The Van Cittert-Zernike Theorem:
Theory, History, and Applications

Kaitlyn Shavelle, Princeton University

Grad Seminar (AST 542)

This *was* a talk on Radiative Transfer codes...

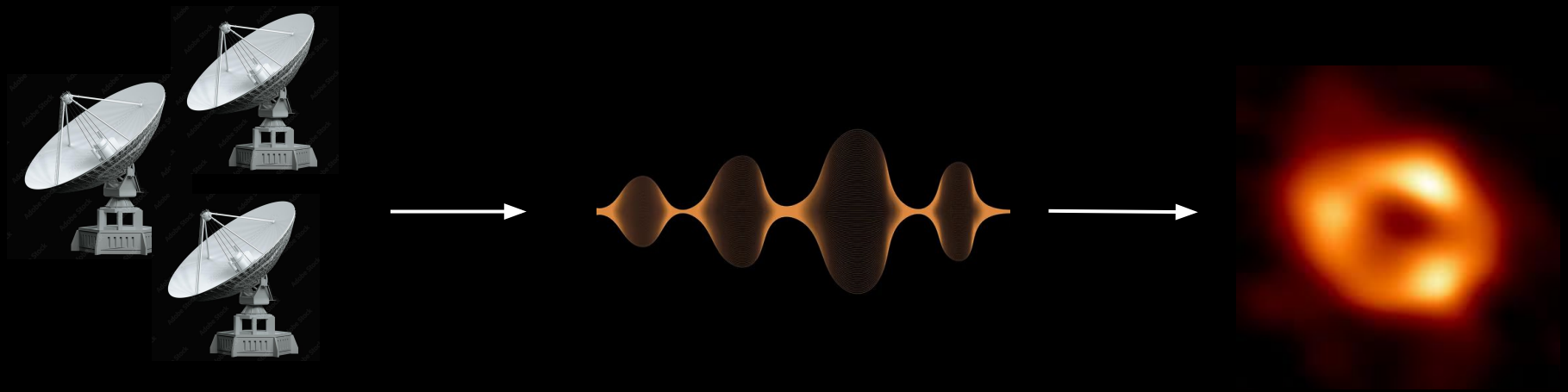
(but Nick and Philippe did such good jobs that I had to pivot.)

- **Background:** basics of interferometry and coherence theory.
- **Mathematical Foundations:** the van Cittert-Zernike Theorem.
- **Broader Applications:** the van Cittert-Zernike Theorem for an example interferometric array.
- **Resources:** simple coding example and how to find interferometric data.

Warning: includes a lot (and I mean *a lot*) of clickable links.

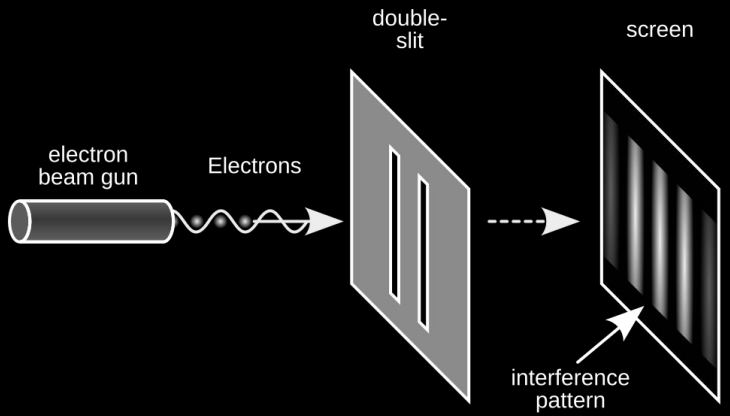
The golden rule: multiple telescopes are stronger than one.

- Simply, interferometry works by combining two or more signals to create a pattern to extract information about objects.



Coherence and interference...

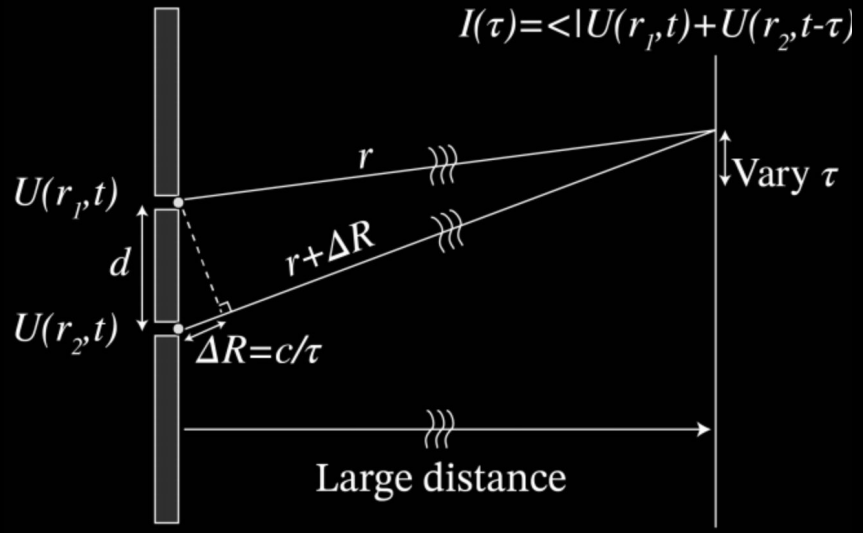
- **Young's Double-Slit Experiment:** interference pattern shows that light is a wave.
  - Temporal and Spatial.



