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Evolution of Single Stars. I.

Stellar Evolution from Main Sequence to White Dwarf or Carbon Ignition

by

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ABSTRACT

Model evolutionary calculations have been made for Population I stars ($X = 0.7$, $Z = 0.03$) with masses of 0.8, 1.5, 3, 5, 7, 10, and 15 $M_{\odot}$. Neutrino losses were taken into account. First the computations were made under the assumption that the stars do not lose mass. These computations were started on the main sequence and carried out through the phases of hydrogen and helium exhaustion in the center. Models of 10 and 15 $M_{\odot}$ ignited carbon in the center nonviolently. Stars of 3, 5, and 7 $M_{\odot}$ ignited carbon in the center at the density of 3×10^{9} g/cm^3. This will probably lead to the type of thermonuclear supernova explosion suggested by Arnett. Also, the 1.5 $M_{\odot}$ model is expected to ignite carbon under similar conditions. When dynamical instability of the red supergiant envelopes was taken into account, the models of 0.8, 1.5, and 3 $M_{\odot}$ lost their envelopes during the double shell burning phase. The masses of the remaining cores were 0.6, 0.8, and 1.2 $M_{\odot}$ respectively. These cores evolved through the part of the H — R diagram occupied by the nuclei of planetary nebulae, and were finally cooling down and evolving towards the white dwarf region.

Given our present knowledge of physics and modern computers it is possible to follow stellar evolution from the main sequence to the white dwarf stage or a supernova explosion. In this paper we describe the most essential results of such computations as well as the computer program used. A preliminary report of this work was presented at the 129th Meeting of the American Astronomical Society (Paczyński 1969*b*).

The evolutionary computations were performed with a Henyey type code for stars with masses in the range of 0·8 — 15 $M_\odot$. A detailed description of the code, and the code itself are available on request from the Joint Institute for Laboratory Astrophysics.[1] Here we shall describe the most important features of this code.

The model of a star was divided into the interior and the envelope. The outer boundary condition for the interior was obtained from a set of envelope integrations. In most cases the envelope contained 10 per cent of the stellar mass. In some models, however, only $2 \times 10^{-7} M_\odot$ was included in the envelope (model nuclei of planetary nebulae). In others (red supergiants), hydrogen, and even helium shell sources were included in the envelope integrations. The general rule used was that the part of a star that can be adequately described with ordinary differential equations was considered to be the envelope. In the envelope the mass fraction (or radius) was the only independent variable. In most cases these were static envelopes computed with a code described elsewhere (Paczyński 1969*a*). The ionization zones of hydrogen and helium and nonadiabatic convection were treated with this code. The shell sources were included in the envelope integrations when their thickness was considered to be sufficiently small, about 1 per cent of the core mass.

When the shell source is sufficiently narrow one can describe its structure with ordinary differential equations (Eggleton 1967). A time derivative of any physical quantity u can be replaced with its space derivative according to the formula

$$\left(\frac{\partial u}{\partial t}\right)_{M_r} = -\left(\frac{dM_c}{dt}\right) \times \left(\frac{\partial u}{\partial M_r}\right)_t \quad , \tag{1}$$

where $\frac{dM_c}{dt}$ is the rate at which the mass of the core increases, or in other words, the rate of mass flow through the shell source. The thinner the shell source, the more accurate this formula becomes, which makes it possible to obtain the precise distribution of chemical elements within the shell and to include a gravitational energy term in the ordinary differential equations describing the shell. This technique does not allow thermal instabilities in shells to appear.

When static envelopes are used the outer boundary condition for the interior is described by two parameters: the radius and the luminosity of the star. When the shell source is included in the envelope we have a third parameter: the rate of mass flow from the envelope to the core.

[1] Requests should be directed to Dr. J. Cox, Joint Institute for Laboratory Astrophysics, University of Colorado, Boulder, Colorado.

Equation of state. Envelope models were always deep enough to cover the hydrogen and helium ionization zones. The matter in the interior was assumed to be completely ionized. A mixture of perfect gas and radiation was taken into account. The electron gas could be arbitrarily degenerate and/or relativistic, but no effects of particle interactions such as crystalization or pair formation were taken into account. All the thermodynamic quantities needed for the evolutionary calculations were stored in tables in the computer memory.

