Helioseismology

Figure 1: Doppler image of the solar disk, taken by the Michelson Doppl
Solar and Heliospheric Observatory [1].

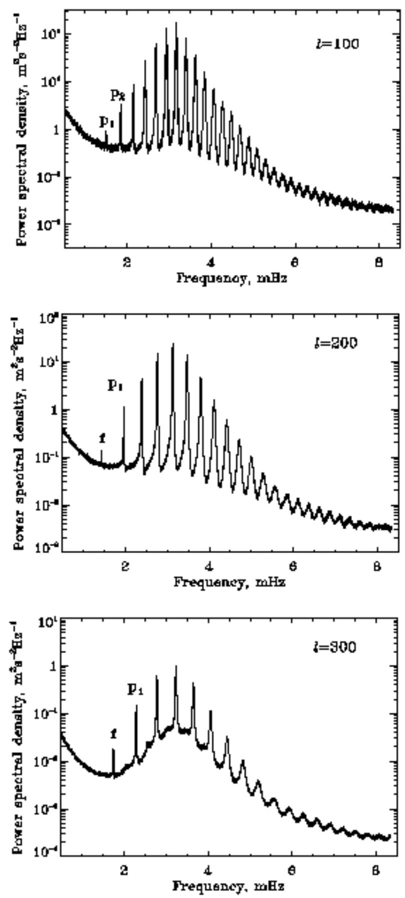


Figure 2: Temporal power spectra from MDI/SOHO at selected values of spherical-harmonic degree l [1]. Note 3.33 mHz = 1 cycle/(5 min).

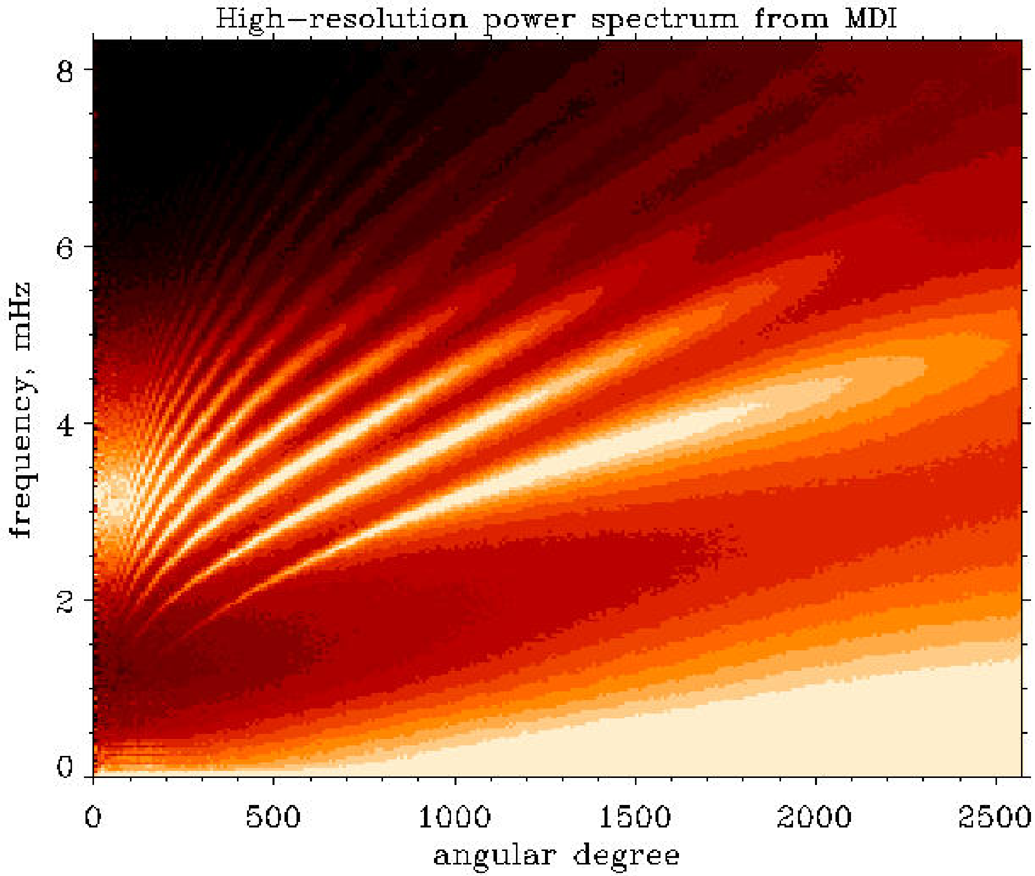


Figure 3: Observed power vs. frequency and l. Each ridge line corresponds to a particular n. [1].

Perturbation around spherical star:

p + f modes $\quad \psi(r,\theta,\phi,t) = \frac{\delta P}{\sqrt{\rho}} = e^{-i\omega t}\, Y_{\ell m}(\theta,\phi)\, R(r)$

Sound!

$$\frac{1}{r^2}\frac{d}{dr} r^2 \left[\frac{dR}{dr}\right] + \underbrace{\left[\frac{\omega^2 - \omega_c^2}{C_s^2} - \frac{\ell(\ell+1)}{r^2}\right]}_{k_r^2} R = 0$$

$$k_h^2 = k_{h\ell}^2 = \frac{\ell(\ell+1)}{r^2}$$

$\omega_c^2 \sim \frac{C_s}{2H}$ (near surface)

cutoff!! waveguide

$$\frac{\omega^2 - \omega_c^2}{c_s^2} - k_h^2 = k_r^2$$

$$\frac{\omega^2}{C_s^2} = \underbrace{k_r^2 + k_h^2}_{k^2} \left(+ \frac{\omega_c^2}{C_s^2}\right)$$

Dispersion relation "Sound"

$$C_s = \sqrt{\frac{\gamma P}{\rho}}$$

$$\boxed{k^2 = \frac{\omega^2 - \omega_c^2}{c_s^2}}$$

waveguide cutoff

$\omega_c \sim 4\times 10^{-3}\ \frac{\text{rad}}{s}$

(25 min)

Helioseismology: p, f, + g modes Several 10^6 frequencies

$\boxed{\text{p modes:}}$ 3 – 12 minutes → "5-minute" oscillations

$f \sim 3000\,\mu Hz$ $V_{n,l,m} \sim 3\ cm/s \overset{\text{superposition}}{\Longrightarrow} \sim 1\ km/s$

Measure Doppler shift or intensity variations

→ SDO, GONG, GOLF, SOHO...

Rotation – 440 nHz (splitting) ↑!

Quantum numbers: n, l, m Eigenmodes + Eigenfrequencies

$\boxed{\text{g modes:}}$

Brunt: $N^2 = -g\left[\frac{\gamma-1}{\gamma}\frac{d\ln P}{dr} - \frac{d\ln T}{dx}\right] \sim \frac{g}{H}(\nabla - \nabla_a)$

