Stellar timescales

One way to come to grips with the physics of stars is to look at the timescales on which they change, or on which important internal processes come to equilibrium.

Dynamical timescale

Suppose the internal pressure of the sun were suddenly removed. The outer radius, R, would collapse under gravity according to

$$\frac{d^2R}{dt^2} = -\frac{GM_\odot}{R^2(t)} \qquad R(0) = R_\odot \quad \frac{dR}{dt}(0) = 0. \tag{1}$$

The radius would shrink to zero after an elapsed "free-fall" time

$$t_{\rm ff} = \frac{\pi}{\sqrt{8}}\left(\frac{R_\odot^3}{GM_\odot}\right)^{1/2}. \tag{2}$$

Dimensional analysis gives a similar result: it is clear that the collapse time should involve the initial surface gravity $g_\odot \equiv GM_\odot/R_\odot^2$ and radius $R_\odot$ in the combination $\sqrt{R_\odot/g_\odot}$. Ignoring all dimensionless factors of order unity, the dynamical time is therefore

$$t_{\rm dyn} \equiv \left(\frac{R_\odot^3}{GM_\odot}\right)^{1/2} \approx 1600\,\mathrm{s}, \tag{3}$$

a bit less than half an hour. This differs from $t_{\rm ff}$ by only 11%.

While the sun is in no such danger of sudden collapse, this is the characteristic period on which the solar interior vibrates in response to small mechanical disturbances such as solar flares, convection, or even the impacts of infalling comets. It is roughly the time required for a sound wave to cross the sun.

Microscopic collision timescales

Photons created in the core of the sun scatter many times on their way out to the surface; actually they are absorbed and re-emitted more than scattered, but never mind that for now. The mean-free path between scatterings is $\lambda = (n_e\sigma)^{-1}$, where n_e is the number of electrons per unit volume, and σ is the scattering cross section per electron. Since the sun is mostly hydrogen, $n_e \approx \rho/m_H$. The mass density averaged over the volume of the sun is

$$\bar\rho_\odot = \frac{3M_\odot}{4\pi R_\odot^3} = 1.4\,\mathrm{g\,cm^{-3}} \tag{4}$$

(similar to that of the human body!), hence $\bar n_e \approx 10^{24}\,\mathrm{cm^{-3}}$. The cross section is generally of order the Thomson cross section,

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{m_ec^2}\right)^2 \approx 6\times10^{-25}\,\mathrm{cm}^2, \tag{5}$$

so $\bar\lambda \approx 1\,\mathrm{cm}$. The corresponding collision time is

$$t_{e\gamma} = \frac{\bar\lambda}{c} \approx 10^{-10}\,\mathrm{s}. \tag{6}$$

It can be shown that the proton-electron and proton-proton collision times are similarly short, or even shorter, compared to $t_{\rm dyn}$ and the other relevant macroscopic timescales discussed below. Importantly therefore, all regions of the interior quickly relax to local thermodynamic equilibrium (LTE). Thermal equilibrium can never be perfect, however, since the surface, which radiates freely to space, is inevitably colder than the core. The temperature gradient drives an outward flux of heat.

Photon-diffusion and Kelvin-Helmholtz (thermal) times

Photons escape by a random walk of step length λ. The root-mean-square distance after N steps is $d_N = N^{1/2}\lambda$. Setting this distance equal to $R_\odot$ gives the typical number of steps needed to travel from center to surface: $N \approx (R_\odot/\lambda)^2 \approx 10^{22}$. The corresponding photon diffusion time is

$$t_{\rm diff} = N t_{e\gamma} \approx 10^{12}\,{\rm s} \approx 3\times 10^4\,{\rm yr}. \tag{7}$$

A related timescale is the time required to radiate the current gravitational binding energy of the sun at its current luminosity; this is the Kelvin-Helmholtz time:

$$t_{\rm KH} \approx \frac{GM_\odot^2/R_\odot}{L_\odot} \approx 3\times 10^7\,{\rm yr}. \tag{8}$$

This is the timescale on which the sun would contract if its nuclear energy sources were turned off. It is much longer than the dynamical time (3) because pressure support is lost only gradually, as heat escapes. It is also significantly longer than the photon-diffusion time above. The reason for the latter is that most of the sun's thermal energy is *stored* in the random motions of electrons and ions rather than photons, even though photons dominate the outward *transport* of energy. Photons carry a larger fraction of the thermal energy of stars more massive than the sun, as will be shown.

Nuclear timescale

The heat released by fusing a mass ΔM of hydrogen into helium is approximately $0.007\Delta M c^2$. Therefore the time required to exhaust all the sun's hydrogen at its current luminosity would be

$$t_{\rm nuc} = \frac{0.007 M_\odot c^2}{L_\odot} \approx 10^{11}\,{\rm yr}. \tag{9}$$

The actual lifetime of the sun will be about one tenth of this because its luminosity will increase greatly when it becomes a red giant.

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Stellar structure and evolution are quantitatively predictable in large part because of the disparity of timescales,

$$t_{\rm nuc} \gg t_{\rm KH},\, t_{\rm diff} \gg t_{\rm dyn} \gg t_{\rm collisions}.$$

When analyzing processes associated with one of these timescales, one can usually ignore the slower processes and assume that the more rapid ones are at equilibrium. Most unsolved problems in stellar theory have to do with breakdown in these assumptions: for example, convection and wind-borne mass loss are departures from dynamical equilibrium that have thermal and nuclear consequences.