MASS – LUMINOSITY RELATION

Within the Eddington model $\beta \equiv P_g/P = \text{const}$, and a star is an $n = 3$ polytrope. Large mass stars have small β, and hence are dominated by radiation pressure, and the opacity in them is dominated by electron scattering. Let us consider the outer part of such a star assuming it is in a radiative equilibrium. We have the equation of hydrostatic equilibrium:

$$\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho, \tag{s2.1}$$

and the equation of radiative equilibrium

$$\frac{dP_r}{dr} = -\frac{\kappa\rho L_r}{4\pi c r^2}. \tag{s2.2}$$

Dividing these equations side by side we obtain

$$\frac{dP_r}{dP} = \frac{\kappa L_r}{4\pi c G M_r}, \tag{s2.3}$$

Near the stellar surface we have $M_r \approx M$ and $L_r \approx L$, and adopting $\kappa \approx \kappa_e = \text{const}$, we may integrate equation (s2.3) to obtain

$$P_r \sim \frac{\kappa_e L_r}{4\pi c G M_r}\left(P\right). \tag{s2.4}$$

We obtain:

$$1 - \beta = \frac{P_r}{P} = \frac{\kappa_e L}{4\pi c G M} = \frac{L}{L_{Edd}} = \frac{1+X}{65300}\frac{M_\odot}{L_\odot}\frac{L}{M}, \tag{s2.5}$$

where L_{Edd} is the **Eddington luminosity**. More interestingly

$$L = \frac{4\pi c G M}{\kappa}\left(1-\beta\right) \tag{s2.5b}$$

$$= 1.25 \times 10^{38}\left(\frac{0.4}{\kappa}\right)\left(\frac{M}{M_\odot}\right)\left(1-\beta\right) \tag{s2.5c}$$

$$= 3.7 \times 10^{35}\left(\frac{0.4}{\kappa}\right)\left(\frac{M}{M_\odot}\right)^3\left(\mu\beta\right)^4, \tag{s2.5d}$$

where the luminosities are in cgs.

Equations (s2.5) and following give a relation between stellar mass, luminosity and β. The Eddington model gives a relation between stellar mass and β :

$$\frac{M}{M_\odot} = \frac{18.1}{\mu^2} \frac{(1-\beta)^{1/2}}{\beta^2}, \tag{s2.6}$$

where μ is a mean molecular weight in units of mass of a hydrogen atom, $\mu^{-1} = 2X + 0.75Y + 0.5Z$, and X, Y, Z, are the hydrogen, helium, and heavy element abundance by mass fraction. Combining equations (s2.5) and (s2.6) we obtain a mass-luminosity relation

$$\frac{M}{M_\odot} = \frac{18.1}{\mu^2} \; \frac{(L/L_{Edd})^{1/2}}{(1-L/L_{Edd})^2}, \qquad L_{Edd} \equiv \frac{4\pi c G M}{\kappa_e}. \tag{s2.7}$$

For Massive Stars

For very massive stars this gives approximate relation

$$L \approx L_{Edd} = \frac{65300 L_\odot}{(1+X)\, M_\odot} M, \qquad M \gg 100 M_\odot. \tag{s2.8}$$

For lower mass stars $L \ll L_{Edd}$, and (s2.7) or (s2.5c) give

$$L \approx L_{Edd}\, \mu^4 \left(\frac{M}{18.1 M_\odot}\right)^2 \approx 30\; L_\odot \left(\frac{M}{M_\odot}\right)^3, \qquad X = 0.7, \quad Z = 0.02. \tag{s2.9}$$

Note:

$$L \sim \frac{\mu^4}{\kappa_e} = (1+X)^{-1} \, (2X + 0.75Y + 0.5Z)^{-4} \sim X^{-2.6} \qquad \text{for} \;\; X \approx 0.7, \tag{s2.10}$$

i.e. the less hydrogen there is the brighter is a hydrogen burning star!