$\left[H = \frac{dr}{d\ln P}\right]$

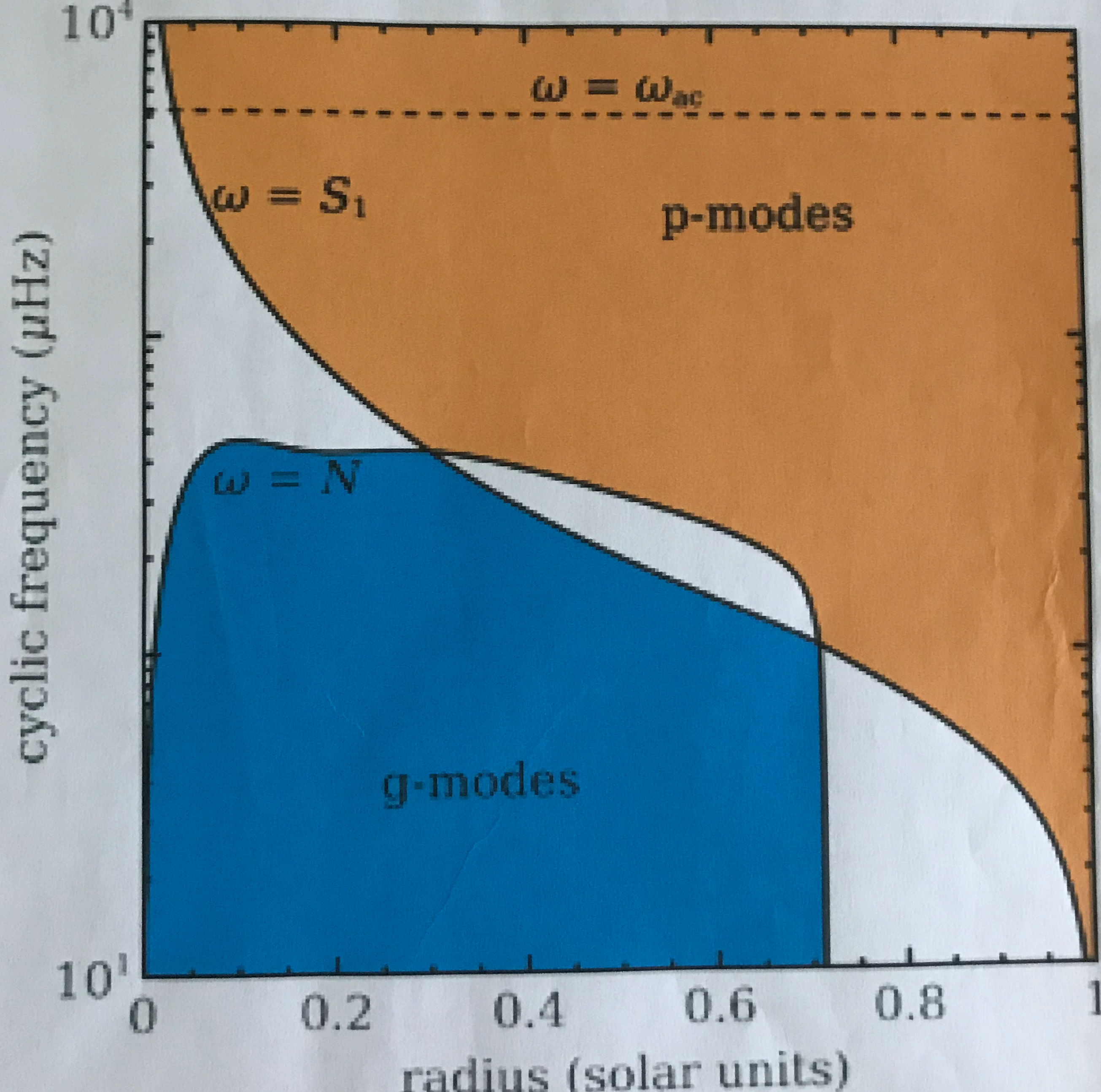
$$k_r^2 \sim \frac{\omega^2}{c_s^2}\left(1 - \frac{c_s^2}{\omega^2}\frac{l(l+1)}{r^2}\right)\left(1 - \frac{N^2}{\omega^2}\right) > 0\ !$$

$$\sim \frac{\omega^2}{c_s^2}\left(1 - \frac{S_r^2}{\omega^2}\right)\left(1 - \frac{N^2}{\omega^2}\right) \qquad \left[S_r^2 = c_s^2\frac{l(l+1)}{r^2}\right]$$

p modes: $\left\{\begin{matrix}\omega > S_r \\ \omega > N\end{matrix}\right\}$

Otherwise k_r → evanescent

g modes: $\left\{\begin{matrix}\omega < S_r \\ \omega < N\end{matrix}\right\}$



10^4
$\omega = \omega_{ac}$
$\omega = S_1$
p-modes
$\omega = N$
g-modes
10^1
0
0.2
0.4
0.6
0.8
1
radius (solar units)
cyclic frequency (μHz)

Sound travel time^{-1}

(Frequency) $f \sim \left(n + \frac{\ell}{2}\right) \frac{c_s}{2R_\odot}$; p modes $(\ell >> 1)$

(Period) $P \sim \frac{N_0}{\sqrt{\ell(\ell+1)}}\left(n + \alpha_{\ell,i}\right)$; g-modes

Rotation: $\Delta f \sim \underline{\underline{m\Omega}}(1 - C_{\ell,n})$; $N_0 \sim 2\pi^2 \int N \frac{dr}{r}$

* g-modes: $f < N$, decrease with n

* p-modes: $f > \omega_c$, increase with n, ℓ

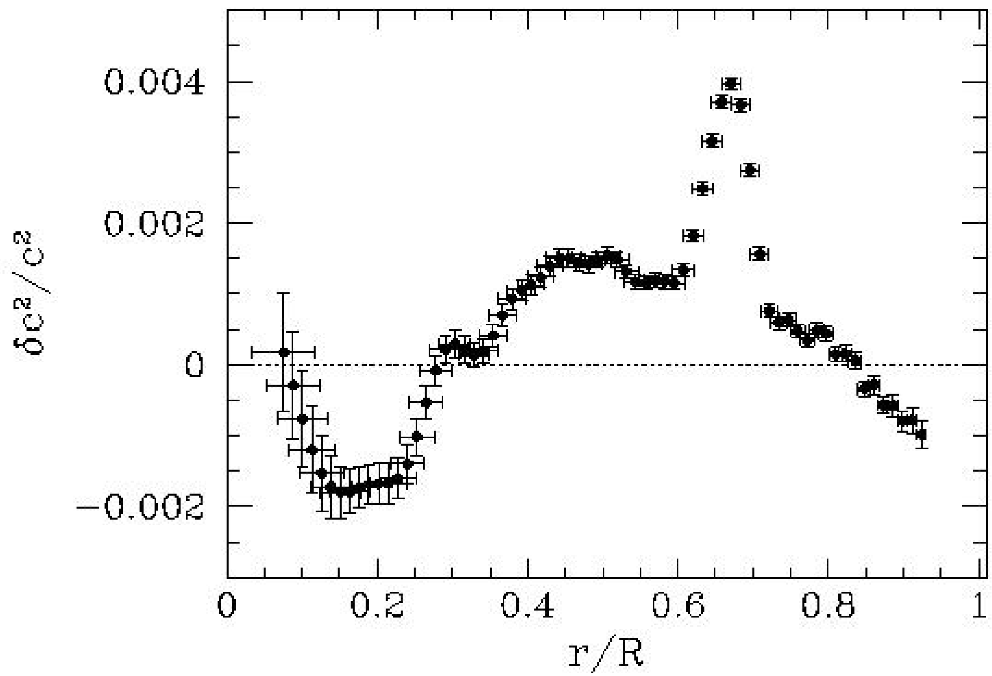


Figure 4: Deviations between sound speed predicted by solar model and sound speed by inversion of helioseismological data [1, 10].

