Name: ____ Due September 23, 2003 9:20 a.m.

Physics/Astronomy/Planetary Sciences 589 Homework 2

This homework is due on the 23th of September in class and is based on the diffusion lectures and notes of the week of September 8th.

1) A sphere of U-235 of radius R is presented to you. The neutron density in the sphere is described by a diffusion equation with a source term that is given by:

$$j = -\sigma_a nvN + (\nu - 1)\sigma_f nvN \, ,$$

where σ_a is the neutron absorption cross section ($\sim$0.1 barns), σ_f is the fission cross section ($\sim$1 barn), ν is the average number of neutrons emitted per fission ($\sim$2.5), v is the mean speed of the neutrons ($\sim 10^5$ cm s^{-1}), n is the neutron density, and N is the number density of the uranium nuclei ($\sim 5 \times 10^{22}$ cm^{-3}). Determine the critical radius (R) above which any stray neutrons would precipitate a run-away chain reaction. What is the mass of this critical sphere (assume that the mass density of uranium is 18.5 gm cm^{-3})? Assume that the neutron density at the outer boundary is $\sim$zero and that the scattering cross section for the purposes of calculating the diffusion coefficent is $\sim$10 barns. (Hint: This is actually an eigenvalue problem and is independent of the number of stray neutrons that might have initiated the chain reaction. Think steady-state.)

2) In the diffusion limit, the radiation energy flux in a static medium is:

$$F = -\frac{c}{3\kappa\rho}\frac{\partial E_R}{\partial r} \, ,$$

where E_R is the radiation energy density and κ is the total opacity. In general, the opacity can be a function of temperature and density. If $\kappa \propto T^n \rho^m$, one can calculate the diffusion of energy by radiation using the energy conservation equation. The relevant equation involves the total energy density (E_T) in the time derivative, but the radiation energy density ($E_R = aT^4$) in the equation for the flux. If $\kappa\rho$ were a constant and the specfic heat (C_V) of the material were a constant, and if E$_R$ were proportional to T (very wrong), this would be linear diffusion and $< r^2 >$ for the approximate extent of the hot region would be $\propto t$ (time) ("linear" diffusion). Using dimensional analysis, derive the corresponding relation for the non-linear diffusion of radiation when the opacity is of the power-law form above and the specific heat of matter+radiation is proportional to T^q. Discuss some interesting example cases. (Hint: The result is not a function of local temperature.)

3 At what mass density (ρ) would the baryonic rest mass-energy density equal the energy density of blackbody radiation at T =1 MeV? Comment.