Opacities. Radiative opacities (including lines) were interpolated from tables calculated by Cox and Stewart (1968) for 20 chemical compositions. Conductive opacities calculated by Hubbard and Lampe (1969) and by Canuto (1970) were used. In the course of an evolutionary calculation three opacity tables were stored in the computer memory: one for a hydrogen rich mixture, another for a helium rich mixture, and the third for a carbon and oxygen rich mixture. This sequence corresponded to the evolutionary changes in the chemical composition. In the model envelopes water vapor opacities computed by Auman (1967) were taken into account, and the H_2O abundance was calculated with a code based on Mihalas' (1967) description.

Nuclear reaction rates. Energy generation in the p-p and CNO cycles was calculated with formulae given by Reeves (1965), helium burning rates (nitrogen + helium, triple alpha, carbon + helium) were adopted from the review by Fowler, Caughlan, and Zimmerman (1967), and the carbon burning rate given by Patterson, Winkler, and Zaidins (1969) was used. For helium and carbon burning weak and strong screening corrections were calculated following Salpeter and Van Horn (1969). Reaction rates were stored in the computer memory as a function of temperature and density.

Neutrino emission. Neutrino emission was taken into account throughout the hydrogen-exhausted cores of all model stars. The rate of neutrino emission was calculated with formulae given by Beaudet, Petrosian, and Salpeter (1967) and stored in a table in a computer memory.

As practically all the information about the input physics is stored in tables and calculated by means of linear interpolation in the course of an evolutionary computation, the program is rather fast. On the CDC 6400 computer, it requires 10 msec per zone per iteration and less than one hour of computing time is sufficient to get an evolutionary track from the main sequence to helium exhaustion in the core and formation of a double shell source model. Another hour is required to evolve a small or intermediate mass star to the white dwarf or carbon ignition state.

Our evolutionary calculations were started with homogeneous main sequence models having a metal content $Z = 0{\cdot}03$, and a hydrogen content $X = 0{\cdot}70$. The calculations were continued up to the helium flash (owing to the helium + nitrogen reaction) in the core for 0·8 and 1·5 $M_{\odot}$, and up to carbon ignition for 3, 5, 7, 10, and 15 $M_{\odot}$. In