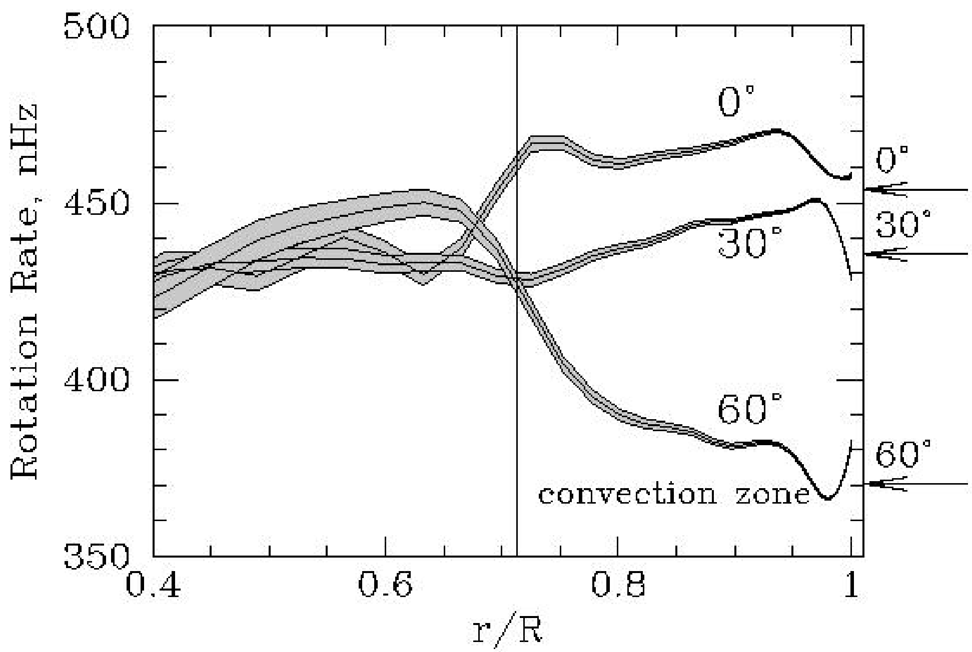
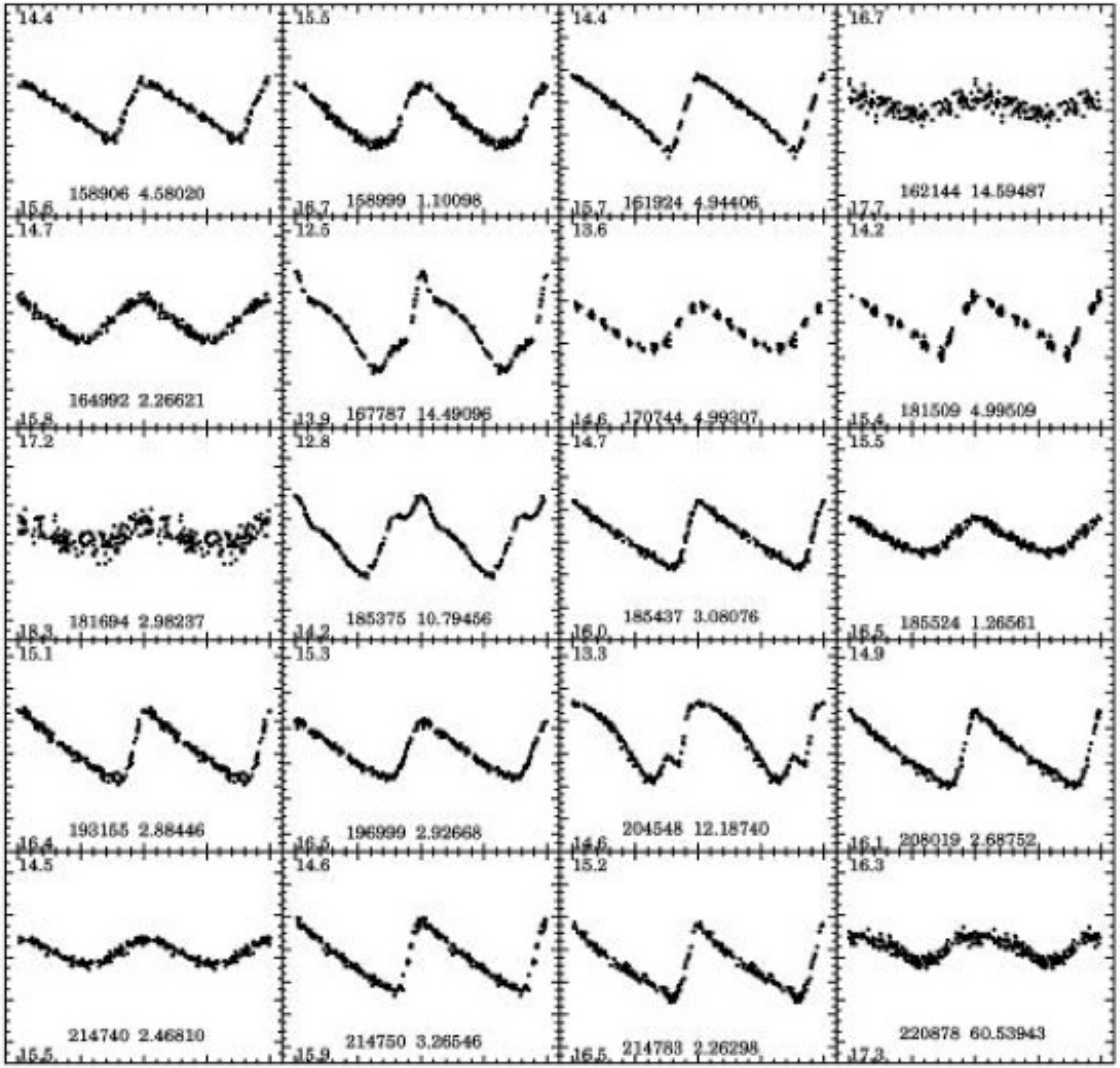
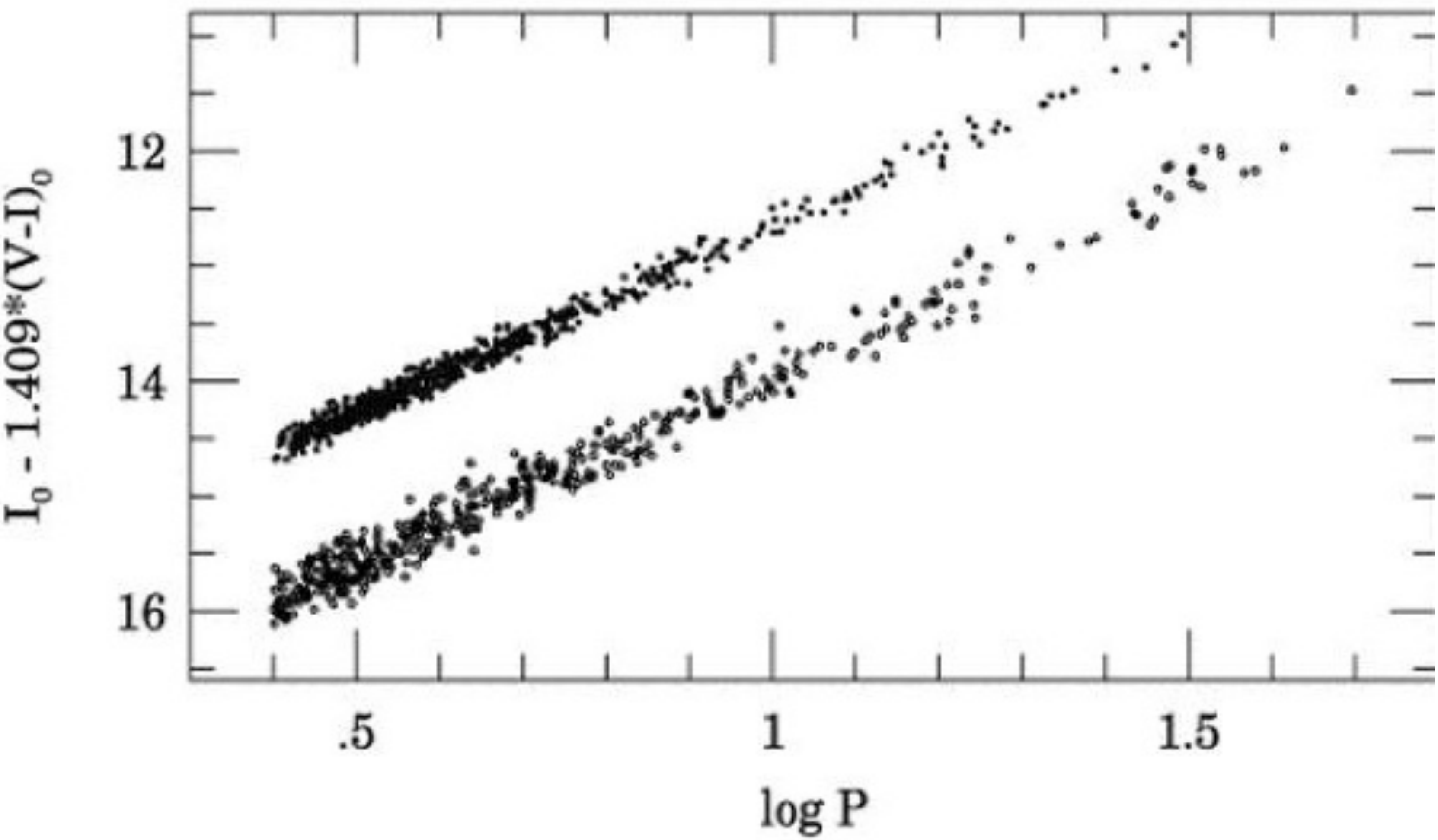


Figure 5: Internal angular velocity at three solar latitudes[1, 11]. Note: 400 nHz = 1 cycle/(30 d).

