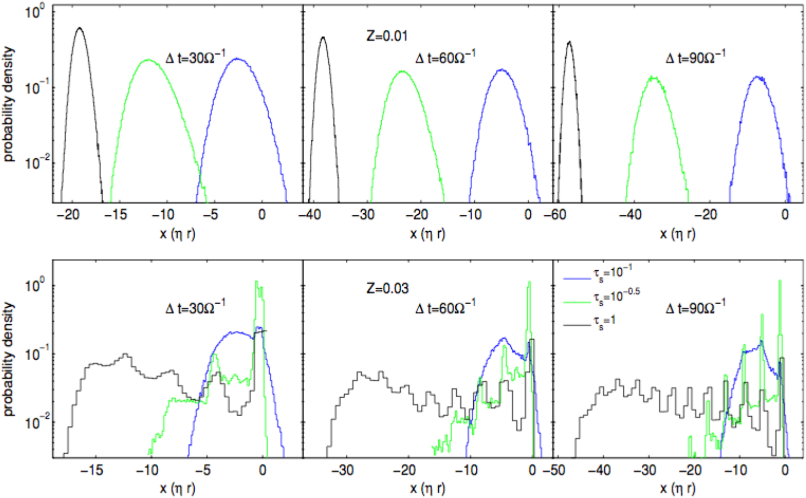
Short stopping times. (Bai and Stone 2010)



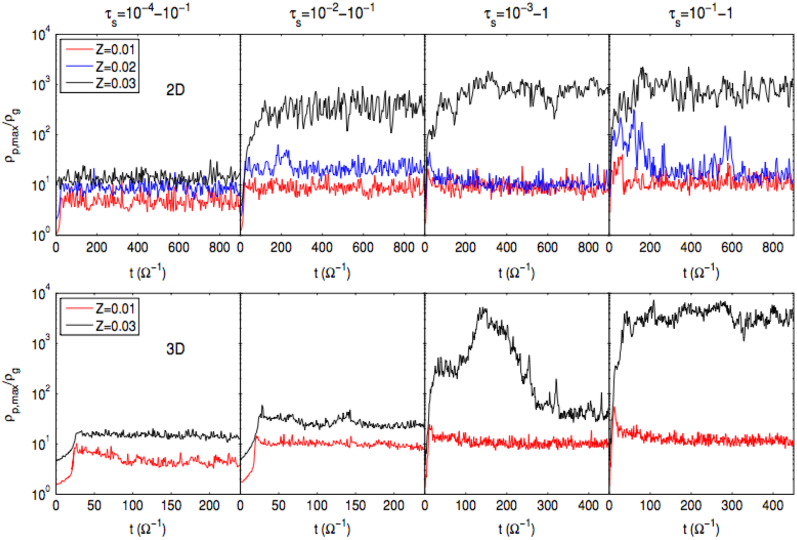
Long stopping times. (Bai and Stone 2010)

Simulation Results

- Streaming instability
    - $\tau_{\mathrm{s}} \gtrsim 10^{-2}$
    - Prevents Kelvin-Helmholtz
    - Large, abundant particles
- Range of particle sizes
    - Clumping counters radial drift
    - Reduced collisional velocity
- Formation in dead zone



Radial drift. (Bai and Stone 2010)



Particle density. (Bai and Stone 2010)

Conclusion

- Need mechanism to grow to km sizes
- Drag hurts and helps
- Streaming instability critical
- Vortical structure may play role
- Seen in simulations
- Generality?

References

Xue-Ning Bai and James M. Stone.
Dynamics of Solids in the Midplane of Protoplanetary Disks: Implications for Planetesimal Formation.
The Astrophysical Journal, 722:1437–1459, October 2010.

P. H. Chavanis.
Trapping of dust by coherent vortices in the solar nebula.
Astronomy and Astrophysics, 356:1089–1111, April 2000.

J. Goodman and B. Pindor.
Secular Instability and Planetesimal Formation in the Dust Layer.
Icarus, 148:537–549, December 2000.

Anders Johansen, Jeffrey S. Oishi, Mordecai-Mark Mac Low, Hubert Klahr, Thomas Henning1, and Andrew Youdin.
Rapid planitesimal formation in turbulent circumstellar disks.
Nature, 448:1022–1025, August 2007.

Andrew N. Youdin and Jeremy Goodman.
Streaming Instabilities in Protoplanetary Disks.
The Astrophysical Journal, 620:459–469, February 2005.