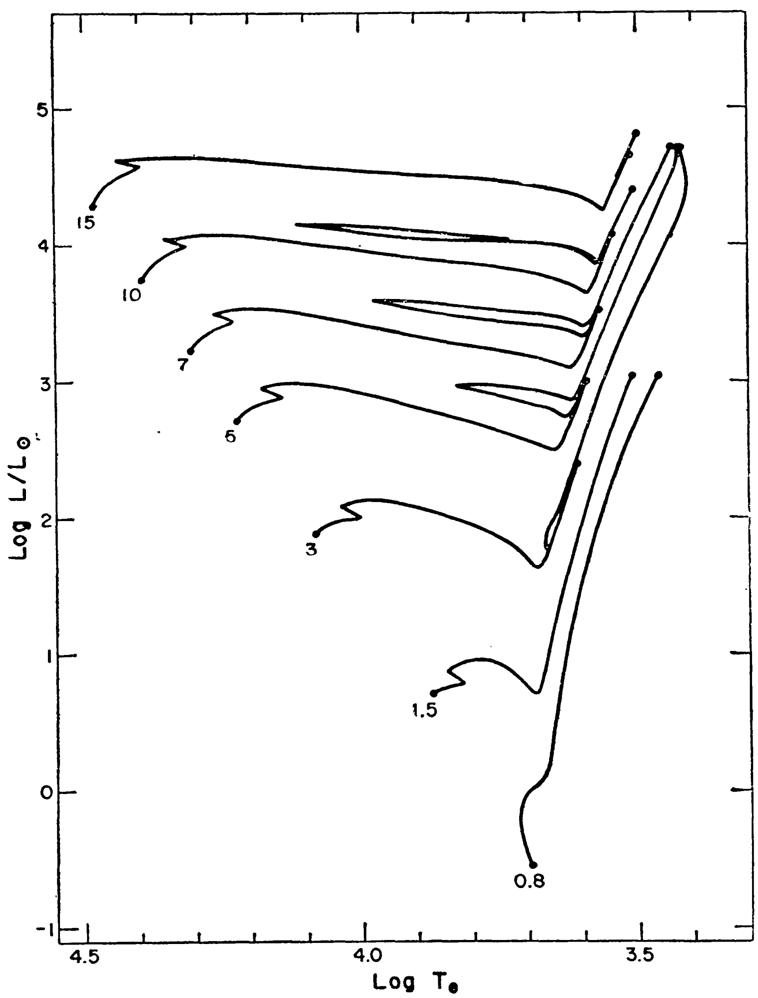

Fig. 1. Evolutionary tracks of Population I ($X = 0{\cdot}7$, $Z = 0{\cdot}03$) stars on the H—R diagram. The numbers at the beginning of each track give stellar mass in units of solar mass. Large dots indicate the position of the homogeneous main sequence models and the position of the models at the times of helium and carbon ignition in their cores.

the latter models the helium + nitrogen reaction was neglected. The evolution of those stars on the H—R diagram is shown in Figure 1. Large dots indicate the positions of models on the main sequence, and at the times of helium and carbon ignition. Evolutionary tracks for the 5, 7, and 10 $M_\odot$ models show the familiar loop on the H—R diagram during core helium burning, but the 15 $M_\odot$ model spent all the core helium burning phase in the red giant region. This problem will be discussed in a separate paper. Fortunately the evolution of the stellar cores is hardly affected by the existence or nonexistence of the loops.

The evolution of the centers of stars on the logarithm density — logarithm temperature plane is shown in Figure 2. Also, a line along

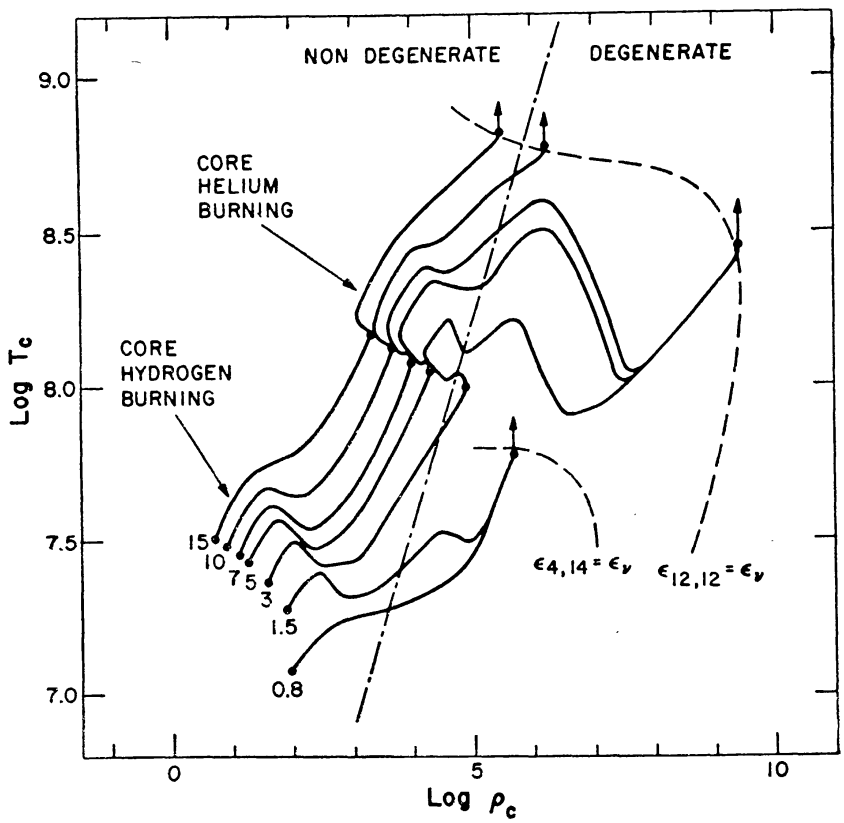

Fig. 2. Evolutionary track of the centers of stellar models in the temperature-density plane. The numbers at the beginning of each track give stellar mass in units of the solar mass. Large dots indicate the position of the centers of the models on the main sequence and at the times of helium and carbon ignition. Along the broken lines neutrino energy losses balance either nitrogen + helium, or carbon burning.