Simple Theory of Stellar Pulsation

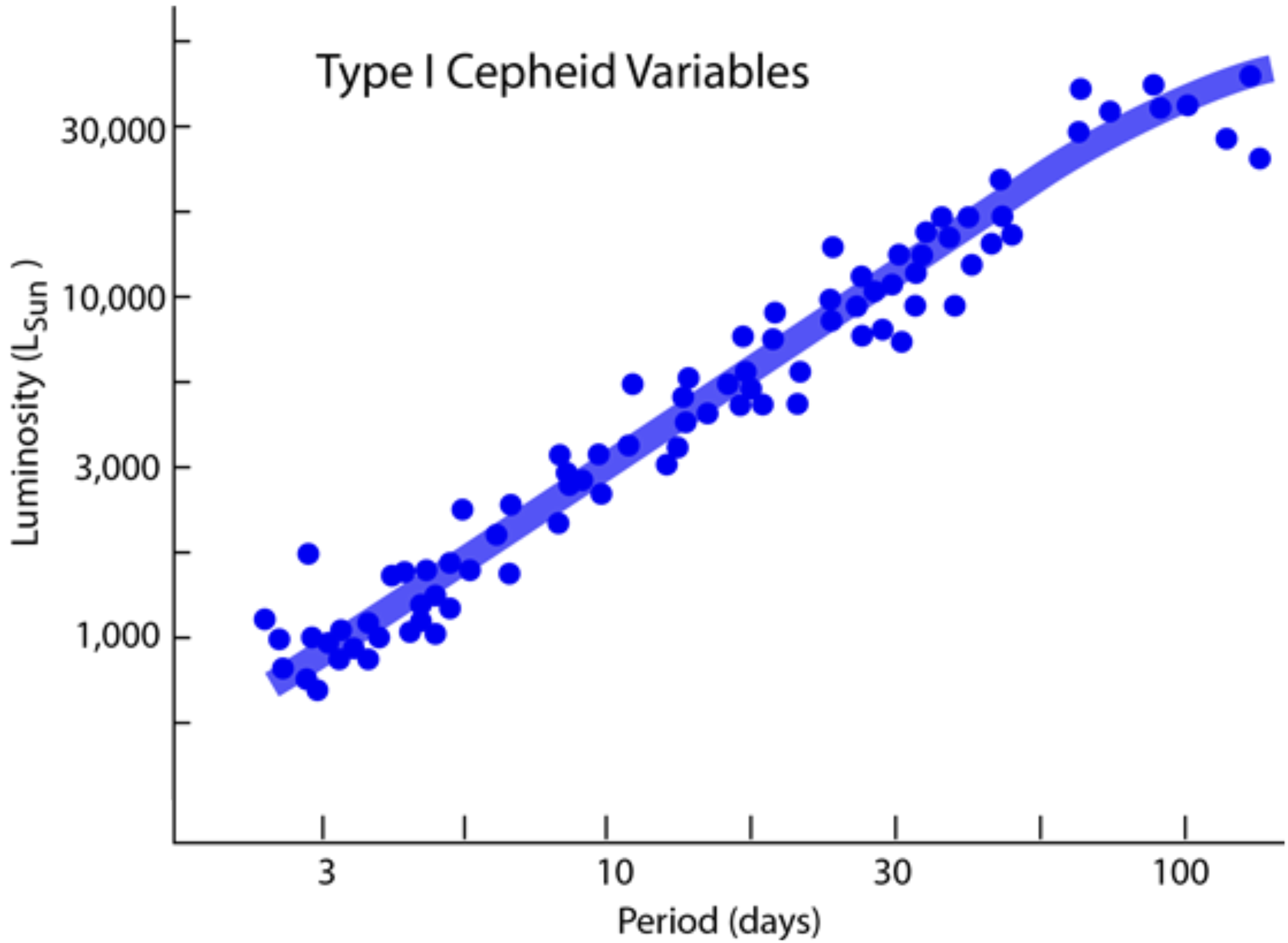


14.4
158906 4.58020
15.6
15.5
158999 1.10098
16.7
14.4
161924 4.94406
15.7
16.7
162144 14.59487
17.7
14.7
164992 2.26621
15.8
12.5
167787 14.49096
13.9
13.6
170744 4.99307
14.6
14.2
181509 4.99509
15.4
17.2
181694 2.98237
18.3
12.8
185375 10.79456
14.2
14.7
185437 3.08076
16.0
15.5
185524 1.26561
16.5
15.1
193155 2.88446
16.4
15.3
196999 2.92668
16.5
13.3
204548 12.18740
14.6
14.9
208019 2.68752
16.1
14.5
214740 2.46810
15.5
14.6
214750 3.26546
15.9
15.2
214783 2.26298
16.5
16.3
220878 60.53943
17.3