More on Young's Double Slit Experiment

A very serious portrait of Thomas Young

Mutual Spatial Coherence Function



Mutual Spatial Coherence Function

- Understanding the spatial coherence of two points.

$$\tau = \frac{\Delta R}{c}.$$

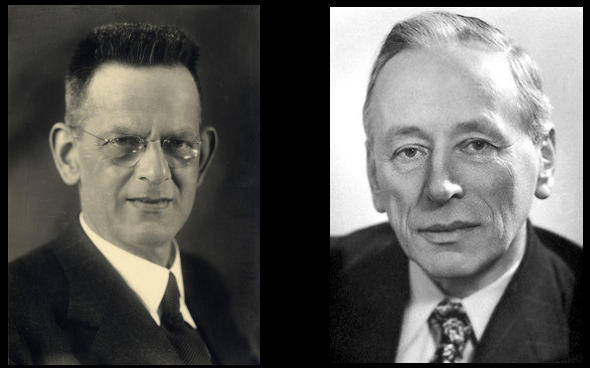
$$I(\tau) = \left\langle |U_1(\mathbf{r}, t) + U_2(\mathbf{r}, t)|^2 \right\rangle$$

$$= \left\langle |U(\mathbf{r}_1, t)|^2 \right\rangle + \left\langle |U(\mathbf{r}_2, t - \tau)|^2 \right\rangle + 2\,\mathrm{Re}\langle U(\mathbf{r}_1, t)\, U(\mathbf{r}_2, t - \tau)^* \rangle.$$

$$\Gamma_{12}(\tau) = \langle U(\mathbf{r}_1, t)\, U(\mathbf{r}_2, t - \tau)^* \rangle,$$

The Van Cittert-Zernike Theorem

- The “heart” of interferometry.
- **Spoiler alert:** there exists a relationship between the mutual spatial coherence function and the sky intensity distribution.
- Ducks?



Very serious portraits of
Pieter Hendrik van Cittert and Frits Zernike

The Van Cittert-Zernike Theorem

The Van Cittert-Zernike Theorem

$$\mathcal{V}(u,v) = \iint I(l,m)e^{-2\pi i(ul+vm)}dldm$$

Mutual spatial coherence function or the Visibility function

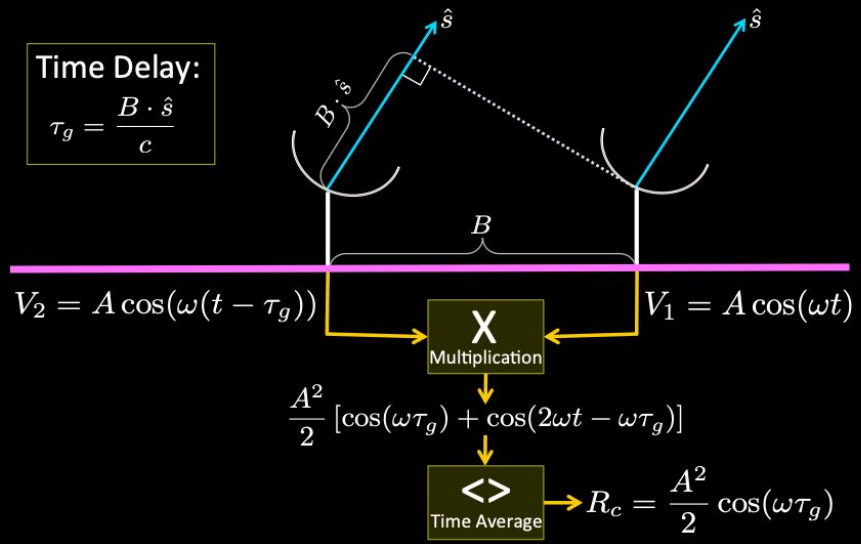
Sky intensity distribution

A less serious portrait of Joseph Fourier

Van Cittert-Zernike Theorem Explained

Van Cittert-Zernike Theorem in Interferometry

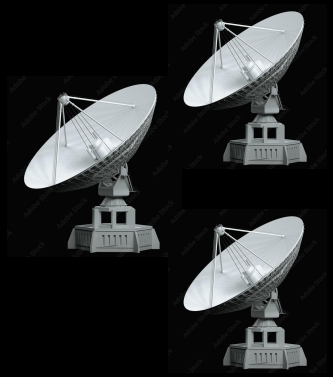
- Correlator: calculates the cross-correlation between signals received by different antennas in an array.
- Gives the "complex visibility."