which the carbon burning rate equals neutrino energy losses is shown. Notice that carbon ignition takes place when the center of the star crosses this line. Carbon ignition is nonviolent in models of 10 and 15 $M_\odot$. The cores of the 3, 5, and 7 $M_\odot$ models are smaller and do not contract too rapidly after the helium exhaustion. As a result, neutrino emission cools them down (see also Weigert, 1966), and carbon ignition takes place when the density at the center reaches 3×10^9 g/cm^3, as suggested by Arnett (1968, 1969). This ignition is most likely explosive.

At the density of 3×10^9 g/cm^3 our cores were dynamically stable. However, the effects of crystallization, electron capture, and general relativity were not taken into account. The strong screening correction to the carbon burning rate may also be uncertain. It is not impossible that if all of these effects were properly taken into account, our cores could become dynamically unstable and collapse prior to carbon ignition. In that case carbon burning would probably be unable to cause a thermonuclear explosion, but rather the core would collapse all the way to neutron star densities (Wheeler 1969). In any case a star with a mass equal to or smaller than $7\,M_\odot$ cannot have a nonviolent carbon burning phase if the neutrino emission due to "universal Fermi interaction" exists.

None of our models ignited carbon in a shell. Extrapolating the behavior of our 5 and $7\,M_\odot$ models we may suspect that carbon ignition in the shell may be possible for $8\,M_\odot$. Also if the carbon burning rate were 100 times larger than that used in this study, our $7\,M_\odot$ model would ignite carbon in a shell soon after helium exhaustion in the core. It should be mentioned that Sugimoto (1969) got carbon ignition in a shell in his model of about $6\,M_\odot$.

Let us define the core as the part of a star interior to the hydrogen shell source. The mass of the core is given as a function of stellar mass in Table 1 for three evolutionary phases: 1) helium ignition,

Table 1.

Mass of the core as a function of stellar mass at three evolutionary phases. See text for details.

$M_{star}/M_\odot$	$M_{core}/M_\odot$		
	(1)	(2)	(3)
0.8	0.39	—	—
1.5	0.40	—	—
3.0	0.35	0.51	1.39
5.0	0.56	0.95	1.39
7.0	0.83	1.45 (1.02)	1.39
10.0	1.35	2.32	2.32
15.0	2.54	3.89	3.91

2) helium exhaustion, and 3) carbon ignition. In the case of the $7\,M_\odot$ model the convective envelope penetrated the core after helium

exhaustion and reduced its mass from 1·45 to 1·02 $M_\odot$. The difference in the core mass of the 0·8 and 1·5 $M_\odot$ models at the time of helium flash is probably caused by numerical inaccuracy in the present computations.

Perhaps the most striking feature of Figure 2 is the convergence of the evolutionary tracks for 3, 5, and 7 $M_\odot$ models into a common