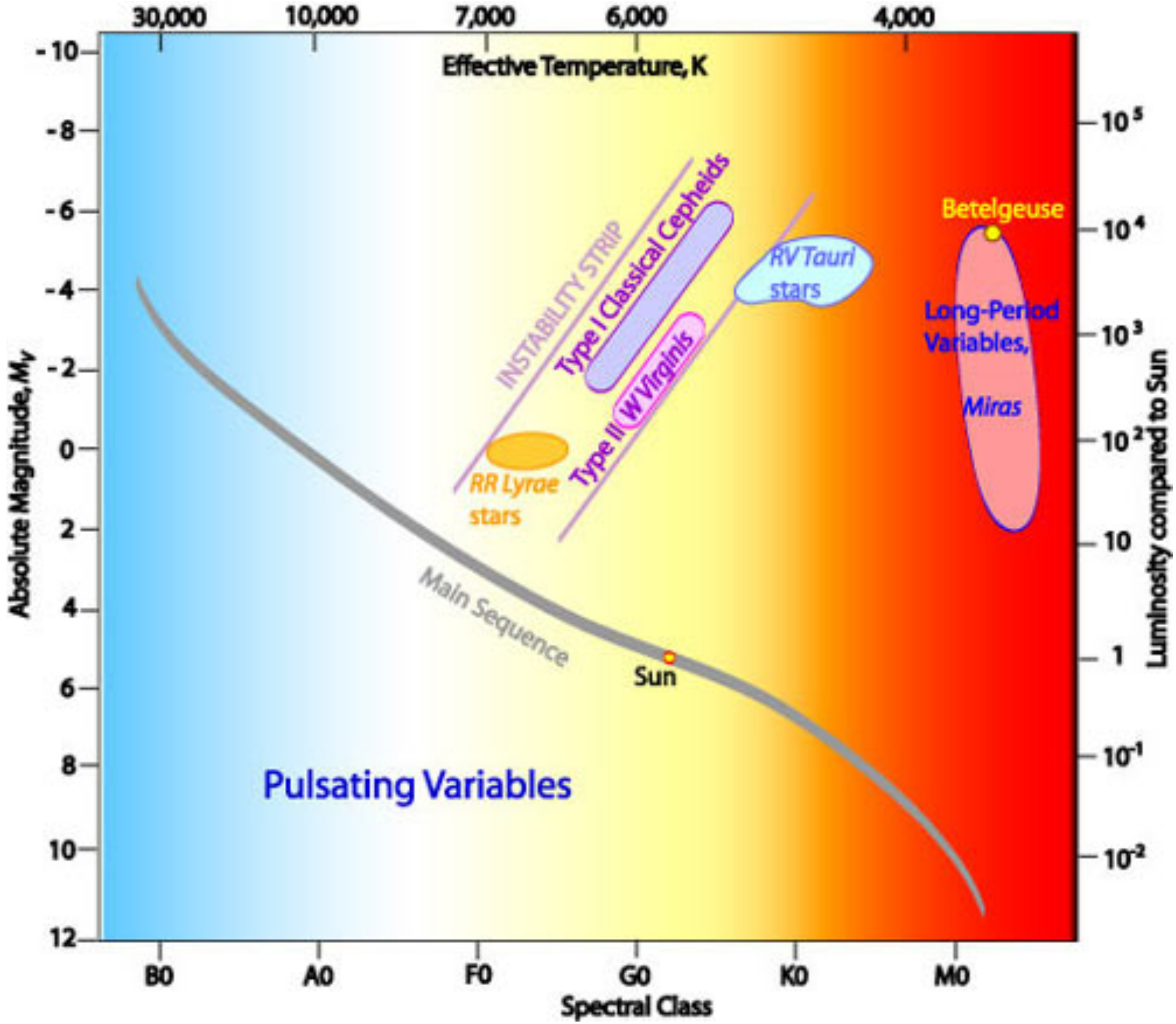


12
14
16
I_0 - 1.409*$(V\text{-}I)_0$
.5
1
1.5
log P

Classical Cepheids



Instability Strip



- Equation of hydrodynamics
- Energy equation: perturbed $\Rightarrow$ $\frac{d(\delta P)}{dt} - \Gamma_1 \frac{P}{\rho} \frac{d\delta\rho}{dt} = (\Gamma_3 - 1)\rho\, \delta\left(\varepsilon_n - \frac{1}{\rho}\operatorname{div}\vec{F}\right)$

$\downarrow \quad \overset{c_s^2}{\sim} \quad \downarrow$

$\left[\frac{1}{\rho}\operatorname{div}\vec{F} = \frac{\partial L}{\partial M}\right]$

(= 0 if adiabatic)

* Assume adiabatic to get frequencies + eigenmodes, then explore sources + sinks of heat.

$\left[\varepsilon_\rho = \frac{d \ln \varepsilon_n}{d \ln \rho}\Big|_T\right]$

$\left[\varepsilon_T = \nu = \frac{d \ln \varepsilon_n}{d \ln T}\Big|_\rho\right]$

$(\varepsilon_n \propto \rho T^\nu)$

$$\frac{\delta\varepsilon_n}{\varepsilon_n} = \frac{\delta\rho}{\rho} + \nu \frac{\delta T}{T}$$

$$= \left[1 + \nu(\Gamma_3 - 1)\right]\frac{\delta\rho}{\rho} \quad ; \quad \Gamma_3 - 1 = \frac{\partial \ln T}{\partial \ln \rho}\Big|_S$$

$$= \left(\frac{1}{(\Gamma_3 - 1)} + \nu\right)\frac{\delta T}{T}$$

$$\frac{dE}{dt} = \int_0^M \delta T \frac{d\,\delta S}{dt}\, dM \quad ; \quad \underline{\underline{\text{Work}}} = \oint^{T\,(\text{period})} \frac{dE}{dt}\, dt = \text{“}\oint P dV\text{”}$$

<u>Nuclear</u> $\Rightarrow \int_0^M \frac{\delta T}{T}\, \delta\varepsilon_n\, dM = \int_0^M dM\, \varepsilon_n \left(\overset{\nu}{\varepsilon_T} + \frac{\overset{\text{“}1\text{”}}{\varepsilon_\rho}}{\Gamma_3 - 1}\right)\left(\frac{\delta T}{T}\right)^2$

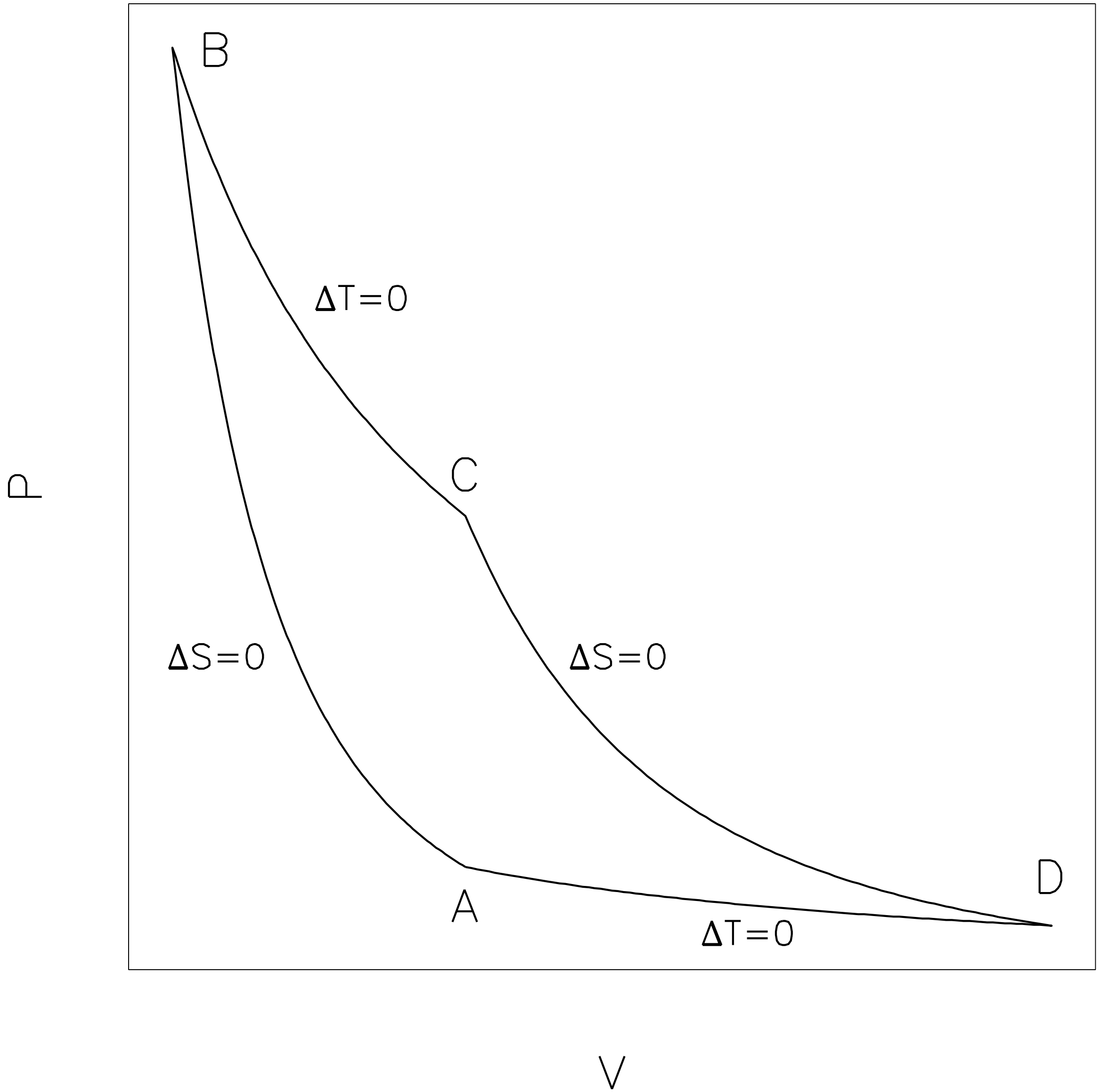
<u>Flux</u> $\Rightarrow -\int_0^M \frac{\delta T}{T}\, \delta\left[\frac{1}{\rho}\operatorname{div}\vec{F}\right] dM = \int_0^M dM \frac{dL}{dM}\left(K_T + \frac{K_\rho}{\Gamma_3 - 1}\right)\left(\frac{\delta T}{T}\right)^2$

