OPACITY

Opacities (κ) are defined through the expression: $\kappa\rho = \Sigma_i n_i \sigma_i$. In cgs, they are in cm^2 g^{-1}. Multiplying κ by the mass per gram in a slab ("grammage") gives us the optical depth of the slab. **Rosseland opacities** are calculated and tabulated by the Opacity project ("OP", Seaton et al.), the OPAL project (Rogers & Iglesias), and classically (though not the best) by the LANL group (Cox et al.).

The important sources of opacity in stars are **Thomson scattering off electrons, free-free opacity (inverse bremsstrahlung), bound-free opacity (the photoelectric effect), bound-bound transitions, and absorption by the H^- atom**. Bound-free, free-free, and bound-bound opacities are collectively known as **Kramers** opacity and have similar ρ and T dependences ($\sim \rho T^{-3.5}$). The bound-free opacity generally dominates, but Thomson is important at high temperatures and low densities (e.g., in massive star envelopes), and H^- opacity is important at low temperatures ($\leq 10^4$ K). In general, the Rosseland integral favors the $h\nu \sim 4kT$ region of the spectrum.

A low density, relatively high temperature fully ionized gas has opacity dominated by **Thomson** photon-electron ($\gamma + e^- \rightarrow \gamma + e^-$) scattering. This is given as

$$\kappa_{Th}\rho = n_e \sigma_{Th}, \tag{op.1}$$

where $n_e = \rho(1+X)/2m_u$ is the number of free electrons per cubic centemeter, and $\sigma_{Th} = 8\pi r_e^2/3$ $= 0.665 \times 10^{-24}\,\mathrm{cm}^{\,2}$ is Thomson scattering cross-section per electron. Putting in numerical values for all constants we obtain

$$\kappa_{Th} = 0.2\,(1+X)\,, \qquad \left[\mathrm{cm}^{\,2}\,\mathrm{g}^{\,-1}\right]. \tag{op.2}$$

If one writes the opacity in the form $\sim \rho^n T^{-s}$, then $n = s = 0$ in this case.

The Rosseland opacity due to free-free, bound-free, and bound-bound electronic transitions can together be approximated with the **Kramers** formula:

$$\kappa_K \approx 4 \times 10^{25}\ (1+X)\ (Z+0.001)\ \frac{\rho}{T^{3.5}} \qquad \left[\mathrm{cm}^{\,2}\,\mathrm{g}^{\,-1}\right]. \tag{op.4}$$

For Kramers opacity, writing it in the form $\sim \rho^n T^{-s}$, we obtain $n = 1$ and $s = 3.5$. This formula is important when hydrogen and helium and other elements are partly ionized, roughly speaking for $T \geq 2 \times 10^4\ K$ and is generally dominated by the bound-free term (hence, the $(1+X)Z$ factor). The peak contribution of the bound-bound component is $\sim$50% and occurs for $T \leq 10^6$ K. Above 10^7 K, the bound-bound contribution is no more than $\sim$10% of the total. Note that for free-free absorption, we can derive the form of the opacity by observing that the total bremsstrahlung emissivity (j) per volume is $\propto \rho^2 T^{1/2}$. Using **Kirchhoff's law** ($\kappa \sim (j/\rho)/B(T)$) in an integral form, we find $\kappa \sim \rho T^{1/2}/T^4 \sim \rho T^{-3.5}$. Note also that the cross section for the photoelectric effect goes as $\sim Q^4 \nu^{-3}$, where Q is the nuclear charge. The stiff dependence on Q is the reason X-ray machines used in medicine provide the contrasts they do.

The Rosseland opacity due to the **negative hydrogen ion**, H^-, with an ionization energy of a scant 0.754 eV ($\equiv$ 1.655 μm in the IR) is the dominant opacity source in the solar atmosphere, in general for $4 \times 10^3 \leq T \leq 8 \times 10^3$ K:

$$\kappa_{H^-} \approx 2.5 \times 10^{-31} \left(\frac{Z}{0.02}\right) \rho^{0.5} T^9 \qquad \left[\mathrm{cm}^2\,\mathrm{g}^{-1}\right] . \tag{op.5}$$

Notice the extremely steep temperature dependence of the H^- opacity, but this steepness is a function of temperature and decreases with increasing T. If one writes the opacity in the form $\sim \rho^n T^{-s}$, then $n = 0.5$ and $s = -9$. but at even lower temperatures s is $\sim$ -11. Since the production of H^- depends on the presence of free electrons often available only because of the ease of ionization of Na, K, Cs, Ca, etc., there is a metallicity dependence. Above $\sim 10^4$ K, H^- is ionized away and H^- opacity is almost gone.

At very low temperature ($T \leq 3500$ K) **molecules** dominate the opacity (see Sharp & Burrows 2007).

At very high density and low temperature the radiative opacity becomes very high, as long as the gas remains substantially ionized and non-degenerate [eq. (op.4)], and the **electron thermal conductivity** becomes a much more efficient heat carrier. A reasonable approximation to the electron conductivity in electron degenerate matter expressed in a form of "opacity" is:

$$\kappa_{cond} \approx 4.0 \times 10^{-8} \, \langle Q^2 \rangle \, \frac{T^2}{\rho^2} \left(1 + \left(\frac{\rho}{2 \times 10^6}\right)^{2/3}\right) \qquad \left[\mathrm{cm}^2\,\mathrm{g}^{-1}\right] , \tag{op.7}$$

where $\langle Q^2 \rangle$ is the average of the square of the electric charge per ion. Electron conduction is important in **white dwarf cores, neutron star crusts, and in the cores of old, massive brown dwarfs**.

Finally, the total effective opacity, which includes both heat carriers: photons and electrons, can be calculated as:

$$\kappa = \left(\kappa_{rad}^{-1} + \kappa_{cond}^{-1}\right)^{-1} . \tag{op.8}$$