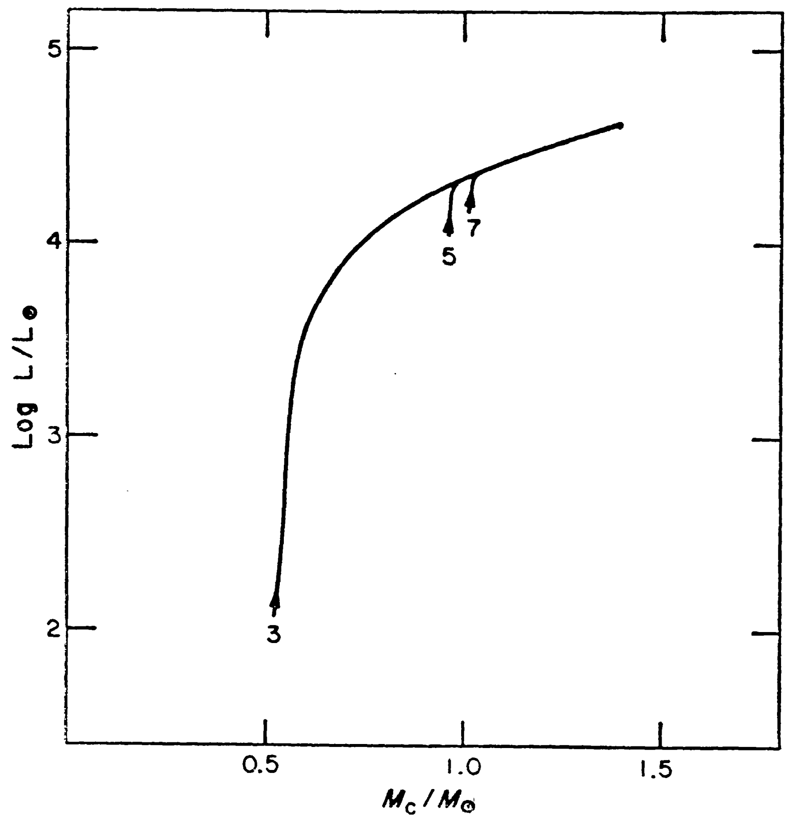

Fig. 3. Variation of the luminosity of 3, 5, and 7 $M_\odot$ models as a function of core mass during the double shell source burning phase.

track after the exhaustion of helium in the core. The same phenomenon may be seen for these models on the mass of the core — luminosity of a star diagram (Fig. 3). Even more surprising, this common mass — luminosity relation can be written as

$$L/L_\odot = 59250 \times M_{core}/M_\odot - 30950 \quad (2)$$

for $0{\cdot}57 < M_{core}/M_\odot < 1{\cdot}39$, with an accuracy better than 2 per cent. In a core of 0·57 $M_\odot$, hydrogen burning in a shell contributes 86 per cent of the total luminosity, helium burning in a shell 13 per cent, and gravitational contraction 1 per cent. In the core of 1·39 $M_\odot$ the corresponding numbers are 80, 10, and 10 per cent.

We expect that the evolutionary tracks of the cores of our 0˙8 and 1˙5 $M_\odot$ models would also merge into the common evolutionary track after the formation of the helium shell source. Therefore, if there is no mass loss during the evolution carbon can be ignited explosively in our 1˙5 $M_\odot$ model. In general, explosive carbon ignition is possible in a star that can form a degenerate carbon-oxygen core