Luminosity

$$K_T = \frac{\partial \ln K}{\partial \ln T}\Big|_\rho$$

$$K_\rho = \frac{\partial \ln K}{\partial \ln \rho}\Big|_T$$

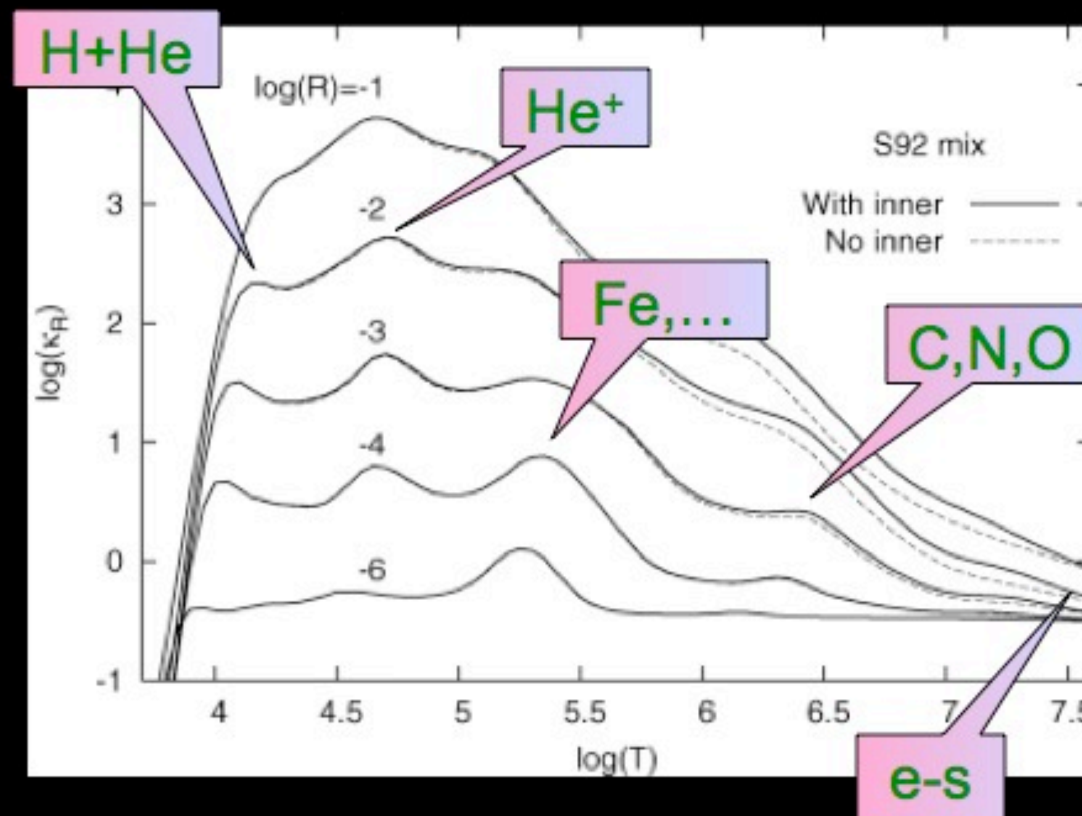
Simple Theory of Stellar Pulsation



6.6 what do real opacities look like?

In regions where specific atoms are ionized, opacity - due to bound-free transitions - has a local maximum.

$$\kappa = \kappa(\rho, T, X_i)$$



If $\boxed{\frac{d}{dr}\left(\kappa_T + \frac{\kappa_\rho}{\Gamma_3 - 1}\right) > 0}$ ⇒ driving region
⇒ pulsation

"Kappa Mechanism"
(valve mechanism)

$\boxed{\left(\varepsilon_T + \frac{\varepsilon_\rho}{\Gamma_3 - 1}\right) > 0}$
Epsilon Mechanism

- Heat engine

- Ionization zones of H + He (He^+, He) — 1–1.5×10^4 K; He^+ : 4×10^4 K

in outer layers of stars

e.g. ⇒ <u>Classical Cepheids</u> (He^+ ionization zone)
<u>Instability strip</u> on HR diagram

<u>β-Cephei</u> (B stars on MS ; 7–20 $M_\odot$)
radial + non-radial modes

opacity peak near <u>2×10^5 K</u> of <u>Fe</u> ionization zone

(γ Doradus ; δ Scuti) [He opacity bump] <u>RR Lyrae</u> (Horizontal branch stars ; A–F) (giants)

$\left[P \sim 0.057 \left(\frac{L}{L_\odot}\right)^{0.6} \text{days}\right]$

Instability strip $5500\,\text{K} < T_{eff} < 7500\,\text{K}$

convection reaches the region

ion. zones too close to surface

— — — — — —

<u>RR Lyraes</u> : $T_{eff} \sim 6 \pm 1 \times 10^3$ K

(T_{eff} = const.) $L \sim R^2$; $P \sim \left(\frac{M}{R^3}\right)^{-\frac{1}{2}}$

$$\Rightarrow \underline{P \sim L^{3/4} M^{-\frac{1}{2}}}$$

(actually $\propto L^{0.9}$ for Cepheids)

Cooling of White Dwarfs

White Dwarf Cooling: Simple Mestel Model

Atmos.: radiative ⇒ match to degenerate core ⇒ assume core is isothermal ⇒ $C_V \doteq$ const.

$$L = -MC_V \frac{dT_c}{dt} \quad ; \quad \frac{dT}{dr} = -\frac{3\kappa\rho/c \, L}{4\pi r^2 (4aT^3)} \quad ; \quad \frac{dP}{dr} = -\frac{GM\rho}{r^2}$$

P in atmosphere → Ideal gas $= \frac{\rho R_G T}{\mu}$ $\quad R_G =$ "gas constant"

$$\frac{\frac{dT}{dr}}{\frac{dP}{dr}} = \frac{dT}{dP} = \left[\frac{3\kappa_0 \mu L}{4\pi(4ac)R_G}\right]\frac{P}{T^{15/2}} \quad ; \quad \underline{\kappa = \kappa_0 \rho / T^{7/2}}$$

Non-deg. - deg. boundary: $K_1(\rho Y_e)^{5/3} \sim \rho Y_e R_G T$

$$\Rightarrow P_e \sim 2T^{5/2}$$

$$P = P_e + P_i \sim \frac{\mu_e}{\mu} P_e \sim 2\frac{\mu_e}{\mu} T^{5/2}$$

$$\Rightarrow L \sim 4\times10^7 \, T_c^{7/2} \frac{M}{M_\odot}$$

$$\frac{dT_c}{dt} = -1.6\times10^{-34} T_c^{7/2} \Rightarrow T_c \sim 2.3\times10^{10}\left(\frac{t}{1\,yr}\right)^{-2/5}$$

$$\Rightarrow \boxed{L \sim 1.86\times10^{10}\left(\frac{M}{M_\odot}\right)\left(\frac{t}{1\,yr}\right)^{-7/5}}$$

$$L = 4\pi R^2 \sigma T_e^4 \Rightarrow T_e \propto t^{-7/20} \quad [R \sim \text{const.}]$$

Extensions: Convective atmosphere

Debye cooling theory : $T \lesssim T_{Debye}$ $C_V \sim T_c^3$

Crystalization : <u>Latent heat</u>

Phase separation : <u>gravitational settling</u> (energy source)

