Ignition Mass for Low-Mass Stars

Low mass stars are supported by gas pressure, while radiation pressure is unimportant. Electron gas may be partly degenerate. Numerical models demonstrate that very low mass stars, with $M < 0.3 M_\odot$ are fully convective, and may be very well approximated with $n = 1.5$ polytropes. We shall approximate the non.-rel. equation of state with the following formula

$$P \approx \left[\left(\frac{k}{\mu H} \rho T \right)^2 + \left(K_1 \rho^{5/3} \right)^2 \right]^{1/2}, \tag{lms.1}$$

where $k/H = 0.825 \times 10^8$ [erg g^{-1} K^{-1}], $K_1 = 0.991 \times 10^{13} \mu_e^{-5/3}$ [erg g$^{-5/3}$ cm^2]. Algebraic approximation to the stellar structure equations gives

$$\rho \approx \frac{M}{R^3}, \qquad P \approx \frac{GM^2}{R^4}. \tag{lms.2}$$

Combining the last two equations we obtain

$$\frac{G^2 M^4}{R^8} \approx \left(\frac{k}{\mu H} \rho T \right)^2 + K_1^2 \rho^{10/3} \approx \left(\frac{kT}{\mu H} \right)^2 \frac{M^2}{R^6} + K_1^2 \frac{M^{10/3}}{R^{10}}, \tag{lms.3}$$

which may be written as

$$T \approx \frac{\mu H}{k} \frac{GM}{R} \left[1 - \left(\frac{K_1}{GRM^{1/3}} \right)^2 \right]^{1/2}. \tag{lms.4}$$

Fully degenerate, the stellar radius satisfies a polytropic ($n = 1.5$) mass-radius relation:

$$R_{min} = \frac{K_1}{0.4242 \; GM^{1/3}}, \qquad K_1 \rho^{5/3} \gg \frac{k}{\mu H} \rho T, \tag{lms.5}$$

where R_{min} is the minimum radius that a star with a mass M may have. This relation is recovered from the equation (lms.4) when we replace the square bracket with $[1 - (R_{min}/R)^2]$. In the limit when degeneracy is negligible we should have

$$T_c = 0.539 \; \frac{\mu H}{k} \frac{GM}{R}, \qquad K_1 \rho^{5/3} \ll \frac{k}{\mu H} \rho T, \tag{lms.6}$$

where T_c is the central temperature of an $n = 1.5$ polytrope with pressure provided by non-degenerate gas. Combining the two limiting cases with the equation (lms.4) we may write it as

$$T_c = 0.539 \; \frac{\mu H}{k} \frac{GM}{R} \left[1 - \left(\frac{R_{min}}{R} \right)^2 \right]^{1/2}, \qquad R_{min} = \frac{K_1}{0.4242 \; GM^{1/3}}. \tag{lms.7}$$

The central temperature reaches its maximum where $dT_c/dR = 0$ for $R = R_{_{Tmax}}$, i.e.

$$R_{_{Tmax}} = 2^{1/2} R_{min}, \tag{lms.8}$$

and the corresponding maximum central temperature is

$$T_{c,max} = \frac{0.539}{2^{1/2}} \frac{\mu H}{k} \frac{GM}{R_{_{Tmax}}} = \frac{0.539}{2} \frac{\mu H}{k} \frac{GM}{R_{min}} = \tag{lms.9}$$

$$0.1143 \; \frac{\mu H G^2}{k K_1} M^{4/3} = 1.56 \times 10^8 \; \mu \mu_e^{5/3} \left(\frac{M}{M_\odot} \right)^{4/3} \approx 6 \times 10^6 \; \left(\frac{M}{0.1 \; M_\odot} \right)^{4/3} \qquad [\mathrm{K}],$$