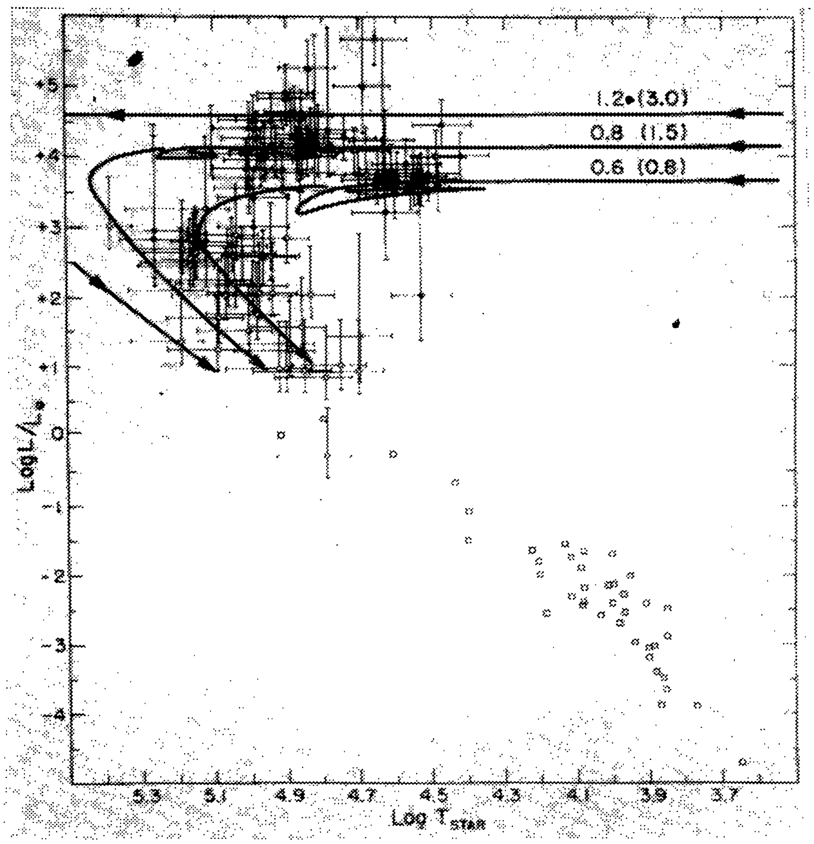

Fig. 4. Evolutionary tracks of the cores of models with initial masses 0˙8, 1˙5, and 3 $M_\odot$ after they have lost their envelopes. Tracks are labeled with the mass of the core and the initial stellar mass. The positions of the nuclei of planetary nebulae (with error bars) and white dwarfs are shown according to O'Dell (1968). (Courtesy of the International Astronomical Union).

of 1˙39 $M_\odot$. In that case we may expect stars in the mass range 1˙4 — 7 $M_\odot$, or 1˙4 — 8 $M_\odot$ to ignite carbon explosively. For masses larger than 10 $M_\odot$ or perhaps larger than 9 $M_\odot$ carbon ignition is not violent. The mass at which transition occurs is somewhere between 7 and 10 $M_\odot$.

Our models with two shell sources have very high luminosities, up to 5˙2 $\times$ 10^4 $L_\odot$. Envelopes of these models are very extended and may become dynamically unstable (Lucy 1967, Paczyński and Ziółkowski 1968*a, b*, Roxburgh 1967). Using Figure 6 of Paczyński and Ziółkowski (1968*b*) to estimate which of our models have dynamically unstable envelopes, we find that instability occurs when the mass of the core is 0·6, 0·8, or 1·2 $M_{\odot}$ compared to a total mass of 0·8, 1·5, or 3·0 $M_{\odot}$, respectively. Here we assume that cores of 0·8 and 1·5 $M_{\odot}$ models follow the mass-luminosity relation given by equation (2). At the onset of dynamical instability the total energy of the envelopes is positive and it is likely that they will be disrupted. It is interesting to see how the remaining core, presumably a nucleus of planetary nebula, will evolve.

The evolution of the cores of 0·6, 0·8, and 1·2 $M_{\odot}$ (with small hydrogen-rich envelopes attached to them) on the H—R diagram is shown in Figure 4. The positions of nuclei of planetary nebulae and white dwarfs are shown using data from O'Dell (1968). The evolution is almost horizontal as long as there is enough mass left in the envelope to support nuclear burning in the two shell sources. As mass flows from the envelope through the shell sources to the core, the mass of the envelope and the radius of the star decrease, and the effective temperature increases. When the mass of the envelope decreases below a certain limit the shell sources die out and the models cool down to

Table 2.

Evolutionary time scales (in years) for model nuclei of planetary nebulae.

0·6 $M_{\odot}$			0·8 $M_{\odot}$			1·2 $M_{\odot}$		
time	$\log T_e$	$\log L/L_{\odot}$	time	$\log T_e$	$\log L/L_{\odot}$	time	$\log T_e$	$\log L/L_{\odot}$
—4·8 (5)	3·50	3·68	—3·5 (5)	3·50	4·21	—4·0 (5)	3·50	4·60
—1·4 (4)	3·70	3·68	—5·4 (3)	3·70	4·21	—6·0 (3)	3·70	4·60
—4·0 (3)	4·00	3·68	—8·0 (1)	4·00	4·21	—3·0	4·00	4·60
0·0	4·40	3·67	0·0	4·40	4·21	0·0	4·40	4·60
4·0 (3)	4·80	3·53	8·0 (1)	4·80	4·21	0·35	4·80	4·60
1·7 (4)	5·00	3·53	1·4 (2)	5·00	4·20	0·8	5·00	4·60
—	—	—	3·6 (2)	5·20	4·08	1·3	5·20	4·60
—	—	—	—	—	—	3·6	5·60	4·60
—	—	—	—	—	—	1·0 (1)	5·83	4·50
—	—	—	9·0 (2)	5·39	4·00	3·2 (1)	5·79	4·00
1·8 (4)	5·02	3·50	1·5 (3)	5·43	3·50	5·6 (1)	5·70	3·50
3·0 (4)	5·15	3·00	2·4 (3)	5·36	3·00	1·4 (2)	5·59	3·00
3·8 (4)	5·09	2·50	5·2 (3)	5·27	2·50	6·0 (2)	5·46	2·50
7·5 (4)	5·02	2·00	1·7 (4)	5·17	2·00	4·5 (3)	5·35	2·00
1·6 (5)	4·94	1·50	7·5 (4)	5·06	1·50	2·0 (4)	5·23	1·50
4·0 (5)	4·84	1·00	2·3 (5)	4·95	1·00	5·0 (4)	5·11	1·00