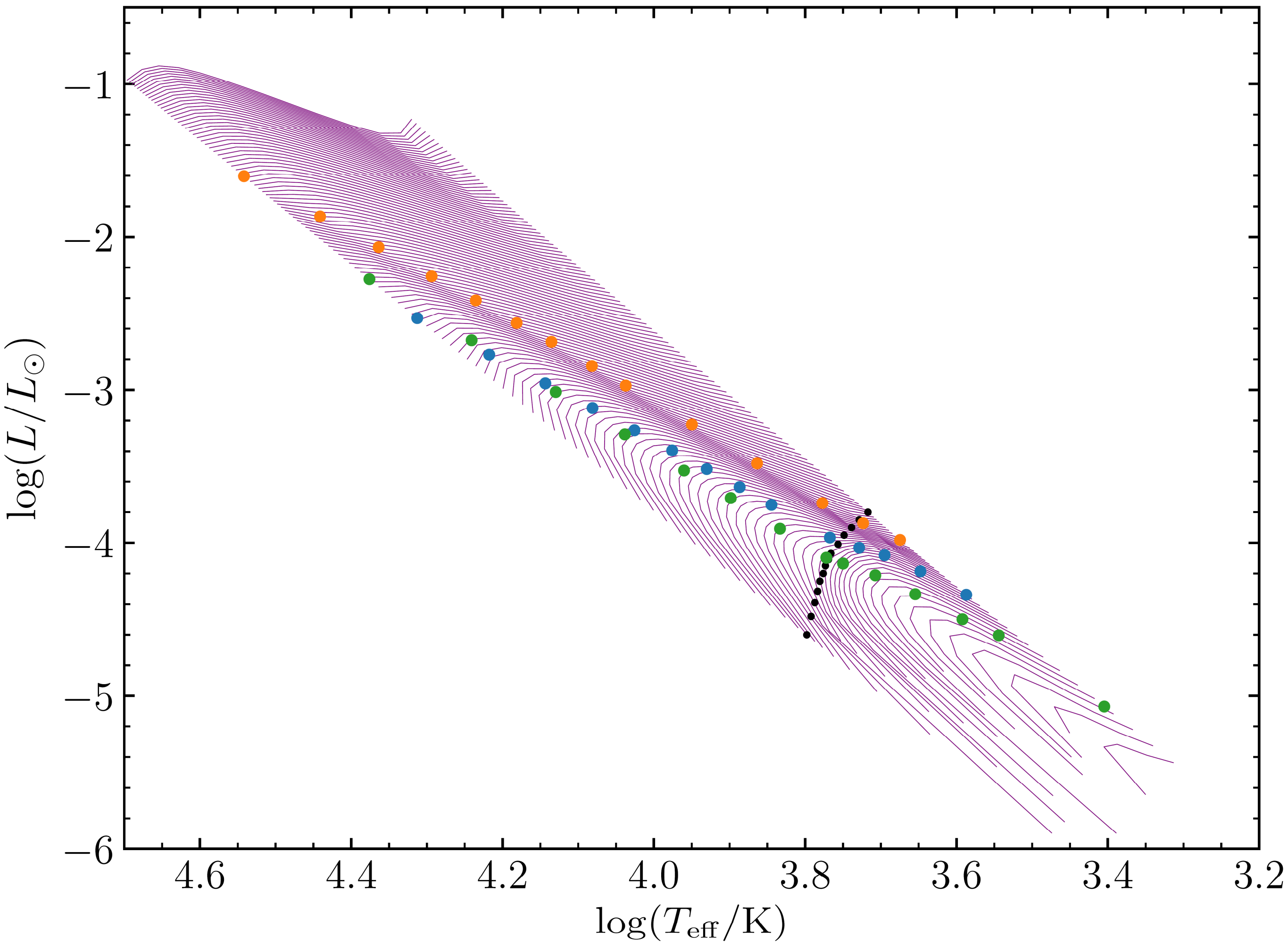
$^{12}C/^{16}O \rightarrow {}^{12}C + {}^{16}O$

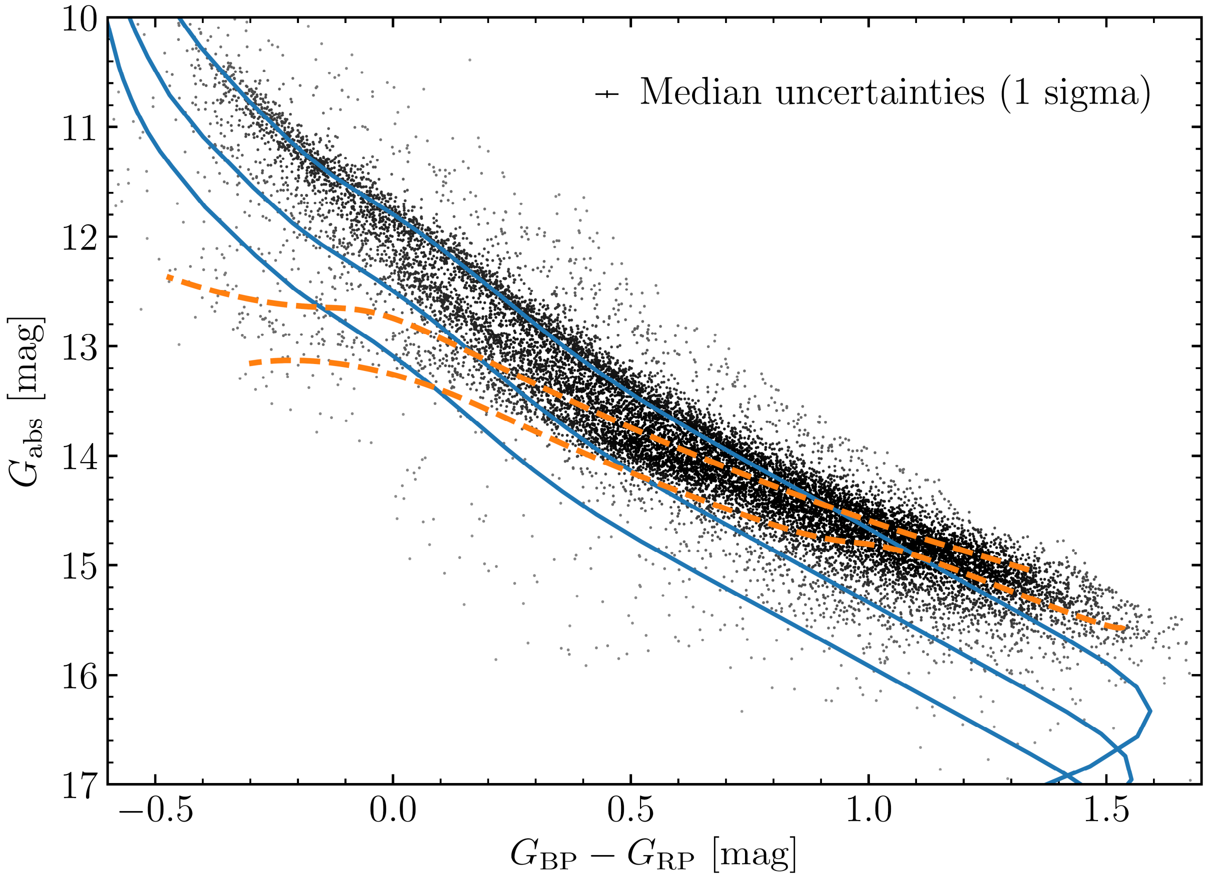
^{20}Ne ?