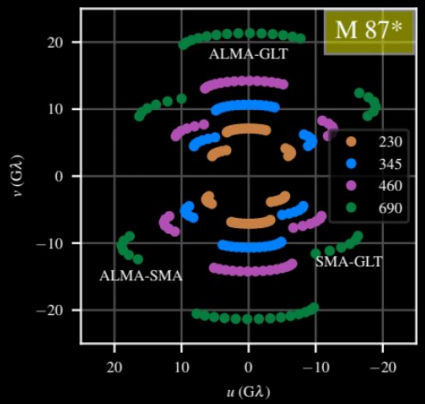
Simplified Interferometry Diagram

Van Cittert-Zernike Theorem in Interferometry

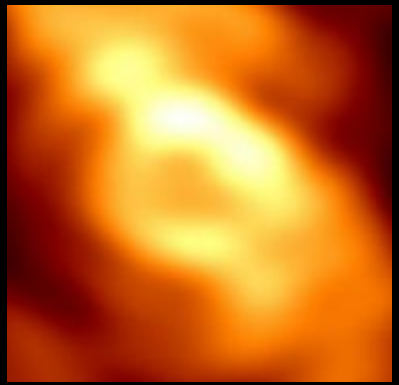
- With Fourier transform of sampled the visibilities, we can create an image.



Measured signal



Measured visabilities



FT of measured visabilities
= “dirty image”

Van Cittert-Zernike Theorem in Interferometry

- Fourier transform of sampled visibilities yields the true sky brightness convolved with the point spread function.
- Big problem: discrete points in the (u,v) plane are sampled.

$$I^D(x,y) = FT^{-1}\{B(u,v) \times V(u,v)\}$$

"Dirty image"

Fourier transform of sampling function at different points in the uv plane ("dirty beam")

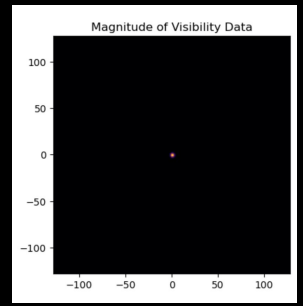
Visabilities

Related resources: try it yourself

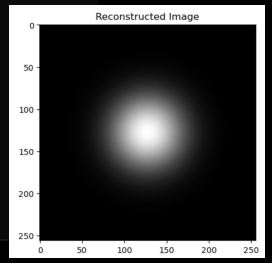
[7]:
```python
import numpy as np
import matplotlib.pyplot as plt

def gaussian_source(size, sigma):
    x = np.linspace(-size/2, size/2, size)
    y = np.linspace(-size/2, size/2, size)
    X, Y = np.meshgrid(x, y)
    return np.exp(-(X**2 + Y**2) / (2 * sigma**2))

size = 256
sigma = 30
source = gaussian_source(size, sigma)

visibility_data = np.fft.fftshift(np.fft.fft2(source))

plt.figure(figsize=(12, 6))

plt.subplot(1, 2, 1)
plt.imshow(np.abs(visibility_data), cmap='inferno', extent=[-size/2, size/2, -size/2, size/2])
plt.title('Magnitude of Visibility Data')
plt.colorbar()


plt.show()

reconstructed_image = np.fft.ifft2(visibility_data)

plt.imshow(np.abs(reconstructed_image), cmap='gray')
plt.title('Reconstructed Image')
plt.colorbar()
plt.show()
```

Defining some random gaussian source

Fourier transform of source to get visibility data



Reconstruction



Related resource: Using Interferometric Data

- **Interferometric Data Simulation:** simulating visibility data to work with the Van Cittert-Zernike theorem.
  - OmniUV - ground and space VLBI observations.
  - VLBI Reconstruction Dataset - VLBI data for a variety of different telescope arrays and targets.
    - EHT Imaging
  - CASA - software used in radio astronomy for processing interferometric data (ALMA and VLA).

Useful Links!

- Overview of the Van Cittert-Zernike theorem
- Coherence and complex signals
- Aperture synthesis explained
- Fourier transforms of images
- Interferometric Data: OmniUV , VLBI Reconstruction Dataset , EHT Imaging , CASA