the white dwarf region. A similar evolutionary track was recently obtained by Rose and Smith (1970) for $0.85\,M_\odot$.

The evolutionary times for our models are given in Table 2. Notice that speed of evolution is extremely sensitive to stellar mass, and is very high for the $1.2\,M_\odot$ model when the effective temperature is in the range of $10^4 - 10^5\,^\circ$K. The zero point of the time scale was arbitrarily chosen to correspond to $\log T_{eff} = 4.4$. We do not know how much mass is left in the envelope after the main part of it has been lost. The surface temperature of a model is very sensitive to the amount of mass left, as is the evolutionary time from the moment when the envelope was ejected to the moment when the effective temperature is $10^{4.4}\,^\circ$K. This temperature seems to be the minimum required to excite the nebula formed from the ejected envelope, as no nucleus of a planetary nebula is cooler than this limit. Notice also that the luminosity of our models is completely insensitive to the mass of the envelope, provided there is any envelope left.

Our model cores of 0.6, 0.8, and $1.2\,M_\odot$ were taken from our evolutionary calculations for $3\,M_\odot$, which were carried out all the way from the main sequence. No attempt was made to choose any parameter so as to fit the observed positions of the nuclei of planetary nebulae with our evolutionary sequences. The structure of our cores is independent of the mechanism which may be responsible for the ejection of the envelope (dynamical instability, stellar wind, thermal instabilities in a helium shell source). In particular we believe that the luminosities of our cores are reliable. However, the relation between the mass of the core, which becomes the nucleus of a planetary nebula, and the total mass of the original star is very uncertain, as this depends on details of the mechanism which disrupts the envelope. If the estimates of dynamical instability made by Paczyński and Ziółkowski (1968*b*) are used in conjunction with relation (2) above, the planetary nebulae may be formed from stars with masses up to $3.5\,M_\odot$, and their nuclei may have masses up to $1.39\,M_\odot$. The lower mass limit is established by the age of the galaxy, and is likely to be about $0.8\,M_\odot$ for the original mass of the star, and $0.6\,M_\odot$ for the nucleus of a planetary nebula.

In our calculations of the evolution of model nuclei of planetary nebulae both shell sources were included in the interior which was evolved with the usual Henyey-type technique. As a result, thermal pulses in the helium shell source developed and gave rise to rather large loops on the H—R diagram, which can be seen in Fig. 4 for the 0.6 and $0.8\,M_\odot$ models. The $1.2\,M_\odot$ model has a loop which falls to the left of the H—R diagram shown in Fig. 4. It took the $0.6\,M_\odot$

model 10^3 years to go through the loop. This time was 10^2 years for the $0.8\,M_\odot$ model, and 10 years for the $1.2\,M_\odot$ model. The position of these loops on the H—R diagram depends on the time when the evolutionary calculations are switched into the ordinary Henyey-type technique, i. e., when the shell sources are included in the interior rather than in the envelope. The loops appear only when the mass of the hydrogen-rich envelope is sufficiently small. If the envelope is too massive, the star remains in the red giant region during the whole thermal pulse of the helium shell (Schwarzschild and Härm 1965).