Note: <u>gravitational settling + thermal evolution</u>:

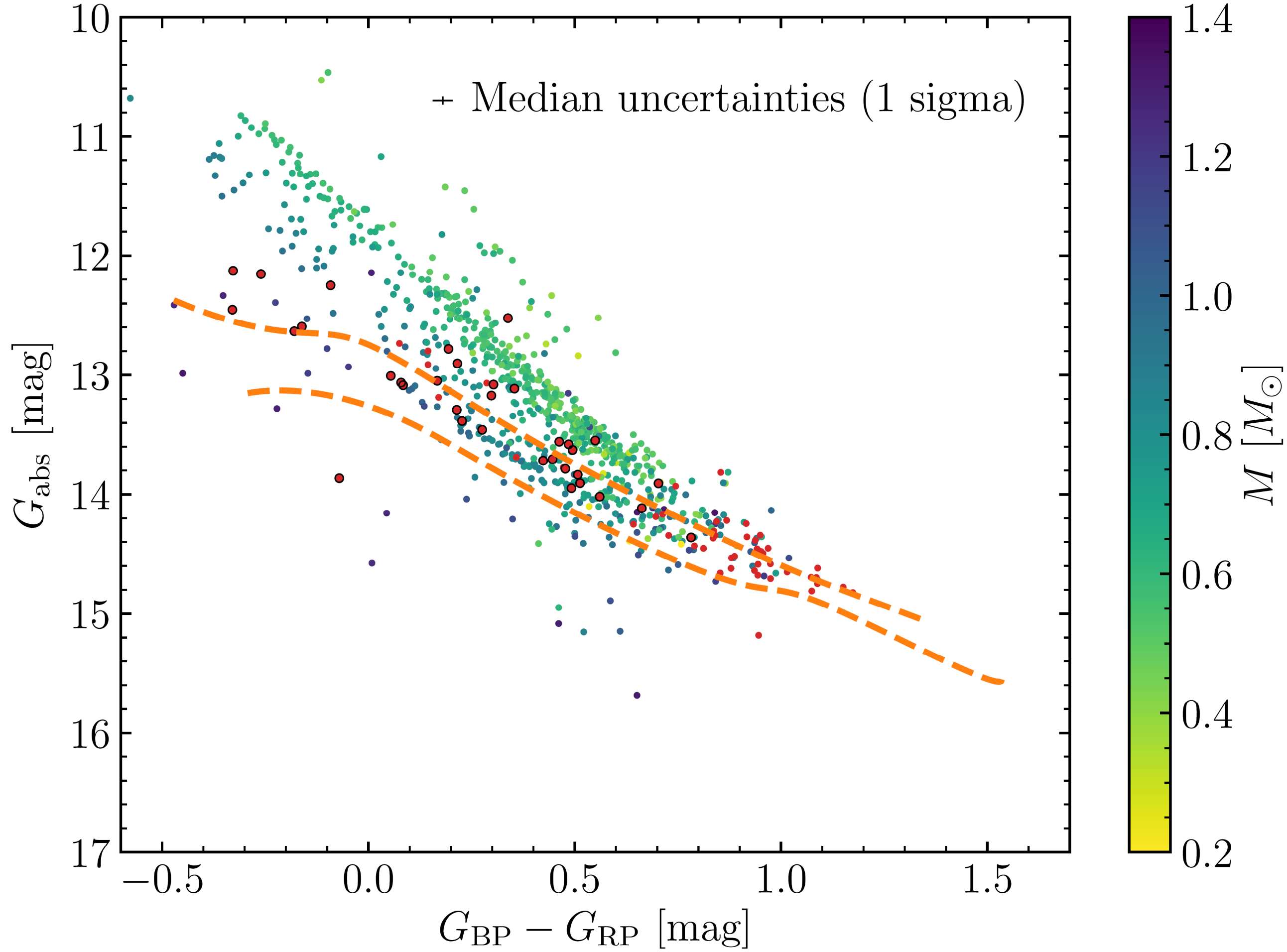
* White Dwarfs
* Jupiter/Saturn H_2/He immiscibility
  ^{20}Ne (!?)
* Earth core formation : Fe + siderophile

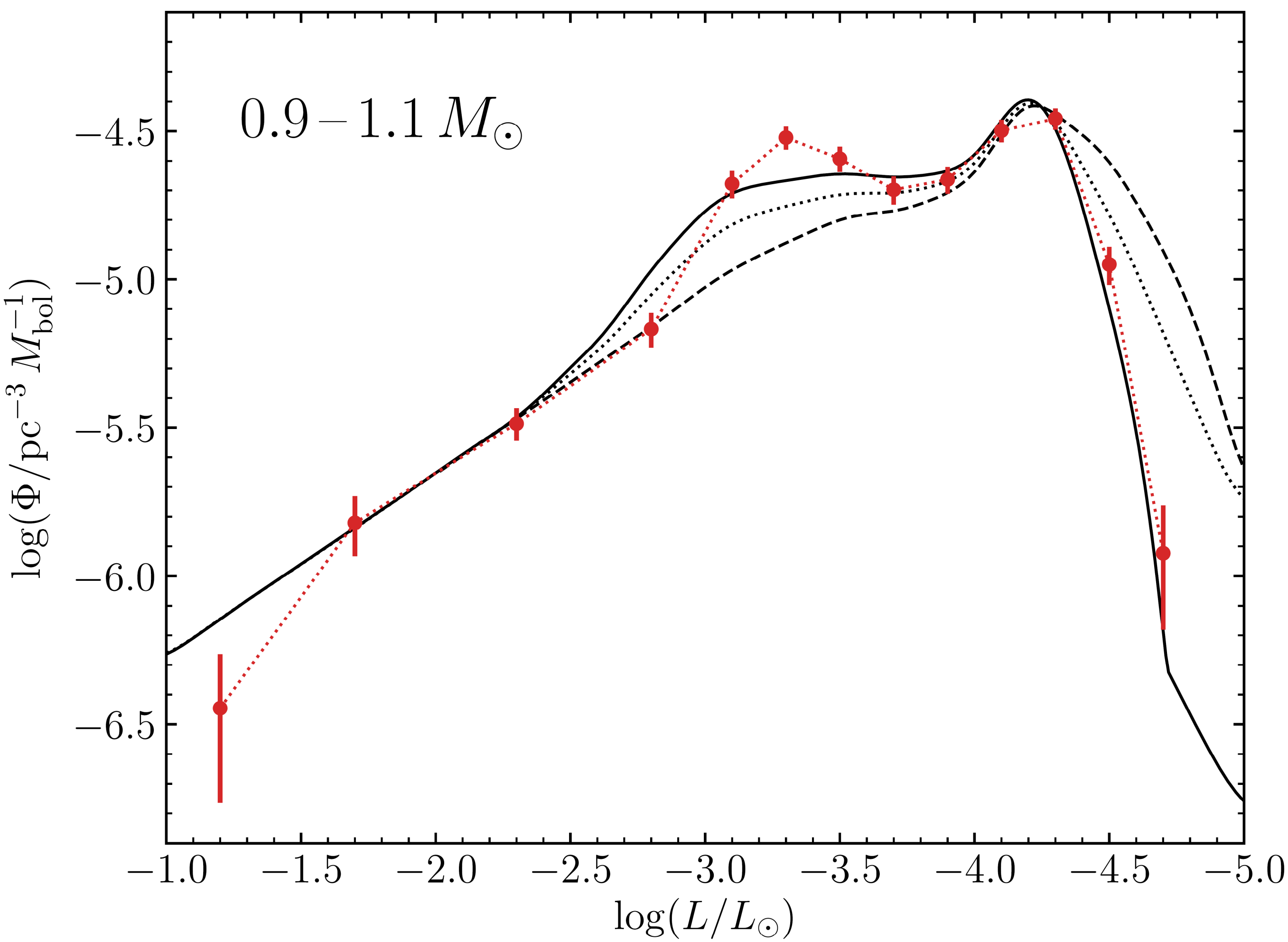
Tremblay et al. 2019 (Nature)





Median uncertainties (1 sigma)
$G_{\rm abs}$ [mag]
$G_{\rm BP} - G_{\rm RP}$ [mag]
10
11
12
13
14
15
16
17
−0.5
0.0
0.5
1.0
1.5


Median uncertainties (1 sigma)
$G_{\rm abs}$ [mag]
$G_{\rm BP} - G_{\rm RP}$ [mag]
M [$M_\odot$]
10
11
12
13
14
15
16
17
−0.5
0.0
0.5
1.0
1.5
1.4
1.2
1.0
0.8
0.6
0.4
0.2



$0.9 - 1.1\,M_\odot$
$\log(\Phi/\mathrm{pc}^{-3}\,M_{\mathrm{bol}}^{-1})$
$\log(L/L_\odot)$
−4.5
−5.0
−5.5
−6.0
−6.5
−1.0
−1.5
−2.0
−2.5
−3.0
−3.5
−4.0
−4.5
−5.0