EDDINGTON MODEL

The model was proposed by Eddington in the 1920s, when very little was known about physical properties of matter in stellar interiors. The model assumes, that pressure is provided by perfect, fully ionized gas and radiation, and that throughout a whole star the ratio of gas pressure to total pressure is constant:

$$P = P_g + P_r, \qquad P_g = \frac{k}{\mu H}\rho T, \qquad P_r = \frac{a}{3}T^4, \qquad \beta \equiv \frac{P_g}{P} = \text{const.} \tag{ed.1}$$

We may express temperature in two different ways:

$$T = \frac{\mu H}{k}\frac{P_g}{\rho} = \frac{\mu H}{k}\frac{\beta P}{\rho}, \qquad T^4 = \frac{3P_r}{a} = \frac{3\,(1-\beta)\,P}{a} \qquad i.e., \frac{P_r}{P} = 1-\beta. \tag{ed.2}$$

These two expressions may be used to eliminate temperature, and find a relation between pressure and density:

$$P = K\rho^{4/3}, \qquad K = \left[\frac{3}{a}\left(\frac{k}{\mu H}\right)^4 \frac{1-\beta}{\beta^4}\right]^{1/3}, \tag{ed.3}$$

i.e. we have a polytropic relation with $n = 3$. K is constant throughout a star, because β was assumed to be constant. Of course, β as well as K may vary from one star to another.
Other useful Eddington model expressions are:

$$T = \left(\frac{3R}{\mu a}\right)^{1/3}\left(\frac{1-\beta}{\beta}\right)^{1/3}\rho^{1/3}$$

and

$$\frac{1-\beta}{\beta} = \frac{\mu S_\gamma}{4R},$$

where $S_\gamma = \frac{4aT^3}{3\rho}$ is the specific radiation entropy. Entropy is an important quantity in stellar structure and evolution studies, even though one does not encounter it in texts as often as one should.

We know that $n = 3$ polytrope is a special case: the total stellar mass is uniquely determined by the value of K :

$$M = \left(\frac{K}{0.3639\;G}\right)^{1.5} = (0.3639\;G)^{-1.5}\left[\frac{3}{a}\left(\frac{k}{\mu H}\right)^4 \frac{1-\beta}{\beta^4}\right]^{1/2}. \tag{ed.4}$$

Another way of writing the above equation is

$$1-\beta == 0.003\left(\frac{M\mu^2}{M_\odot}\right)^2 \beta^4 .$$

The last equation gives another relation between stellar mass and β, which may be expressed as

$$\frac{M}{M_\odot} = \frac{18.1}{\mu^2}\,\frac{(1-\beta)^{1/2}}{\beta^2}. \tag{ed.5}$$

According to this equation gas pressure dominates, i.e. $\beta \to 1$ when stellar mass is very small, and radiation pressure dominates, i.e. $\beta \to 0$ when stellar mass is very large. The two contributions to pressure are equal, i.e. $\beta = 0.5$ when the stellar mass is $M/M_\odot = 51/\mu^2$. For a solar mass star, i.e. for $M/M_\odot = 1$, and for the solar chemical composition, i.e. for $\mu \approx 0.62$, we have $\beta \approx 0.9995$, i.e. radiation pressure may be neglected.