The present calculations indicate that we may expect fairly rapid changes in those nuclei of planetary nebulae which happen to be seen while developing a thermal pulse in a helium shell source. It is interesting that at least one planetary nebula has a nucleus, FG Saggittae, which has changed very considerably over a period of the last 70 years (Herbig and Boyarchuk 1968).

If our model computations are correct we expect all stars with mass below about $3.5\,M_\odot$ to lose their envelopes through dynamical instability, form a planetary nebula and a nucleus which will finally cool down to become a white dwarf. These stars will never burn carbon. The more massive stars never achieve a high enough L/M ratio to have dynamically unstable envelopes. Stars in the mass range $3.5-7\,M_\odot$ or $3.5-8\,M_\odot$ are expected to ignite carbon explosively when their central densities reach $3\times10^9\,\mathrm{g/cm^3}$. In this case the type of thermonuclear supernova suggested by Arnett would terminate stellar evolution. Because some details of our models are sufficiently uncertain, the possibility remains that the cores of such stars may become dynamically unstable and collapse before carbon ignition. This would lead to the formation of a neutron star. Stars with a mass above 8 or $9\,M_\odot$ ignite carbon nonviolently, and their final evolution is not too clear.

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REFERENCES

Arnett, W. D. . . . . 1968 *Nature*, **219**, 1344.
. . . . 1969 *Astroph. and Space Science*, **5**, 180.
Auman, J. Jr. . . . . 1967 *Ap. J. Suppl.*, **14**, 171.

Beaudet, G., Petrosian, V., and Salpeter, E. E. . . . . 1967 *Ap. J.*, **150**, 979.

Canuto, V. . . . . 1970 *Ap. J.*, **159**, 641.

Cox, A. N., and Stewart, J. N. 1969 *Astr. Council, Acad. of Sciences, U.S.S.R. Scientific Informations*, 15.

Eggleton, P. P. . . . . . . 1967 *M. N. R. A. S.*, **135**, 243.

Fowler, W.A., Caughlan, G.R., and Zimmerman, B. A. . . 1967 *Ann. Rev. Astr. Ap.*, **5**, 525.

Herbig, G. H., and Boyarchuk, A. A. . . . 1968 *Ap. J.*, **153**, 397.

Hubbard, W. B., and Lampe, M. 1969 *Ap. J.*, **156**, 795.

Lucy, L. B. . . . . 1967 *A. J.*, **72**, 813.

Mihalas, D. . . . . 1967 *Methods in Computational Physics* (New York: Academic Press) Vol. **7**, p. 1.

O'Dell, C. R. . . . . 1968 *Planetary Nebulae, Proceedings of I.A.U. Symposium No. 34* (Dordrecht-Holland: D. Reidel Publ. Co.), p. 369.

Paczyński, B. . . . . . . 1969*a* *Acta Astr.*, **19**, 1.

. . . . . 1969*b* *Bull. A. A. S.*, **1**, 256.

Paczyński, B., and Ziółkowski, J. . . . 1968*a* *Planetary Nebulae, Proceedings of I.A.U. Symposium No. 34* (Dordrecht-Holland: D. Reidel Publ. Co.), p. 396.

. . . 1968*b* *Acta Astr.*, **18**, 255.

Patterson, J. R., Winkler, H., and Zaidins, C. S. . . . . 1969 *Ap. J.*, **157**, 367.

Reeves, H. . . . . 1965 *Stellar Structure*, ed., L. H. Aller, and D. B. McLaughlin (Chicago: The University of Chicago Press), p. 156.

Rose, W. K., and Smith, R. L. . 1969 *Ap. J.*, **159**, 903.

Roxburgh, I. W. . . . . . . 1967 *Nature*, **215**, 838.

Selpeter, E. E., and Van Horn, H. M. . . . 1969 *Ap. J.*, **155**, 183.

Schwarzschild, M., and Härm, R. . . . . 1965 *Ap. J.*, **142**, 855.

Sugimoto, D. . . . . 1969 *preprint*.

Weigert, A. . . . . 1966 *Zs. f. Ap.*, **64**, 395.

Wheeler, J. C. . . . . . . 1969 University of Colorado: *thesis*.

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